

9.1

Solid Figures

Goal

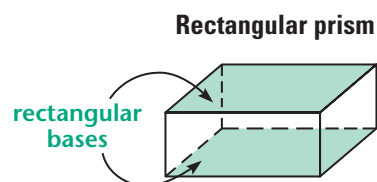
Identify and name solid figures.

Key Words

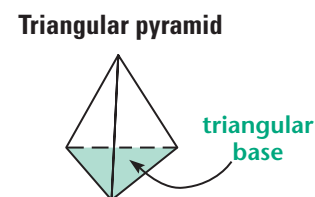
- solid
- polyhedron
- base
- face
- edge

The three-dimensional shapes on this page are examples of *solid figures*, or **solids**. When a solid is formed by polygons, it is called a **polyhedron**.

Polyhedra Prisms and pyramids are examples of polyhedra. To name a prism or pyramid, use the shape of the *base*.

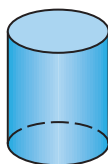
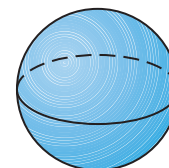


The two **bases** of a prism are congruent polygons in parallel planes.



The **base** of a pyramid is a polygon.

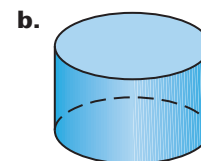
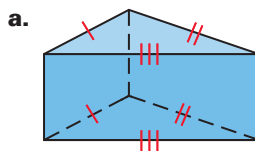
Not Polyhedra Solids with curved surfaces, like the cylinder, cone, and sphere shown below, are not polyhedra.

Cylinder**Cone****Sphere****Student Help****VOCABULARY TIP**

Poly- means “many” and *-hedron* is Greek for “side” or “face.” A polyhedron is a figure with many faces. The plural of polyhedron is *polyhedra*.

EXAMPLE 1 Identify and Name Polyhedra

Tell whether the solid is a polyhedron. If so, identify the shape of the bases. Then name the solid.

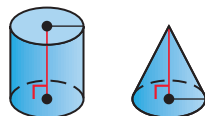
**Solution**

- a. The solid is formed by polygons so it is a polyhedron. The bases are congruent triangles in parallel planes. This figure is a triangular prism.
- b. A cylinder has a curved surface, so it is not a polyhedron.

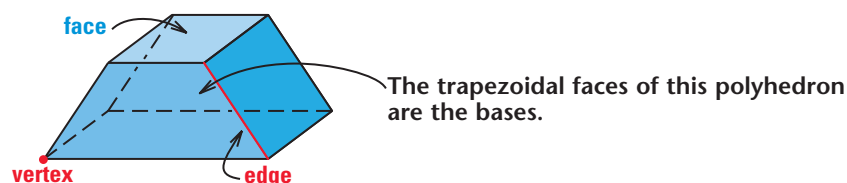
Student Help

STUDY TIP

In this book, all of the solids are *right solids*. This means that the segment representing the *height* is perpendicular to the base(s).



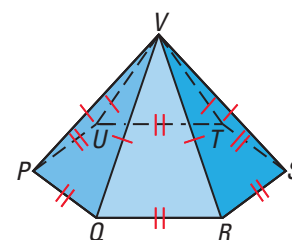
Parts of a Polyhedron To avoid confusion, the word *side* is not used when describing polyhedra. Instead, the plane surfaces are called **faces** and the segments joining the vertices are called **edges**.



EXAMPLE 2 Find Faces and Edges

Use the diagram at the right.

- Name the polyhedron.
- Count the number of faces and edges.
- List any congruent faces and congruent edges.



Solution

- The polyhedron is a hexagonal pyramid.
- The polyhedron has 7 faces and 12 edges.
- Using the markings on the diagram, you can conclude the following:

Congruent faces

$$\triangle PQV \cong \triangle QRV \cong \triangle RSV \cong \triangle STV \cong \triangle TUV \cong \triangle UPV$$

Congruent edges

$$\overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{ST} \cong \overline{TU} \cong \overline{UP}$$

$$\overline{PV} \cong \overline{QV} \cong \overline{RV} \cong \overline{SV} \cong \overline{TV} \cong \overline{UV}$$

Student Help

VISUAL STRATEGY

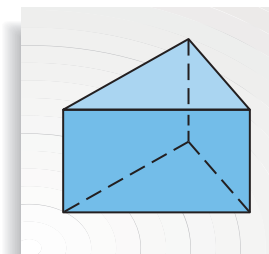
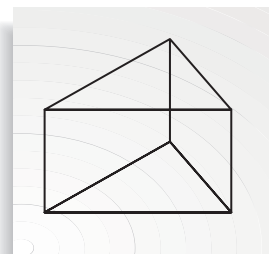
For more help drawing three dimensional figures, see p. 472.

