7.2 Similar Polygons

Goal

Identify similar polygons.

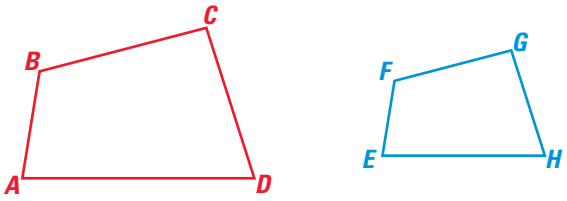
Key Words

- similar polygons
- scale factor

In geometry, two figures that have the same shape are called *similar*.

Two polygons are **similar polygons** if corresponding angles are congruent and corresponding side lengths are proportional.

In the diagram, *ABCD* is similar to *EFGH*. The symbol ~ indicates similarity. So, you can write *ABCD* ~ *EFGH*. When you refer to similar polygons, list their corresponding vertices in the same order.



ABCD* ~ *EFGH

Corresponding Angles

$\angle A \cong \angle E$
$\angle B \cong \angle F$
$\angle C \cong \angle G$
$\angle D \cong \angle H$

Ratios of Corresponding Sides

$$\frac{EF}{AB} = \frac{FG}{BC} = \frac{GH}{CD} = \frac{HE}{DA}$$

Student Help

VOCABULARY TIP
$\triangle PRQ \sim \triangle STU$ is called a *similarity statement*.

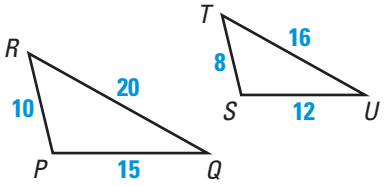
EXAMPLE 1 Use Similarity Statements

$\triangle PRQ \sim \triangle STU$.

a. List all pairs of congruent angles.

b. Write the ratios of the corresponding sides in a statement of proportionality.

c. Check that the ratios of corresponding sides are equal.



Solution

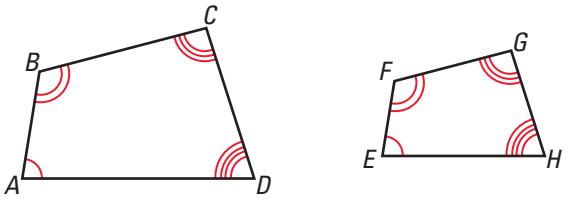
a. $\angle P \cong \angle S$, $\angle R \cong \angle T$, and $\angle Q \cong \angle U$.

b. $\frac{ST}{PR} = \frac{TU}{RQ} = \frac{US}{QP}$

c. $\frac{ST}{PR} = \frac{8}{10} = \frac{4}{5}$, $\frac{TU}{RQ} = \frac{16}{20} = \frac{4}{5}$, and $\frac{US}{QP} = \frac{12}{15} = \frac{4}{5}$.

The ratios of corresponding sides are all equal to $\frac{4}{5}$.

Scale Factor If two polygons are similar, then the ratio of the lengths of two corresponding sides is called the **scale factor**.



scale factor of *EFGH* to *ABCD* $= \frac{EF}{AB} = \frac{FG}{BC} = \frac{GH}{CD} = \frac{HE}{DA}$

Student Help
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MORE EXAMPLES
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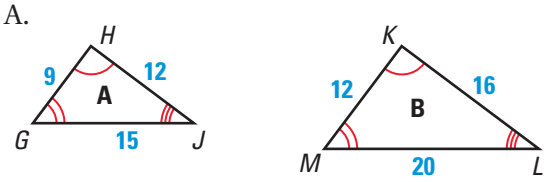
EXAMPLE 2 Determine Whether Polygons are Similar

Determine whether the triangles are similar. If they are similar, write a similarity statement and find the scale factor of Figure B to Figure A.



Solution

1 Check whether the corresponding angles are congruent.

From the diagram, you can see that $\angle G \cong \angle M$, $\angle H \cong \angle K$, and $\angle J \cong \angle L$. Therefore, the corresponding angles are congruent.

