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Properties of Tangents

Goal

Use properties of a tangent to a circle.

Key Words

- point of tangency p. 589
- perpendicular p. 108
- tangent segment

A discus thrower spins around in a circle one and a half times, then releases the discus. The discus forms a path tangent to the circle.

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THEOREMS 11.1 and 11.2

Theorem II.I

Words If a line is tangent to a circle, then it is perpendicular to the radius drawn at the point of tangency.

Symbols If ℓ is tangent to $\odot C$ at B, then $\ell \perp \overline{CB}$.

Theorem 11.2

Words In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

Symbols If $\ell \perp \overline{CB}$, then ℓ is tangent to $\odot C$ at *B*.





Vocabulary TIP Tangent is based on a Latin word meaning "to touch."

EXAMPLE 1 Use Properties of Tangents

 \overrightarrow{AC} is tangent to $\odot B$ at point *C*. Find *BC*.



Solution

 \overrightarrow{BC} is a radius of $\odot B$, so you can apply Theorem 11.1 to conclude that \overrightarrow{BC} and \overrightarrow{AC} are perpendicular.

So, $\angle BCA$ is a right angle, and $\triangle BCA$ is a right triangle. To find *BC*, use the Pythagorean Theorem.

| $(\mathbf{BA})^2 = (\mathbf{BC})^2 + (\mathbf{AC})^2$ | Pythagorean Theorem |
|---|-------------------------------------|
| $13^2 = (BC)^2 + 12^2$ | Substitute 13 for BA and 12 for AC. |
| $169 = (BC)^2 + 144$ | Multiply. |
| $25 = (BC)^2$ | Subtract 144 from each side. |
| 5 = BC | Find the positive square root. |





SILOS are used as storage bins for feed for farm animals. Round silos allow for the feed to be tightly packed, which prevents it from spoiling.

EXAMPLE 2 Find the Radius of a Circle

You are standing at *C*, 8 feet from a silo. The distance to a point of tangency is 16 feet. What is the radius of the silo?



Solution

Tangent \overrightarrow{BC} is perpendicular to radius \overrightarrow{AB} at *B*, so $\triangle ABC$ is a right triangle. So, you can use the Pythagorean Theorem.

| $(\mathbf{AC})^2 = (\mathbf{AB})^2 + (\mathbf{BC})^2$ | Pythagorean Theorem |
|---|---|
| $(r + 8)^2 = r^2 + 16^2$ | Substitute r + 8 for AC, r for AB, and 16 for BC. |
| $r^2 + 16r + 64 = r^2 + 256$ | $(r + 8)(r + 8) = r^2 + 16r + 64$ |
| 16r + 64 = 256 | Subtract r^2 from each side. |
| 16r = 192 | Subtract 64 from each side. |
| r = 12 | Divide each side by 16. |
| | |

ANSWER The radius of the silo is 12 feet.

You can use the Converse of the Pythagorean Theorem to show that a line is tangent to a circle.

EXAMPLE 3 Verify a Tangent to a Circle

How can you show that \overleftarrow{EF} must be tangent to $\bigcirc D$?



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Solution

Use the Converse of the Pythagorean Theorem to determine whether \triangle *DEF* is a right triangle.

 $(DF)^2 \stackrel{?}{=} (DE)^2 + (EF)^2$ $15^2 \stackrel{?}{=} 9^2 + 12^2$ $225 \stackrel{?}{=} 81 + 144$ 225 = 225

| Compare $(DF)^2$ with $(DE)^2 + (EF)^2$. |
|--|
| Substitute 15 for DF, 9 for DE, and 12 for EF. |
| Multiply. |
| Simplify. |
| |

 \triangle *DEF* is a right triangle with right angle *E*. So, \overline{EF} is perpendicular to \overline{DE} . By Theorem 11.2, it follows that \overrightarrow{EF} is tangent to $\bigcirc D$.

Student Help

To review the Converse

of the Pythagorean Theorem, see p. 200.

LOOK ВАСК





Tangent Segment A **tangent segment** touches a circle at one of the segment's endpoints and lies in the line that is tangent to the circle at that point.



Activity 11.2, on page 594, shows that tangent segments from the same exterior point are congruent.

THEOREM 11.3

Words If two segments from the same point outside a circle are tangent to the circle, then they are congruent.

Symbols If \overline{SR} and \overline{ST} are tangent to $\bigcirc P$ at points *R* and *T*, then $\overline{SR} \cong \overline{ST}$.



EXAMPLE 4 Use Properties of Tangents

AB is tangent to $\bigcirc C$ at *B*. \overline{AD} is tangent to $\bigcirc C$ at *D*. Find the value of *x*.



Student Help Skills Review To review solving equations, see p. 673.....