EXAMPLE 3 Sketch a Polyhedron

Sketch a triangular prism.

Solution

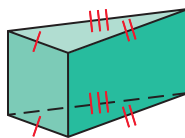
- Draw the triangular bases.
- Connect the corresponding vertices of the bases with vertical lines.
- Partially erase the hidden lines to create dashed lines. Shade the prism.



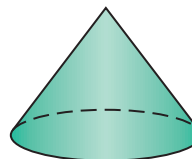
Checkpoint Identify and Sketch Polyhedra

Tell whether the solid is a polyhedron. If so, identify the shape of the base(s). Then name the solid.

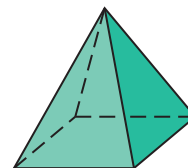
1.



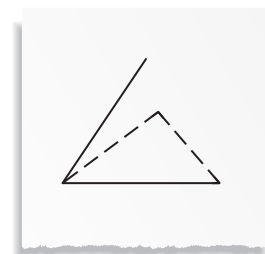
2.



3.

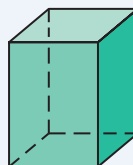


4. Copy the partial drawing of a triangular pyramid. Then complete the drawing of the pyramid.

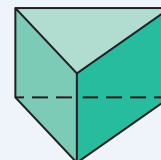


TYPES OF SOLIDS

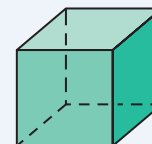
Prism



Rectangular Prism



Triangular Prism

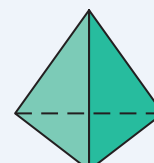


Cube
(Square Prism)

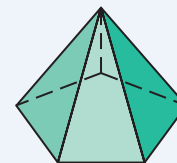
Pyramid



Rectangular Pyramid



Triangular Pyramid



Pentagonal Pyramid

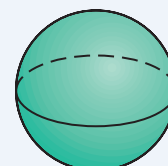
Cylinder



Cone



Sphere



Student Help

STUDY TIP

In this book, the faces of a pyramid, not including the base, are congruent isosceles triangles, unless otherwise noted.

9.1 Exercises

Guided Practice

Vocabulary Check

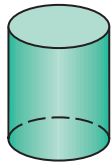
In Exercises 1–3, match the solid with its name.

A. prism

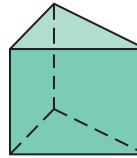
B. pyramid

C. cylinder

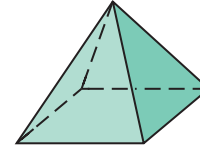
1.



2.

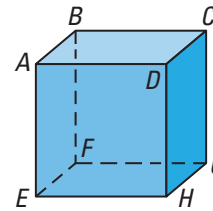


3.



In Exercises 4–9, tell whether the statement is *true* or *false*. Refer to the prism below, if necessary.

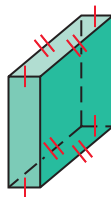
4. $CDHG$ is a face of the prism.
5. A prism has only one base.
6. $ABCD$ and $EFGH$ are possible bases of the prism.
7. An edge of the prism is H .
8. \overline{GC} is an edge of the prism.
9. A prism is a polyhedron.



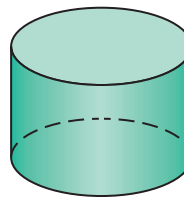
Skill Check

Tell whether the solid is a polyhedron. If so, identify the shape of the base(s). Then name the solid.

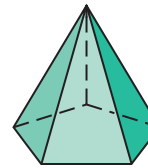
10.



11.

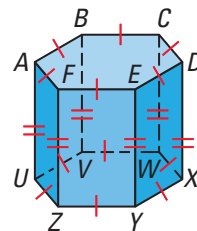


12.

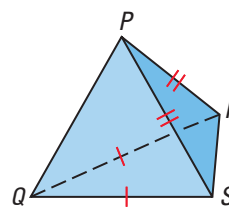


Name the polyhedron. Count the number of faces and edges. List any congruent faces and congruent edges.

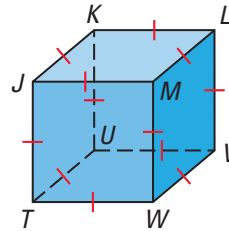
13.



