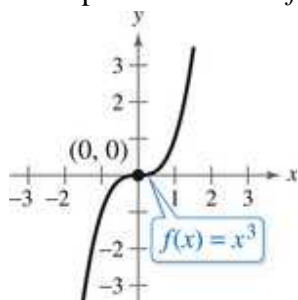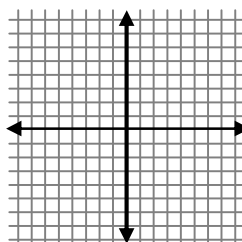**Polynomial Functions of Higher Degree**

Warm-up

Consider the parent function $f(x) = x^3$ graphed below. Answer the following questions.
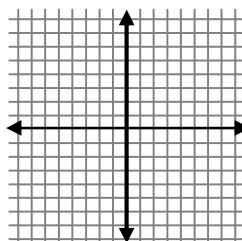

a) What is the domain?

b) What is the range?

c) What is the x-intercept?

d) On what open interval is it increasing?

e) On what open interval is it decreasing?

f) Is it an even function, odd function or neither?

g) It is symmetric to the _____.

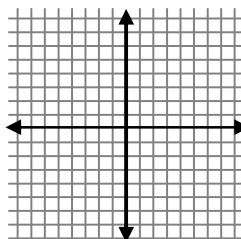**Example 1**: Transformations of the parent function $f(x) = x^3$

    a) Sketch the graph of $g(x) = -x^3$

    b) Sketch the graph of $h(x) = x^3 + 1$

    c) Sketch the graph of $k(x) = (x-1)^3$

Polynomial Functions of Higher Degree
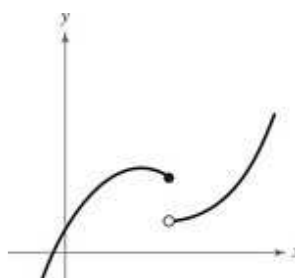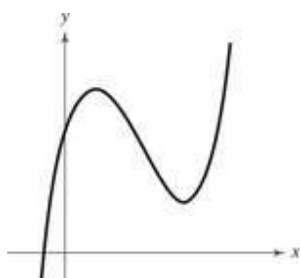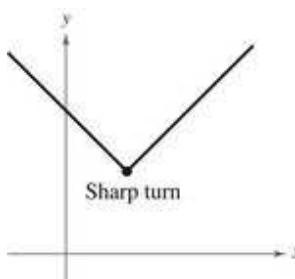
Definitions

Polynomials _____
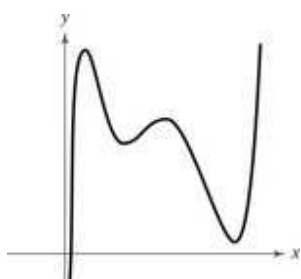
Coefficient _____

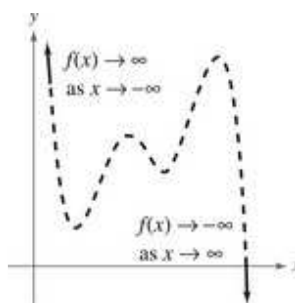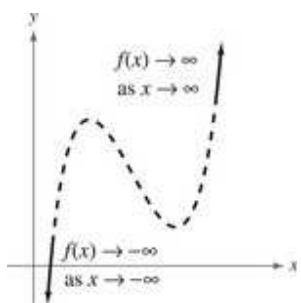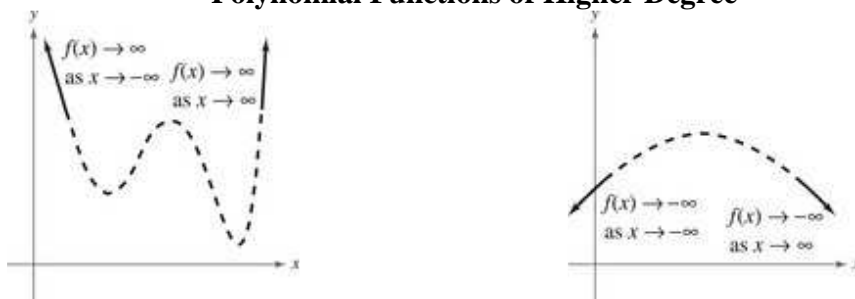Highest Degree _____

Leading Coefficient _____

Zeros _____

Continuous _____

Smooth, Rounded Turns _____

The Leading Coefficient Test for End Behavior

Polynomial Functions of Higher Degree

$f(x) \to \infty$
as $x \to -\infty$ $f(x) \to \infty$
as $x \to \infty$

$f(x) \to -\infty$
as $x \to -\infty$ $f(x) \to -\infty$
as $x \to \infty$

Example 2

a) Use the Leading Coefficient Test to describe the end behavior of the graph of $f(x) = -x^3 + 4x$

b) What is the end behavior of the graph of $f(x) = 2x^4 - 3x + 1$?

Practice Problem 1

a) What is the end behavior of the graph of $g(x) = 5 - 7x - 3x^2$?

b) What is the end behavior of the graph of $h(x) = x^5 - x$?

Zeros of a Polynomial Function

Example 3

Find all real zeros of $f(x) = x^3 - x^2 - 2x$

Practice Problem 2

Find all real zeros of $f(x) = -2x^4 + 2x^3$

Example 4

a) Find a polynomial with zeros -2, 3, and 3.

Polynomial Functions of Higher Degree

b) Find a polynomial with zeros -5, -5, -5, and $\dfrac{1}{2}$

Repeated Zeros

In Example 4 above, there are repeated zeros. This is called _____.

In part a, 3 is a zero of _____.

In part b, -5 is a zero of _____.

Graphically, an odd multiplicity means that the graph _____.

An even multiplicity means that the graph _____.

Practice Problem 3

Find a polynomial that has the zeros 0, -2, -3, -3

Steps for Sketching the Graph of a Polynomial Function
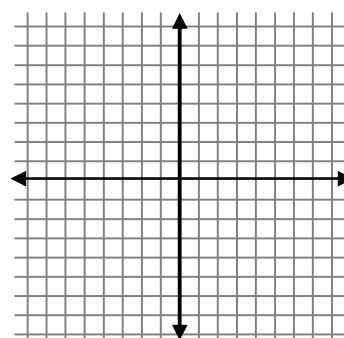
1. _____

2. _____

3. _____

Example 5   Sketch the graph of: $f(x) = 3x^4 - 3x^3$

Step 1: End Behavior

Step 2: Zeros

Step 3: Plot Points

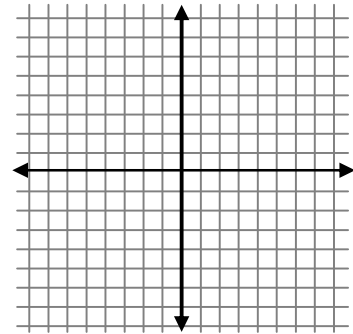| x	-1	0.5	1	1.5
$f(x)$				

Polynomial Functions of Higher Degree

Practice Problem 4: Sketch the graph of $f(x) = -2x^3 + 6x^2 - \dfrac{9}{2}x$

Step 1: End Behavior

Step 2: Zeros

Step 3: Plot Points

x	-0.5	0.5	1	2
$f(x)$				