11.8 Rotations

Goal

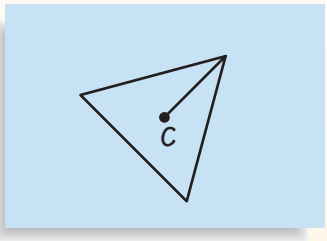
Identify rotations and rotational symmetry.

Key Words

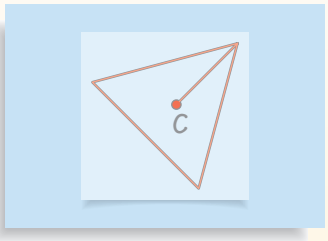
- rotation
- center of rotation
- angle of rotation
- rotational symmetry

Geo-Activity Rotating a Figure

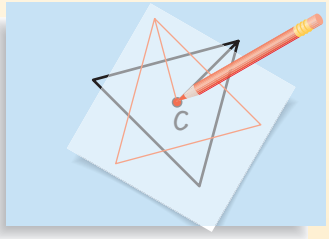
1. Draw an equilateral triangle. Label as shown. Draw a line from the center to one of the vertices.



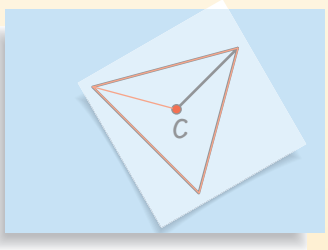
2. Copy the triangle onto a piece of tracing paper.



3. Place a pencil on the center point and turn the tracing paper over the original triangle until it matches up with itself.



4. How many degrees did you turn the triangle? Is there more than one way to turn the triangle so that it matches up with itself?



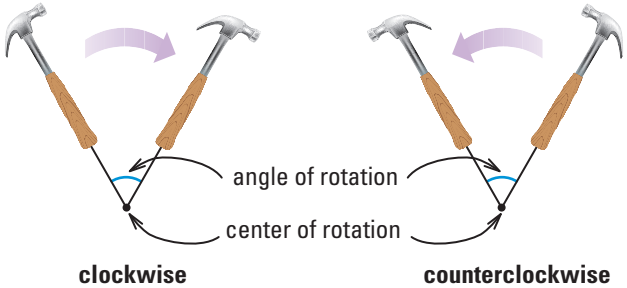
5. Draw a rectangle and a square. Repeat Steps 1 through 4. How many degrees did you turn each figure until it matched up with itself?

Visualize It!

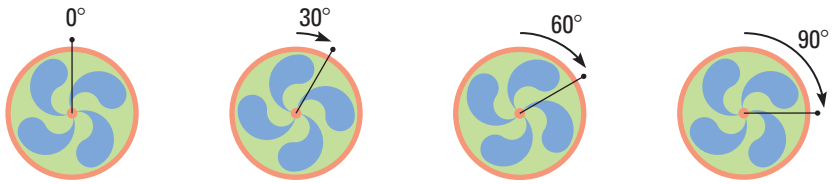
Clockwise means to go in the direction of the hands on a clock.

Counterclockwise means to go in the opposite direction.

A **rotation** is a transformation in which a figure is turned about a fixed point. The fixed point is the **center of rotation**. In the Geo-Activity above, point C is the center of rotation. Rays drawn from the center of rotation to a point and its image form an angle called the **angle of rotation**. Rotations can be clockwise or counterclockwise.



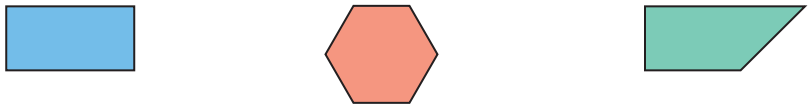
Rotational Symmetry A figure in a plane has **rotational symmetry** if the figure can be mapped onto itself by a rotation of 180° or less. For instance, the figure below has rotational symmetry because it maps onto itself by a rotation of 90°.



EXAMPLE 1 Identify Rotational Symmetry

Does the figure have rotational symmetry? If so, describe the rotations that map the figure onto itself.

a. Rectangle **b.** Regular hexagon **c.** Trapezoid



Solution

a. Yes. A rectangle can be mapped onto itself by a clockwise or counterclockwise rotation of 180° about its center.

