10.2 45°-45°-90° Triangles

Goal

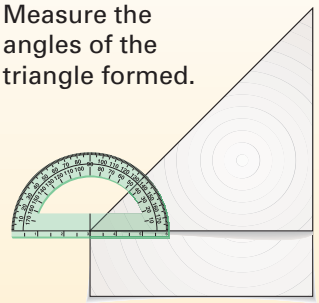
Find the side lengths of 45°-45°-90° triangles.

Key Words

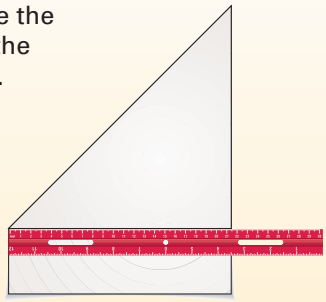
- **45°-45°-90° triangle**
- **isosceles triangle** p. 173
- **leg of a right triangle** p. 192
- **hypotenuse** p. 192

Geo-Activity Exploring an Isosceles Right Triangle

1. Fold a large piece of paper so the top lines up with one side.
2. Measure the angles of the triangle formed.



3. Measure the legs of the triangle.

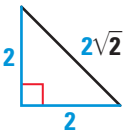


4. Use the Pythagorean Theorem to predict the length of the hypotenuse.
5. Measure the hypotenuse to verify your answer in Step 4.

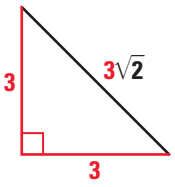
Student Help

LOOK BACK
To review the Pythagorean Theorem, see p. 192.

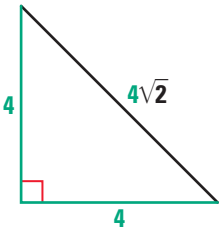
A right triangle with angle measures of 45°, 45°, and 90° is called a **45°-45°-90° triangle**. You can use the Pythagorean Theorem to find the length of the hypotenuse of any 45°-45°-90° triangle.



$$\sqrt{2^2 + 2^2} = \sqrt{4 + 4}$$
$$= \sqrt{4 \cdot 2}$$
$$= \sqrt{4} \cdot \sqrt{2}$$
$$= 2\sqrt{2}$$



$$\sqrt{3^2 + 3^2} = \sqrt{9 + 9}$$
$$= \sqrt{9 \cdot 2}$$
$$= \sqrt{9} \cdot \sqrt{2}$$
$$= 3\sqrt{2}$$



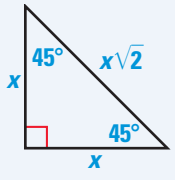
$$\sqrt{4^2 + 4^2} = \sqrt{16 + 16}$$
$$= \sqrt{16 \cdot 2}$$
$$= \sqrt{16} \cdot \sqrt{2}$$
$$= 4\sqrt{2}$$

THEOREM 10.1

45°-45°-90° Triangle Theorem

Words In a 45°-45°-90° triangle, the length of the hypotenuse is the length of a leg times $\sqrt{2}$.

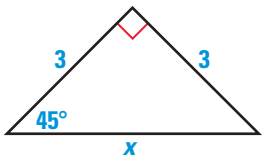
Symbols hypotenuse = leg $\cdot \sqrt{2}$



Student Help
CLASSZONE.COM
MORE EXAMPLES
More examples at classzone.com

EXAMPLE 1 Find Hypotenuse Length

Find the length x of the hypotenuse in the 45°-45°-90° triangle shown at the right.



Solution

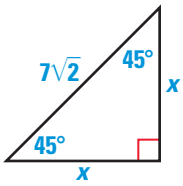
By the 45°-45°-90° Triangle Theorem, the length of the hypotenuse is the length of a leg times $\sqrt{2}$.

$$\text{hypotenuse} = \text{leg} \cdot \sqrt{2} \qquad \text{45°-45°-90° Triangle Theorem}$$
$$= 3 \cdot \sqrt{2} \qquad \text{Substitute.}$$

ANSWER ▶ The length of the hypotenuse is $3\sqrt{2}$.

EXAMPLE 2 Find Leg Length

Find the length x of each leg in the 45°-45°-90° triangle shown at the right.



Student Help
READING TIP
The expression $x\sqrt{2}$ is equivalent to $\sqrt{2}x$.

Solution

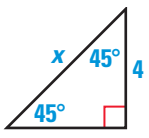
By the 45°-45°-90° Triangle Theorem, the length of the hypotenuse is the length of a leg times $\sqrt{2}$.

$$\text{hypotenuse} = \text{leg} \cdot \sqrt{2} \qquad \text{45°-45°-90° Triangle Theorem}$$
$$7\sqrt{2} = x\sqrt{2} \qquad \text{Substitute.}$$
$$\frac{7\sqrt{2}}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}} \qquad \text{Divide each side by } \sqrt{2}.$$
$$7 = x \qquad \text{Simplify.}$$

ANSWER ▶ The length of each leg is 7.

Checkpoint Find Hypotenuse and Leg Lengths

Find the value of x.