2 Check whether the corresponding side lengths are proportional.

$$\frac{MK}{GH} = \frac{12}{9} = \frac{12 \div 3}{9 \div 3} = \frac{4}{3}$$

$$\frac{KL}{HJ} = \frac{16}{12} = \frac{16 \div 4}{12 \div 4} = \frac{4}{3}$$

$$\frac{LM}{JG} = \frac{20}{15} = \frac{20 \div 5}{15 \div 5} = \frac{4}{3}$$

All three ratios are equal, so the corresponding side lengths are proportional.

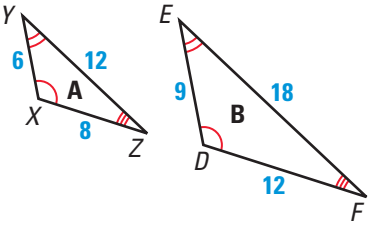
ANSWER By definition, the triangles are similar. $\triangle GHJ \sim \triangle MKL$.

The scale factor of Figure B to Figure A is $\frac{4}{3}$.

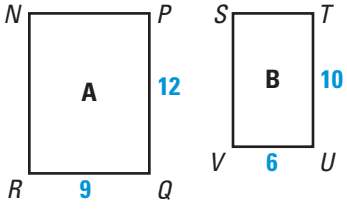
Checkpoint Determine Whether Polygons are Similar

Determine whether the polygons are similar. If they are similar, write a similarity statement and find the scale factor of Figure B to Figure A.

1.

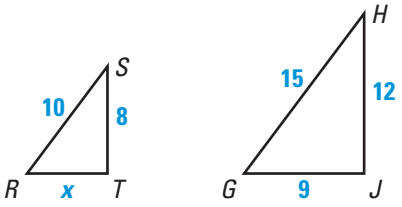


2.



EXAMPLE 3 Use Similar Polygons

$\triangle RST \sim \triangle GHJ$.
Find the value of x.



Solution

Because the triangles are similar, the corresponding side lengths are proportional. To find the value of x, you can use the following proportion.

$\frac{GH}{RS} = \frac{JG}{TR}$ Write proportion.

$\frac{15}{10} = \frac{9}{x}$ Substitute 15 for GH, 10 for RS, 9 for JG, and x for TR.

$15 \cdot x = 10 \cdot 9$ Cross product property

$15x = 90$ Multiply.

$\frac{15x}{15} = \frac{90}{15}$ Divide each side by 15.

$x = 6$ Simplify.

Student Help

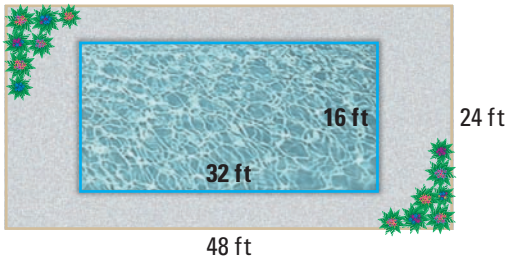
STUDY TIP
You can find x using other proportions that include TR. Another proportion you can use is $\frac{ST}{HJ} = \frac{TR}{JG}$.

EXAMPLE 4 Perimeters of Similar Polygons

The outlines of a pool and the patio around the pool are similar rectangles.

a. Find the ratio of the length of the patio to the length of the pool.

b. Find the ratio of the perimeter of the patio to the perimeter of the pool.



Solution

a. The ratio of the length of the patio to the length of the pool is

$$\frac{\text{length of patio}}{\text{length of pool}} = \frac{48 \text{ feet}}{32 \text{ feet}} = \frac{48 \div 16}{32 \div 16} = \frac{3}{2}.$$

b. The perimeter of the patio is $2(24) + 2(48) = 144$ feet.

The perimeter of the pool is $2(16) + 2(32) = 96$ feet.