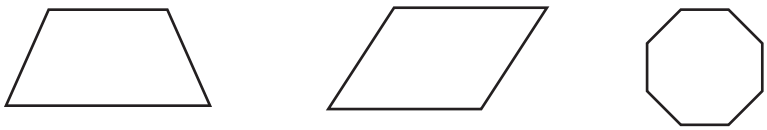
b. Yes. A regular hexagon can be mapped onto itself by a clockwise or counterclockwise rotation of 60°, 120°, or 180° about its center.

c. No. A trapezoid does not have rotational symmetry.

Checkpoint Identify Rotational Symmetry

Does the figure have rotational symmetry? If so, describe the rotations that map the figure onto itself.

1. Isosceles trapezoid **2.** Parallelogram **3.** Regular octagon



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MORE EXAMPLES
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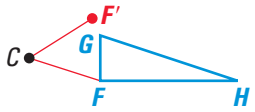
EXAMPLE 2 Rotations

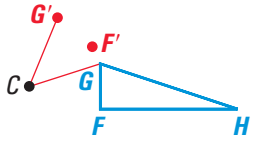
Rotate $\triangle FGH$ 50° counterclockwise about point C.

Solution

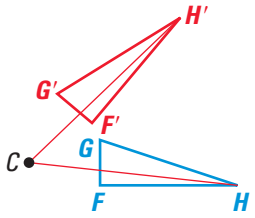
1. To find the image of point F, draw $\overline{CF}$ and draw a 50° angle. Find F' so that $CF = CF'$.



2. To find the image of point G, draw $\overline{CG}$ and draw a 50° angle. Find G' so that $CG = CG'$.



3. To find the image of point H, draw $\overline{CH}$ and draw a 50° angle. Find H' so that $CH = CH'$. Draw $\triangle F'G'H'$.



EXAMPLE 3 Rotations in a Coordinate Plane

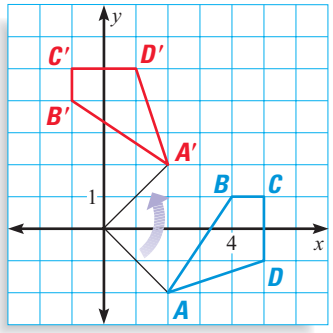
Sketch the quadrilateral with vertices $A(2, -2)$, $B(4, 1)$, $C(5, 1)$, and $D(5, -1)$. Rotate it 90° counterclockwise about the origin and name the coordinates of the new vertices.

Solution

Plot the points, as shown in blue.

Use a protractor and a ruler to find the rotated vertices.

The coordinates of the vertices of the image are $A'(2, 2)$, $B'(-1, 4)$, $C'(-1, 5)$, and $D'(1, 5)$.



Checkpoint *Rotations in a Coordinate Plane*

4. Sketch the triangle with vertices $A(0, 0)$, $B(3, 0)$, and $C(3, 4)$. Rotate $\triangle ABC$ 90° counterclockwise about the origin. Name the coordinates of the new vertices A', B', and C'.

11.8 Exercises

Guided Practice

Vocabulary Check

1. What is a *center of rotation*?
2. Explain how you know if a figure has *rotational symmetry*.

Skill Check

Does the figure have rotational symmetry? If so, describe the rotations that map the figure onto itself.

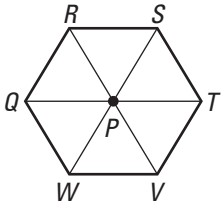
3. 

4. 

5. 

The diagonals of the regular hexagon shown form six equilateral triangles. Use the diagram to complete the statement.

6. A clockwise rotation of 60° about P maps R onto __?__.
7. A counterclockwise rotation of 60° about __?__ maps R onto Q.
8. A clockwise rotation of 120° about Q maps R onto __?__.
9. A counterclockwise rotation of 180° about P maps V onto __?__.

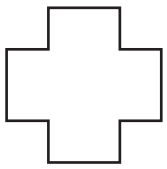


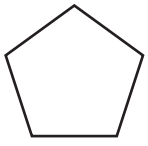
Practice and Applications

Extra Practice

See p. 696.

