Precalculus – Chapter 8 Test Review

1. Write the first 5 terms of the sequence whose *n*th term is $a_n = \dfrac{n!}{(n+3)!}$.

2. Simplify: $\dfrac{(n)!}{(n-2)!}$

3. Find a formula for the *n*th term of the sequence: $\frac{2}{1}, \frac{4}{1}, \frac{6}{2}, \frac{8}{6}, \frac{10}{24}, \cdots$

4. Use sigma notation to write the sum: $\frac{2}{3}+\frac{4}{4}+\frac{6}{5}+\frac{8}{6}+\cdots\frac{14}{9}$

5. Which of the following is an arithmetic sequence?

   (a) 2, 4, 8, 16, 32,… (b) –2, 4, –8, 16, 32,…

   (c) 3, 6, 9, 12, 15,… (d) all of these

6. Find the first 4 terms of the arithmetic sequence with $a_1 = 7$ and $d = 2$.

7. Find a_n for the arithmetic sequence with $a_1 = 12, \quad d = \frac{1}{3}$, and $n = 52$.

8. Find the sum of the first 50 positive integers that are multiples of 3.

9. Find the sum of the first 30 terms of the sequence: $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, 5\sqrt{2}, \ldots$

10. Find the sum of the first *n* terms of the arithmetic sequence:–7, 1, 9, 17, …

11. Find the sum of the first 14 terms of the arithmetic sequence whose *n*th term is $a_n = 2n+3$.

12. Find a formula for a_n for the arithmetic sequence with $a_3 = 8$ and $d = -3$.

13. Find the sum: $\sum_{n=1}^{200}(2n+3)$

14. Find the sum: $\sum_{i=1}^{9}3(i-2)$

15. Determine the seating capacity of an auditorium with 30 rows of seats if there are 25 seats in the first row, 28 seats in the second row, 31 seats in the third row, and so on.

16. Which of the following is a geometric sequence?

(a) –1, –3, –5, –7, –9, … (b) 2, 3, 5, 7, 11, …

(c) 1, 2, 4, 7, 11, 16, … (d) –2, 4, –8, 16, –32, …

17. Determine the common ratio for the geometric sequence: $4, -2, 1, -\frac{1}{2}, \frac{1}{4}, \cdots$

18. Write the first five terms of the geometric sequence with $a_1 = 5$ and $r = \frac{1}{2}$.

19. Find the 10^{th} term of the geometric sequence with $a_1 = 3$ and $r = 2.2$

20. Find the 28^{th} term of the geometric sequence: 2, 2.4, 2.88, 3.456, 4.1472, …

21. Find the sum and round your answer to four decimal places: $\sum_{n=0}^{10} 2\left(\frac{1}{3}\right)^n$

22. Find the sum and round your answer to two decimal places: $\sum_{n=1}^{13} 3\left(\frac{3}{2}\right)^n$

23. Find the sum of the first six terms of the geometric sequence with $a_1 = 3$ and $a_2 = \frac{3}{2}$.

24. Evaluate: $\sum_{n=0}^{\infty} 4\left(\frac{1}{4}\right)^n = 4 + 1 + \frac{1}{4} + \frac{1}{16} + \cdots$

25. Evaluate: $\sum_{n=0}^{\infty} 5\left(-\frac{1}{3}\right)^n$

26. In a geometric sequence, $a_4 = 125$ and $a_{10} = \frac{125}{64}$. Find a_{14}.

PreCalculus – Chapter 9 Test Review – Solutions: Work shown, answers highlighted.

1. Write the first 5 terms of the sequence whose *n*th term is $a_n = \dfrac{n!}{(n+3)!}$.

$\dfrac{1!}{4!}, \dfrac{2!}{5!}, \dfrac{3!}{6!}, \dfrac{4!}{7!}, \dfrac{5!}{8!} = \dfrac{1}{4\cdot3\cdot2}, \dfrac{1}{5\cdot4\cdot3}, \dfrac{1}{6\cdot5\cdot4}\cdots =$ $\dfrac{1}{24}, \dfrac{1}{60}, \dfrac{1}{120}, \dfrac{1}{210}, \dfrac{1}{336}$

2. Simplify: $\dfrac{(n)!}{(n-2)!}$ $\dfrac{n\cdot(n-1)\cdot\cancel{(n-2)\cdots}}{\cancel{(n-2)\cdots}}$ = n(n–1) = $n^2 - n$

3. Find a formula for the *n*th term of the sequence: $\dfrac{2}{1}, \dfrac{4}{1}, \dfrac{6}{2}, \dfrac{8}{6}, \dfrac{10}{24}, \cdots$ $\dfrac{2n}{(n-1)!}$

4. Use sigma notation to write the sum: $\dfrac{2}{3}+\dfrac{4}{4}+\dfrac{6}{5}+\dfrac{8}{6}+\cdots\dfrac{14}{9}$ $\displaystyle\sum_{n=1}^{7}\frac{2n}{n+2}$