14.



15.



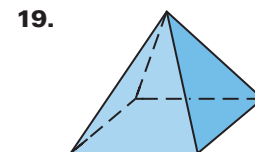
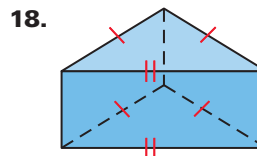
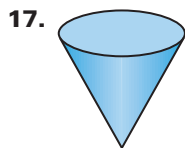
16. **Visualize It!** ➔ How many faces and edges does a box of cereal have?

Practice and Applications

Extra Practice

See p. 691.

Name Bases and Solids Tell whether the solid is a polyhedron. If so, identify the shape of the base(s). Then name the solid.

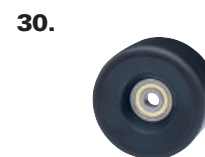


Logical Reasoning Tell whether the statement is *true* or *false*.

20. A rectangular pyramid has two bases.
21. A triangular prism has two bases.
22. The bases of a prism are congruent polygons.
23. A cone has two bases.
24. A sphere is a polyhedron.

Identify Solids Match the solid with its name.

- | | | |
|----------------------|------------|-------------|
| A. cone | B. pyramid | C. cylinder |
| D. rectangular prism | E. cube | F. sphere |



Homework Help

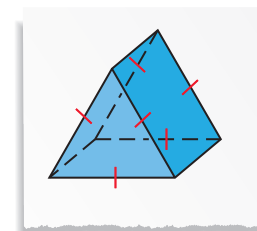
Example 1: Exs. 17–19,
25–35

Example 2: Exs. 36–38

Example 3: Exs. 43–51

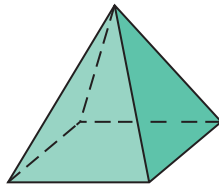
Error Analysis Julie incorrectly identified the solid below as a pyramid with a square base.

31. Correctly identify the solid.
32. What would you say to Julie to help her tell the difference between this solid and a pyramid?

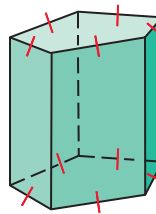


Identify Polyhedra Tell whether the solid is a polyhedron. If so, identify the shape of the base(s). Then name the solid.

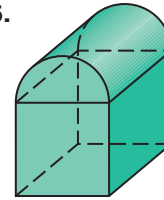
33.



34.

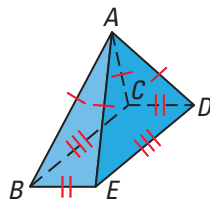


35.

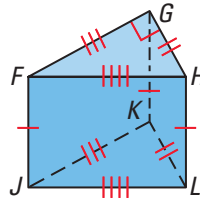


Counting Faces and Edges Name the polyhedron. Then count the number of faces and edges. List any congruent faces and congruent edges.

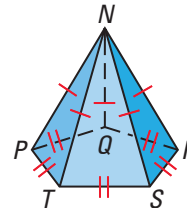
36.



37.



38.



Logical Reasoning Determine whether the statement is *true* or *false*. Explain your reasoning.

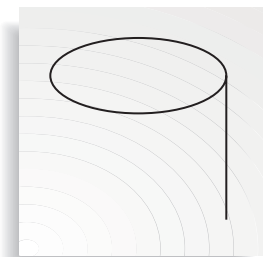
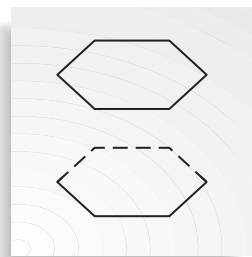
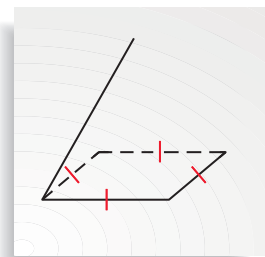
- 39. Prisms, pyramids, cylinders, cones, and spheres are all solids.
- 40. Prisms, pyramids, cylinders, cones, and spheres are all polyhedra.
- 41. Every face of a prism is also a base of the prism.
- 42. Every base of a prism is also a face of the prism.

Visualize It! Copy the partial drawing. Then complete the drawing of the solid.

43. square pyramid

44. hexagonal prism

45. cylinder



Student Help

STUDY TIP

Use a pencil when drawing solids so that you can erase hidden lines easily.

Sketching Solids Sketch the solid described.