Rotational Symmetry **Does the figure have rotational symmetry? If so, describe the rotations that map the figure onto itself.**

10. 

11. 

12. 

Wheel Hubs **Describe the rotational symmetry of the wheel hub.**

13. 

14. 

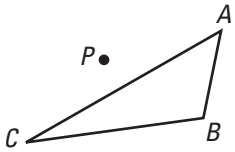
15. 

Homework Help

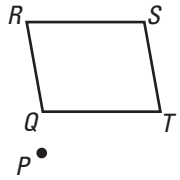
Example 1: Exs. 10–15
Example 2: Exs. 16–26
Example 3: Exs. 27–30

Rotating a Figure Trace the polygon and point P on paper. Use a straightedge and protractor to rotate the polygon clockwise the given number of degrees about P.

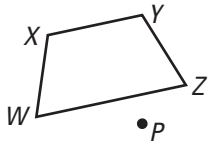
16. 150°



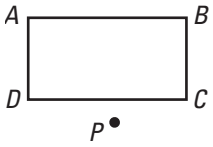
17. 135°



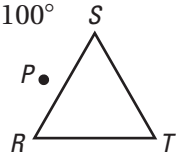
18. 60°



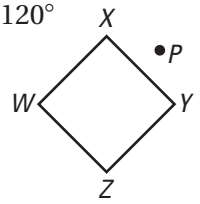
19. 40°



20. 100°



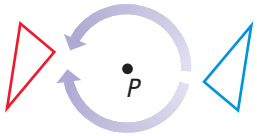
21. 120°



Visualize It!

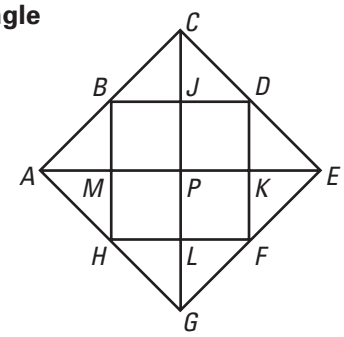
Rotating a figure 180° clockwise is the same as rotating a figure 180° counterclockwise.

180° counterclockwise



180° clockwise

Describing an Image State the segment or triangle that represents the image.



22. 90° clockwise rotation of $\overline{AB}$ about P

23. 90° clockwise rotation of $\overline{KF}$ about P

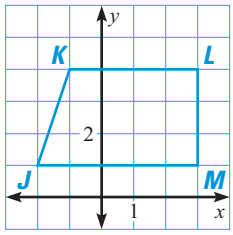
24. 180° rotation of $\triangle BCJ$ about P

25. 180° rotation of $\triangle KEF$ about P

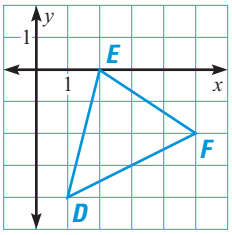
26. 90° counterclockwise rotation of $\overline{CE}$ about E

Finding a Pattern Use the given information to rotate the figure about the origin. Find the coordinates of the vertices of the image and compare them with the vertices of the original figure. Describe any patterns you see.

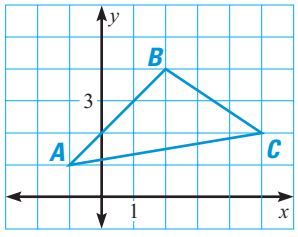
27. 90° clockwise



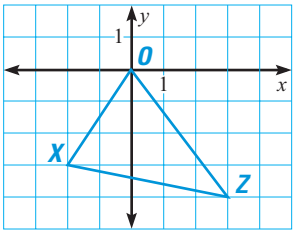
28. 90° counterclockwise



29. 90° counterclockwise



30. 180°



Link to Careers

GRAPHIC DESIGNERS may create symbols to represent a company or organization. These symbols often appear on packaging, stationery, and Web sites.