1.


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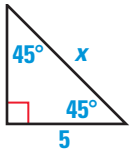
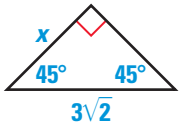
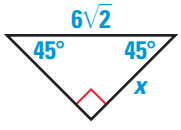


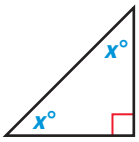
3.


4.


Using Algebra

EXAMPLE 3 Identify 45°-45°-90° Triangles

Determine whether there is enough information to conclude that the triangle is a 45°-45°-90° triangle. Explain your reasoning.



Solution

By the Triangle Sum Theorem, $x° + x° + 90° = 180°$.

So, $2x° = 90°$, and $x = 45$.

ANSWER Since the measure of each acute angle is 45°, the triangle is a 45°-45°-90° triangle.

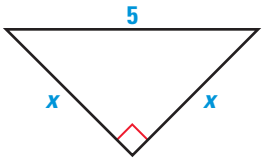
Example 3 shows that whenever a right triangle has congruent acute angles, it is a 45°-45°-90° triangle.

Student Help

LOOK BACK
To review the Base Angles Theorem, see p. 185.

EXAMPLE 4 Find Leg Length

Show that the triangle is a 45°-45°-90° triangle. Then find the value of x.



Solution

The triangle is an isosceles right triangle. By the Base Angles Theorem, its acute angles are congruent. From the result of Example 3, this triangle must be a 45°-45°-90° triangle.

You can use the 45°-45°-90° Triangle Theorem to find the value of x.

$$\text{hypotenuse} = \text{leg} \cdot \sqrt{2} \qquad \text{45°-45°-90° Triangle Theorem}$$

$$5 = x\sqrt{2} \qquad \text{Substitute.}$$

$$\frac{5}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}} \qquad \text{Divide each side by } \sqrt{2}.$$

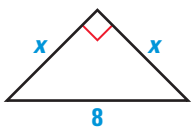
$$\frac{5}{\sqrt{2}} = x \qquad \text{Simplify.}$$

$$3.5 \approx x \qquad \text{Use a calculator to approximate.}$$

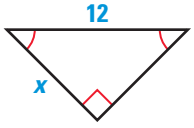
Checkpoint ***Find Leg Lengths***

Show that the triangle is a 45°-45°-90° triangle. Then find the value of x. Round your answer to the nearest tenth.

5.



6.



10.2 Exercises

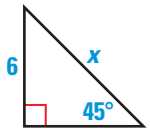
Guided Practice

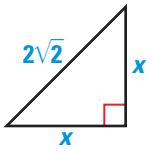
Vocabulary Check

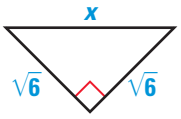
1. How many congruent sides does an *isosceles right triangle* have?
2. How many congruent angles does an isosceles right triangle have? What are the measures of the three angles?

Skill Check

Find the value of x in the 45°-45°-90° triangle. Write your answer in radical form.

3. 

4. 

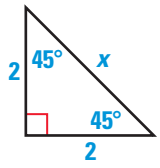
5. 

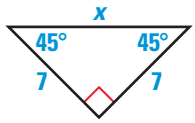
Practice and Applications

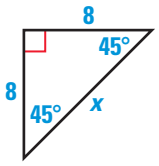
Extra Practice

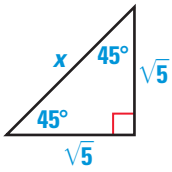
See p. 693.

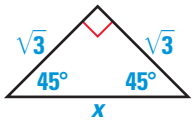
Finding Hypotenuse Lengths **Find the length of the hypotenuse in the 45°-45°-90° triangle. Write your answer in radical form.**

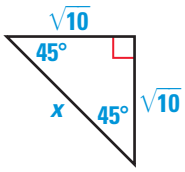
6. 

7. 

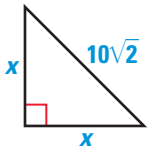
8. 

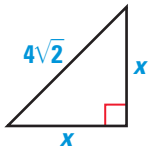
9. 

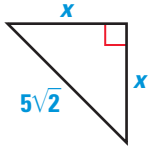
10. 

11. 

Finding Leg Lengths **Find the length of a leg in the 45°-45°-90° triangle.**

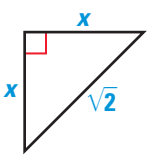
12. 

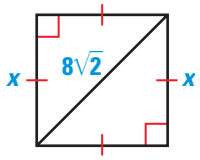
13. 

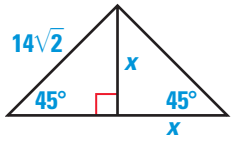
14. 

Homework Help

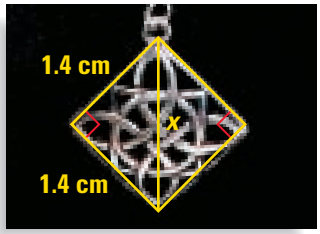
Example 1: Exs. 6–11, 18
Example 2: Exs. 12–17
Example 3: Exs. 19–27
Example 4: Exs. 22–27

15. 