46. rectangular prism

47. rectangular pyramid

48. cube

49. cone

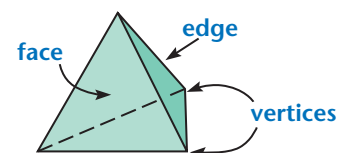
50. cylinder

51. sphere

EXAMPLE Euler's Formula

Mathematician Leonhard Euler proved that the number of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula $F + V = E + 2$.

Use Euler's Formula to find the number of vertices on the *tetrahedron* shown.

**Solution**

The tetrahedron has 4 faces and 6 edges.

$$F + V = E + 2 \quad \text{Write Euler's Formula.}$$

$$4 + V = 6 + 2 \quad \text{Substitute 4 for } F \text{ and 6 for } E.$$

$$4 + V = 8 \quad \text{Simplify.}$$

$$V = 8 - 4 \quad \text{Subtract 4 from each side.}$$

$$V = 4 \quad \text{Simplify.}$$

ANSWER ▶ The tetrahedron has 4 vertices.

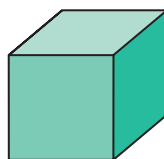
Link to
History



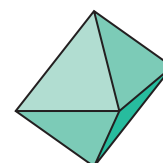
PLATO The solids on this page are called *Platonic solids*, named after the Greek mathematician and philosopher Plato. The image of Plato above is a detail of *The School of Athens* (1509–10) by Raphael.

Platonic Solids A Platonic solid has faces that are congruent, regular polygons. Use the example above to find the number of vertices on the Platonic solid.

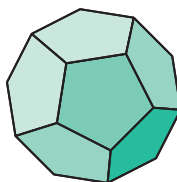
52. cube
6 faces, 12 edges



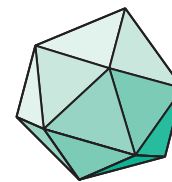
53. octahedron
8 faces, 12 edges



54. dodecahedron
12 faces, 30 edges



55. icosahedron
20 faces, 30 edges



xy **Using Algebra** Use Euler's Formula to find the number of faces, edges, or vertices. Use the example above as a model.

56. A prism has 5 faces and 6 vertices. How many edges does it have?
57. A pyramid has 12 edges and 7 vertices. How many faces does it have?
58. A prism has 8 faces and 12 vertices. How many edges does it have?

Standardized Test Practice

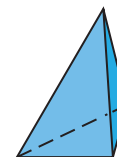
59. Multiple Choice How many faces does the prism below have?

- (A) 2 (B) 3
(C) 4 (D) 5



60. Multiple Choice How many edges does the pyramid at the right have?

- (F) 6 (G) 5
(H) 4 (J) 3



61. Multiple Choice How many vertices does the pyramid above have?

- (A) 6 (B) 5 (C) 4 (D) 3

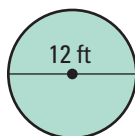
Mixed Review

Finding Measures of Squares and Rectangles Use the given information to find the missing measure. (*Lesson 8.3*)

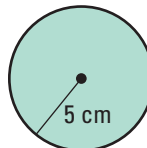
- 62.** A square has a side length of 9 centimeters. Find its area.
63. A rectangle has a height of 4 meters and a base length of 7 meters. Find its area.
64. A rectangle has an area of 60 square inches and a height of 6 inches. Find the length of its base.
65. A square has an area of 169 square feet. Find its side length.

Finding Circumference and Area of a Circle Find the circumference and the area of the circle. Round your answers to the nearest whole number. (*Lesson 8.7*)

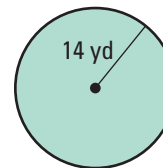
66.



67.



68.



Algebra Skills

Evaluating Expressions Evaluate the expression.

(*Skills Review, p. 670*)

- 69.** $92 - (12 + 39)$ **70.** $8 + 4 \cdot 3 - 5$ **71.** $(7 - 5) \cdot 14$
72. $10 - (5 - 2)^2 + 8$ **73.** $14 + 4^2 - 26$ **74.** $3(10 - 3)^2$

Substituting and Simplifying Expressions Evaluate the expression when $l = 3$, $h = 5$, and $w = 2$. Write your answer in terms of π , if appropriate. (*Skills Review, p. 674*)

- 75.** $l \cdot w \cdot h$ **76.** $2l + 2w + 2h$ **77.** $2\pi h$
78. $\pi w^2 h$ **79.** πl^2 **80.** $2l + 2w$