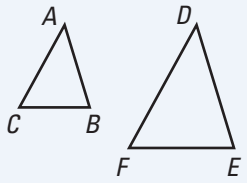
The ratio of the perimeter of the patio to the perimeter of the pool is

$$\frac{\text{perimeter of patio}}{\text{perimeter of pool}} = \frac{144 \text{ feet}}{96 \text{ feet}} = \frac{144 \div 48}{96 \div 48} = \frac{3}{2}.$$

In Example 4 on the previous page, notice that the ratio of the perimeters of the similar figures is equal to the ratio of the side lengths. This observation is generalized in the following theorem.

THEOREM 7.1

Perimeters of Similar Polygons



Words If two polygons are similar, then the ratio of their perimeters is equal to the ratio of their corresponding side lengths.

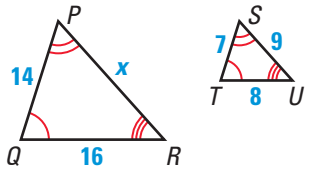
Symbols If $\triangle ABC \sim \triangle DEF$, then $\frac{DE + EF + FD}{AB + BC + CA} = \frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA}$.

Checkpoint ✓ **Use Similar Polygons**

In the diagram, $\triangle PQR \sim \triangle STU$.

3. Find the value of x.

4. Find the ratio of the perimeter of $\triangle STU$ to the perimeter of $\triangle PQR$.



7.2 Exercises

Guided Practice

Vocabulary Check

1. If two triangles are *congruent*, must they be *similar*? Explain.

2. If two triangles are *similar*, must they be *congruent*? Explain.

Skill Check

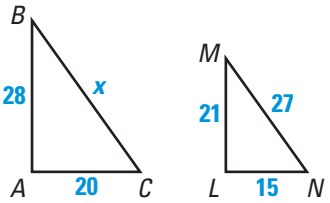
In Exercises 3–6, $\triangle ABC \sim \triangle LMN$.

3. List all pairs of congruent angles.

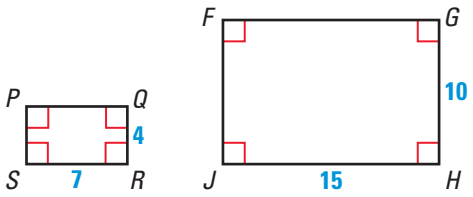
4. Write the ratios of the corresponding sides in a statement of proportionality.

5. Find the scale factor of $\triangle LMN$ to $\triangle ABC$.

6. Find the value of x.



7. Are the two rectangles shown at the right similar? Explain your reasoning.



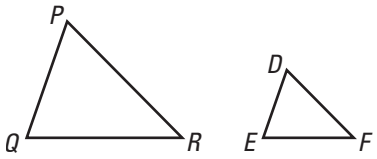
Practice and Applications

Extra Practice

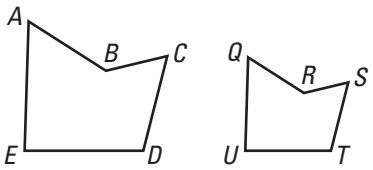
See p. 687.

Using Similarity Statements **List all pairs of congruent angles. Then write the ratios of the corresponding sides in a statement of proportionality.**

8. $\triangle PQR \sim \triangle DEF$



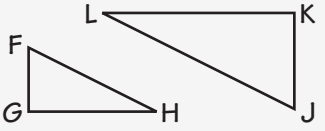
9. $ABCDE \sim QRSTU$



10. Error Analysis $\triangle FGH \sim \triangle JKL$. A student was asked to list all pairs of congruent angles and write the ratios of the corresponding sides in a statement of proportionality. Copy the diagram and correct the student's errors.