16. 

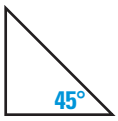
17. 

18. Jewelry Use a calculator to find the length x of the earring shown at the right. Round your answer to the nearest tenth.

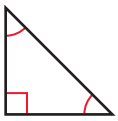


You be the Judge **Determine whether there is enough information to conclude that the triangle is a 45°-45°-90° triangle. Explain your reasoning.**

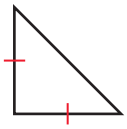
19.



20.

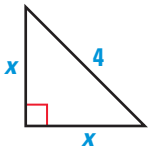


21.

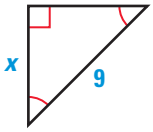


Finding Leg Lengths **Show that the triangle is a 45°-45°-90° triangle. Then find the value of each variable. Round to the nearest tenth.**

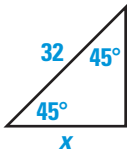
22.



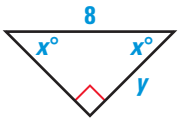
23.



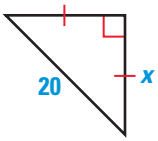
24.



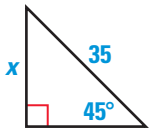
25.



26.

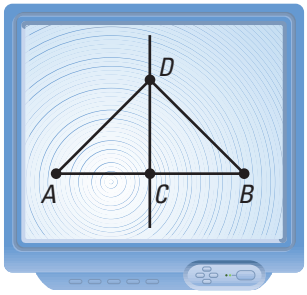


27.



Technology **In Exercises 28–30, use geometry software.**

1. Draw $\overline{AB}$ and construct its midpoint, C.
2. Construct the perpendicular bisector of $\overline{AB}$.
3. Construct point D on the bisector and construct $\overline{AD}$ and $\overline{DB}$.
4. Measure $\angle ADB$. Drag point D until $m\angle ADB = 90°$.

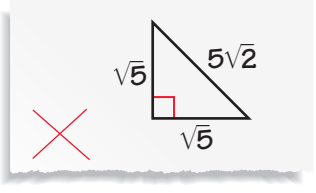


28. Name three 45°-45°-90° triangles. Explain how you know they are 45°-45°-90° triangles.

29. Measure $\overline{AC}$, $\overline{CB}$, and $\overline{CD}$. What do you notice? Explain.

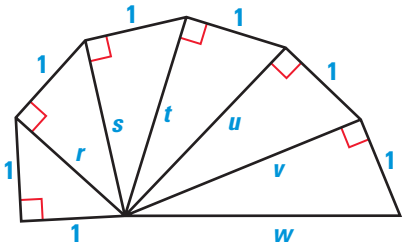
30. Predict the measures of $\overline{AD}$ and $\overline{DB}$. Check your answer by measuring the segments.

31. Error Analysis A student labels a 45°-45°-90° triangle as shown. Explain and correct the error.



Quilt Design **The quilt design in the photo is based on the pattern in the diagram below. Use the diagram in Exercises 32 and 33.**

"Wheel of Theodorus," by Diana Venters



32. Working from left to right, use the Pythagorean Theorem in each right triangle to find the values of r, s, t, u, v, and w.

33. Identify any 45°-45°-90° triangles in the figure.

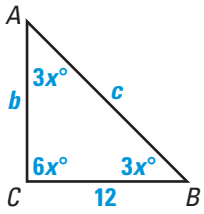
Standardized Test Practice

34. Multi-Step Problem Use the triangle shown below.

a. Find the value of x. Then find $m\angle A$, $m\angle B$, and $m\angle C$.

b. Find the values of b and c.

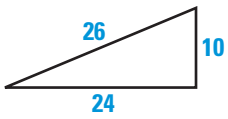
c. Use the Pythagorean Theorem or the 45°-45°-90° Triangle Theorem to justify your answers in part (b).



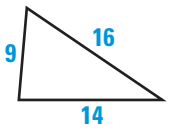
Mixed Review

Classifying Triangles **Classify the triangle as *acute*, *right*, or *obtuse*.** *(Lesson 4.5)*

35.



36.



37.

Simplifying Radicals **Simplify the radical expression.** *(Lesson 10.1)*

38. $\sqrt{24}$ **39.** $\sqrt{63}$ **40.** $\sqrt{52}$ **41.** $\sqrt{64}$

42. $\sqrt{80}$ **43.** $\sqrt{196}$ **44.** $\sqrt{250}$ **45.** $\sqrt{117}$

Algebra Skills

Writing Fractions as Decimals **Write the fraction as a decimal. For repeating decimals, also round to the nearest hundredth.** *(Skills Review, p. 657)*

46. $\frac{9}{10}$ **47.** $\frac{3}{5}$ **48.** $\frac{2}{3}$ **49.** $\frac{33}{100}$

50. $\frac{4}{9}$ **51.** $\frac{3}{20}$ **52.** $\frac{47}{50}$ **53.** $\frac{1}{6}$