Career Links CLASSZONE.COM

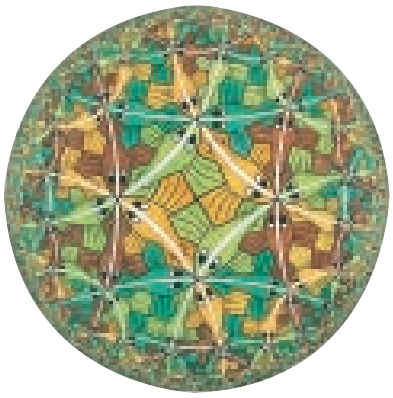
Graphic Design **A music store, Ozone, is running a contest for a store logo. Two of the entries are shown. What do you notice about them?**

31.

ozone

32.

Rotations in Art **In Exercises 33–36, refer to the image below by M.C. Escher. The piece is called *Circle Limit III* and was completed in 1959.**



33. Does the piece have rotational symmetry? If so, describe the rotations that map the image onto itself.

34. Would your answer to Exercise 33 change if you disregard the color of the figures? Explain your reasoning.

35. Describe the center of rotation.

36. Is it possible that this piece could be hung upside down and have the same appearance? Explain.

Standardized Test Practice

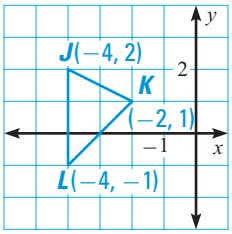
37. Multiple Choice What are the coordinates of the vertices of the image of $\triangle JKL$ after a 90° clockwise rotation about the origin?

Ⓐ $J'(1, 2), K'(4, 2), L'(1, 4)$

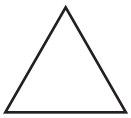
Ⓑ $J'(2, 1), K'(4, 2), L'(1, 4)$

Ⓒ $J'(4, 2), K'(2, 1), L'(4, -1)$

Ⓓ $J'(2, 4), K'(1, 2), L'(-1, 4)$

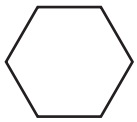


38. Multiple Choice Which of the four polygons shown below does not have rotational symmetry?

Ⓕ 

Ⓖ 

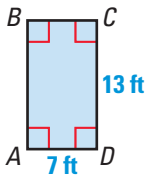
Ⓗ 

Ⓙ 

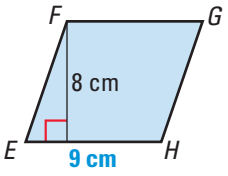
Mixed Review

Area of Polygons **Find the area of the polygon.** *(Lessons 8.3, 8.5)*

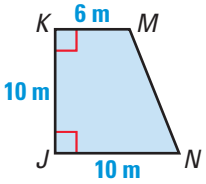
39. rectangle $ABCD$



40. parallelogram $EFGH$



41. trapezoid $JKMN$



Algebra Skills

Evaluating Radicals **Evaluate. Give the exact value if possible. If not, approximate to the nearest tenth.** *(Skills Review, p. 668)*

42. $\sqrt{42}$ **43.** $\sqrt{90}$ **44.** $\sqrt{256}$ **45.** $\sqrt{0}$

Quiz 3

1. What are the center and the radius of the circle whose equation is $(x + 1)^2 + (y - 6)^2 = 25$? *(Lesson 11.7)*

2. Write the standard equation of the circle with center $(0, -4)$ and radius 3. *(Lesson 11.7)*

Graph the equation. *(Lesson 11.7)*

3. $x^2 + (y - 1)^2 = 36$

4. $(x + 2)^2 + (y - 5)^2 = 4$

5. $(x - 3)^2 + (y + 4)^2 = 9$

6. $(x + 1)^2 + (y + 1)^2 = 16$

Does the figure have rotational symmetry? If so, describe the rotations that map the figure onto itself. *(Lesson 11.8)*

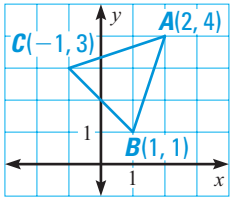
7.

8.

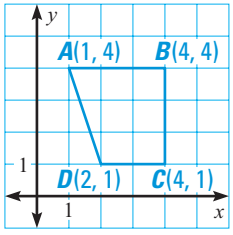
9.

Use the given information to rotate the figure about the origin. Find the coordinates of the vertices of the image and compare them with the vertices of the original figure. Describe any patterns you see. *(Lesson 11.8)*

10. 180°



11. 90° counterclockwise



12. 90° clockwise