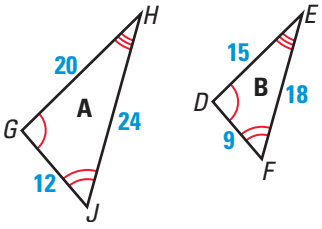
$\angle F \cong \angle J, \angle G \cong \angle L, \angle H \cong \angle K$

$\frac{FG}{JK} = \frac{KL}{GH} = \frac{FH}{JL}$ ✗

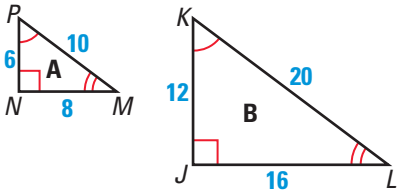


Determining Similarity **Determine whether the polygons are similar. If they are similar, write a similarity statement and find the scale factor of Figure B to Figure A.**

11.



12.



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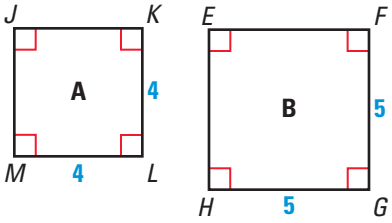

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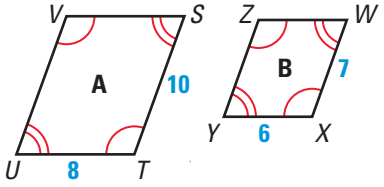

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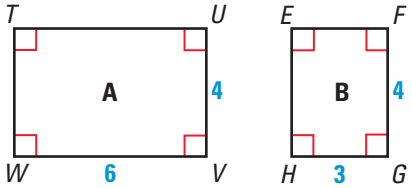
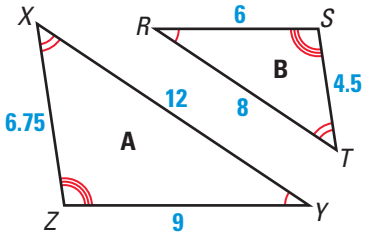


16.

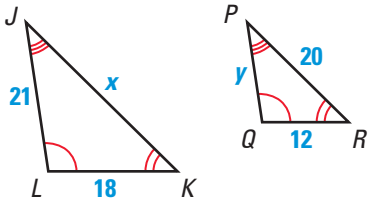


Homework Help

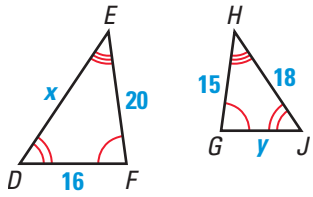
Example 1: Exs. 8–10, 22, 23
Example 2: Exs. 11–16, 29
Example 3: Exs. 17–21, 24, 25
Example 4: Exs. 26, 27

Using Similar Polygons **The two polygons are similar. Find the values of *x* and *y*.**

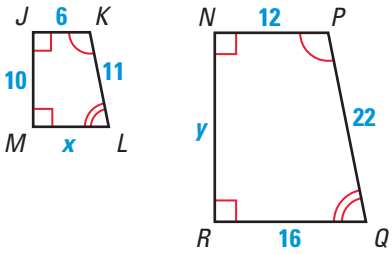
17.



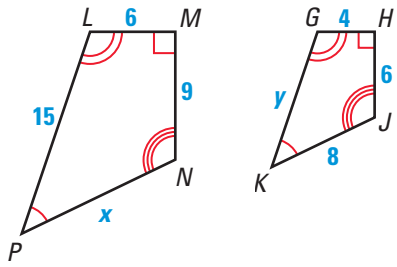
18.



19.



20.



Link to Careers

ARTIST Some artists create work for public, educational, or commercial purposes. Alejandro Romero, above, creates large murals that are displayed in public buildings

Career Links CLASSZONE.COM

21. Mural Alejandro Romero created the mural, *Chicago Federation of Labor*, by enlarging the 56 in. by 21 in. sketch shown below. Romero used a scale factor of about 3.5. What are the dimensions of the mural in inches? In feet?