5. Which of the following is an arithmetic sequence?

(a) 2, 4, 8, 16, 32,… (b) –2, 4, –8, 16, 32,…

(c) 3, 6, 9, 12, 15,… (d) all of these

6. Find the first 4 terms of the arithmetic sequence with $a_1 = 7$ and $d = 2$.

7, 9, 11, 13

7. Find a_n for the arithmetic sequence with $a_1 = 12$, $d = \frac{1}{3}$, and $n = 52$.

$a_1 = 12$ 12 – 1/3 = 11 2/3 $a_n = \frac{1}{3}n + 11\frac{2}{3}$ $\frac{1}{3}(52) + 11\frac{2}{3} =$ 29

8. Find the sum of the first 50 positive integers that are multiples of 3.

3+6+9+… $\frac{n}{2}(a_1 + a_n)$ $a_1 = 3$, $a_{50} = 150$ 25(3 + 150) = 3825

9. Find the sum of the first 30 terms of the sequence: $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, 5\sqrt{2},\ldots$

$a_n = n\sqrt{2}$ $15(\sqrt{2} + 30\sqrt{2}) = 15(31\sqrt{2}) =$ $465\sqrt{2}$

10. Find the sum of the first *n* terms of the arithmetic sequence:–7, 1, 9, 17, …

$a_1 = 7$ $a_n = 8n - 15$ $S = \frac{n}{2}(-7 + 8n - 15) = \frac{n}{2}(8n - 22) =$ $4n^2 - 11n$

11. Find the sum of the first 14 terms of the arithmetic sequence whose *n*th term is $a_n = 2n + 3$.

7(5 + 31) = 252

12. Find a formula for a_n for the arithmetic sequence with $a_3 = 8$ and $d = -3$.

$a_1 = 14$ $a_n = -3n + 17$

13. Find the sum: $\displaystyle\sum_{n=1}^{200}(2n+3)$ 100(5 + 403) = 40,800

14. Find the sum: $\displaystyle\sum_{i=1}^{9}3(i-2)$ $\frac{9}{2}(-3 + 21) =$ 81

15. Determine the seating capacity of an auditorium with 30 rows of seats if there are 25 seats in the first row, 28 seats in the second row, 31 seats in the third row, and so on.

$3n + 22$ $\quad 15(25 + 112)$ $\quad =$ $\quad 2055$

16. Which of the following is a geometric sequence?

(a) –1, –3, –5, –7, –9, … (b) 2, 3, 5, 7, 11, …

(c) 1, 2, 4, 7, 11, 16, … (d) –2, 4, –8, 16, –32, …

17. Determine the common ratio for the geometric sequence: $4, -2, 1, -\frac{1}{2}, \frac{1}{4}, \cdots$ – ½

18. Write the first five terms of the geometric sequence with $a_1 = 5$ and $r = \frac{1}{2}$.

$5 \quad \frac{5}{2} \quad \frac{5}{4} \quad \frac{5}{8} \quad \frac{5}{16}$

19. Find the 10th term of the geometric sequence with $a_1 = 3$ and $r = 2.2$

$3(2.2)^{10-1} =$ 3621.807653

20. Find the 28th term of the geometric sequence: 2, 2.4, 2.88, 3.456, 4.1472, …

r = 1.2 $\quad 2(1.2)^{27} \quad =$ 274.7411039

21. Find the sum and round your answer to four decimal places: $\sum_{n=0}^{10} 2\left(\frac{1}{3}\right)^n$

$2\frac{1-\frac{1}{3}^{11}}{1-\frac{1}{3}} = 2\frac{177146}{177147}\cdot\frac{3}{2} = \frac{531438}{177147} = 2.999982 \approx 3.0000$

22. Find the sum and round your answer to two decimal places: $\sum_{n=1}^{13} 3\left(\frac{3}{2}\right)^n$

$a_1 = 3\cdot\frac{3}{2} = \frac{9}{2}$ $\quad \frac{9}{2}\frac{1-\frac{3}{2}^{13}}{1-\frac{3}{2}} = 1742.575562 \approx 1742.58$

23. Find the sum of the first six terms of the geometric sequence with $a_1 = 3$ and $a_2 = \frac{3}{2}$.

$3\left(\frac{1-\frac{1}{64}}{1-\frac{1}{2}}\right) = 3\left(\frac{\frac{63}{64}}{\frac{1}{2}}\right) = 3\cdot\frac{63}{64}\cdot 2 = \frac{189}{32}$

24. Evaluate: $\sum_{n=0}^{\infty} 4\left(\frac{1}{4}\right)^n = 4 + 1 + \frac{1}{4} + \frac{1}{16} + \cdots$ $\quad \frac{4}{1-\frac{1}{4}} = \frac{4}{\frac{3}{4}} = \frac{16}{3}$

25. Evaluate: $\sum_{n=0}^{\infty} 5\left(-\frac{1}{3}\right)^n$ $\quad \frac{5}{1-\frac{-1}{3}} = \frac{5}{\frac{4}{3}} =$ $\frac{15}{4}$

26. In a geometric sequence, $a_4 = 125$ and $a_{10} = \frac{125}{64}$. Find a_{14}.

$a_{10} = a_4 \cdot r^6$

$\frac{125}{64} = 125 \cdot r^6$

$r^6 = \frac{1}{64}$

$r = \frac{1}{2}$

$a_{14} = a_{10} \cdot r^4$

$a_{14} = \frac{125}{64} \cdot \frac{1}{2}^4$

$\frac{125}{64} \cdot \frac{1}{16} = \frac{125}{1024}$