Solution

AD = AB2x + 3 = 112x = 8x = 4 Two tangent segments from the same point are congruent. Substitute 2x + 3 for *AD* and 11 for *AB*. Subtract 3 from each side. Divide each side by 2.

2.

Checkpoint V Use Properties of Tangents

 \overline{CB} and \overline{CD} are tangent to $\odot A$. Find the value of x.





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II.2 Exercises

Guided Practice

Vocabulary Check

1. Complete the statement: In the diagram at the right, \overline{AB} is _?____ to $\odot C$, and point B is the <u>?</u>.

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- **Skill Check**
- **2.** In the diagram below, \overrightarrow{XY} is tangent to $\odot C$ at point *P*. What is $m \angle CPX$? Explain.
- **3.** In the diagram below, AB = BD = 5 and AD = 7. Is \overline{BD} tangent to $\odot C$? Explain.



 \overline{AB} is tangent to $\odot C$ at A and \overline{DB} is tangent to $\odot C$ at D. Find the value of x.



Practice and Applications

Extra Practice

See p. 695.





Finding Segment Lengths \overline{AB} and \overline{AD} are tangent to $\bigcirc C$. Find the value of *x*.



Homework Help

Example 1: Exs. 7–9, 27 **Example 2:** Exs. 13–19 **Example 3:** Exs. 20–21 Example 4: Exs. 10-12, 22 - 26

Using Algebra Square the binomial.

13. $(x+2)^2$ **14.** $(x+4)^2$ **15.** $(x+7)^2$ **16.** $(x+12)^2$

Finding the Radius of a Circle \overline{AB} is tangent to $\odot C$. Find the

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HOMEWORK HELP

Extra help with problem solving in Exs. 17–19 is at classzone.com



Verifying Tangents Tell whether \overline{AB} is tangent to $\odot C$. Explain your reasoning.



Finding Congruent Parts In Exercises 22–24, \overline{AB} and \overline{AD} are tangent to $\odot C$.

- **22.** Name all congruent segments.
- **23.** Name all congruent angles.
- **24**. Name two congruent triangles.

Visualize It! In Exercises 25 and 26, $\odot L$ has radii \overline{LJ} and \overline{LK} that are perpendicular. \overline{KM} and \overline{JM} are tangent to $\odot L$.

- **25.** Sketch $\odot L$, \overline{LJ} , \overline{LK} , \overline{KM} , and \overline{JM} .
- **26.** Is \triangle *JLM* congruent to \triangle *KLM*? Explain your reasoning.
- **27. Global Positioning System** GPS satellites orbit 12,500 miles above Earth. Because GPS signals can't travel through Earth, a satellite can transmit signals only as far as points *A* and *C* from point *B*. Find *BA* and *BC* to the nearest mile.







GPS Hikers sometimes carry navigation devices which utilize GPS technology. GPS helps hikers calculate where they are and how to get to another location.



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28. Challenge You are cruising away from a fireworks show over a bay at point *A*. The highest point of the fireworks is point *F*. When your ship reaches point *D*, you can no longer see the fireworks over the horizon. You are standing at point *E*. \overline{FE} is tangent to Earth at *B*. Find *FE*. Round your answer to the nearest mile.

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| 29. | Multiple | Choice | In the dia | igram | below, | \overline{EF} | and | \overline{EG} | are | tange | ent |
|-----|-----------------|-----------|------------|-------|--------|-----------------|-----|-----------------|-----|-------|-----|
| | to $\odot C. W$ | hat is th | e value of | x? | | | | r | | | |





30. Multiple Choice In the diagram below, \overline{SR} is tangent to $\bigcirc P$. Find the radius of $\bigcirc P$.

| i illa tilo italiao or or i | | | |
|-----------------------------|-------------|--|--|
| F 18 | G 27 | | |
| H 36 | J 45 | | |



Mixed Review Using the Triangle Inequality Can the side lengths form a triangle? Explain. (Lesson 4.7)

| 31. 5, 11, 14 | 32. 8, 14, 23 | 33. 15, 3, 13 |
|----------------------|----------------------|----------------------|
| 34. 18, 25, 9 | 35. 10, 3, 7 | 36. 22, 6, 29 |

Using the Midsegment Theorem Find the value of x. (Lesson 7.5)



| Algebra Skills | Finding Slope Find the slope of the line that passes through the points. (<i>Skills Review, p. 665</i>) | | | | |
|----------------|--|--------------------------------|--------------------------------|--|--|
| | 40. (0, 0) and (-3, 6) | 41. (2, 4) and (8, 0) | 42. (1, 5) and (-2, 1) | | |
| | 43. (0, -3) and (4, 7) | 44. (-1, 6) and (4, -5) | 45. (-7, 2) and (-1, 4) | | |