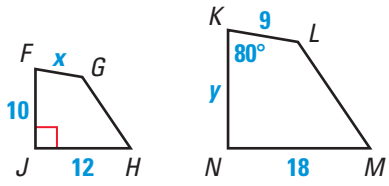
Using Similar Polygons **In Exercises 22–26, use the diagram below, where *FGHJ* ~ *KLMN*.**

22. Find $m\angle N$.

23. Find $m\angle F$.

24. Find the value of x.

25. Find the value of y.

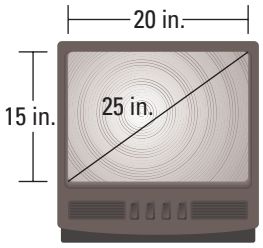


26. Find the ratio of the perimeter of *KLMN* to the perimeter of *FGHJ*.

27. Perimeters of Similar Polygons $\triangle QRS$ is similar to $\triangle XYZ$. The ratio of one side of $\triangle XYZ$ to the corresponding side of $\triangle QRS$ is 6 : 7. What is the ratio of the perimeter of $\triangle XYZ$ to the perimeter of $\triangle QRS$?

28. Challenge $\triangle JKL$ is similar to $\triangle STU$. The ratio of ST to JK is 5 to 2. The perimeter of $\triangle STU$ is 35 feet. Find the perimeter of $\triangle JKL$.

Link to Technology



TELEVISION SCREENS have sizes that are based on the length of their diagonal. The *aspect ratio* of a 25-inch standard definition television screen is 20 in. : 15 in., or 4 : 3.

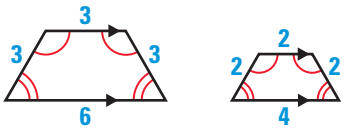
29. Television Screens The *aspect ratio* of a television screen is the length-to-width ratio of the screen. A standard definition television has an aspect ratio of 4 : 3. A high definition projection television has an aspect ratio of 16 : 9. Are the television screens similar rectangles?

EXAMPLE **Logical Reasoning**

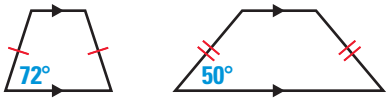
Are two isosceles trapezoids *always*, *sometimes*, or *never* similar?

Solution

It is possible to sketch two similar isosceles trapezoids.



It is also possible to sketch two isosceles trapezoids that are *not* similar.



ANSWER ▶ Therefore, two isosceles trapezoids are sometimes similar.

Logical Reasoning **Are the polygons *always*, *sometimes*, or *never* similar?**

30. Two isosceles triangles

31. Two rhombuses

32. Two equilateral triangles

33. A right and an isosceles triangle

Standardized Test Practice

34. Multiple Choice $\triangle DEF \sim \triangle MNP$. Which statement may be *false*?

Ⓐ $\angle E \cong \angle N$

Ⓑ $\angle P \cong \angle D$

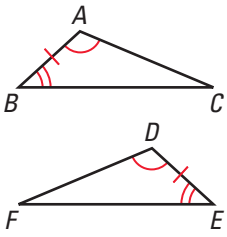
Ⓒ $\frac{MN}{DE} = \frac{NP}{EF}$

Ⓓ $\frac{MN}{DE} = \frac{PM}{FD}$

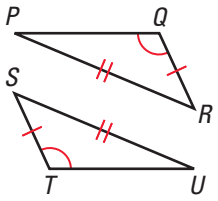
Mixed Review

Showing Triangles are Congruent **Does the diagram give enough information to show that the triangles are congruent? If so, state the postulate or theorem you would use.** *(Lessons 5.3, 5.4)*

35.



36.



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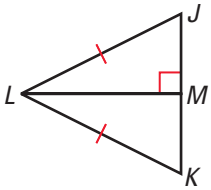

Algebra Skills

Writing Equivalent Fractions **Write two equivalent fractions.** *(Skills Review, p. 656)*

38. $\frac{1}{3}$

39. $\frac{2}{5}$

40. $\frac{4}{7}$

41. $\frac{9}{4}$