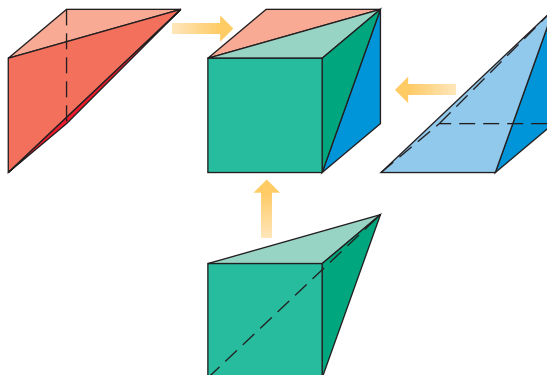# 9.5 Volume of Pyramids and Cones

Goal
Find the volumes of pyramids and cones.

Key Words
- pyramid p. 491
- cone p. 493
- volume p. 500

In the puzzle below, you can see that the square prism can be made using three congruent pyramids. The volume of each pyramid is one-third the volume of the prism.
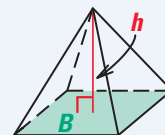
Volume Puzzle

VOLUME OF A PYRAMID

Words Volume = $\frac{1}{3}$(area of base)(height)

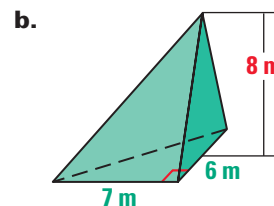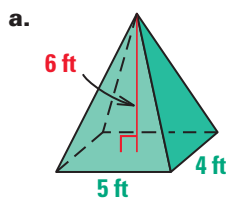**Symbols** $V = \frac{1}{3}Bh$

Student Help

STUDY TIP
In Example 1(b), 6 m is the height of the triangular base and 8 m is the height of the pyramid.

EXAMPLE 1 Find the Volume of a Pyramid

Find the volume of the pyramid.

a.
6 ft
4 ft
5 ft

b.
8 m
6 m
7 m

Solution

a. $V = \frac{1}{3}Bh$ Write the formula for volume.
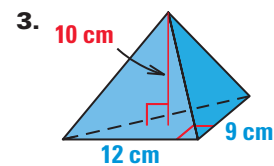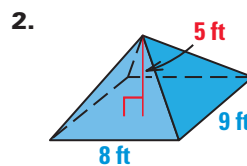
$= \frac{1}{3}(5 \cdot 4)(6)$ Substitute.

$= 40$ Simplify.
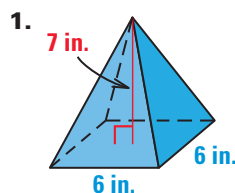
ANSWER ▶ The volume is 40 cubic feet.

b. $V = \frac{1}{3}Bh$

$= \frac{1}{3}\left(\frac{1}{2} \cdot 7 \cdot 6\right)(8)$

$= 56$

ANSWER ▶ The volume is 56 cubic meters.

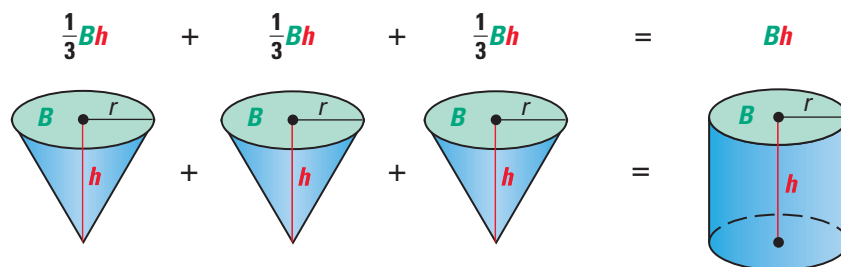**Checkpoint** ✓ *Find the Volume of a Pyramid*

Find the volume of the pyramid.

1. 7 in. / 6 in. / 6 in.

2. 5 ft / 9 ft / 8 ft

3. 10 cm / 9 cm / 12 cm

Volume of a Cone The volume of a cone is related to the volume of a cylinder in the same way that the volume of a pyramid is related to the volume of a prism.
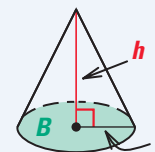
$$\frac{1}{3}Bh \quad + \quad \frac{1}{3}Bh \quad + \quad \frac{1}{3}Bh \quad = \quad Bh$$

VOLUME OF A CONE

Words Volume $= \frac{1}{3}$(**area of base**)(**height**)

Symbols $V = \frac{1}{3}Bh$

$\qquad\quad = \frac{1}{3}\pi r^2 h$

EXAMPLE 2 Find the Volume of a Cone

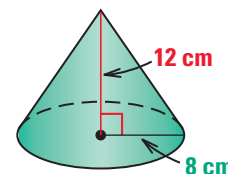Find the volume of the cone. Round your answer to the nearest whole number.

12 cm
8 cm

Solution

The radius of the cone is $r = 8$ cm.

The height of the cone is $h = 12$ cm.

$$V = \frac{1}{3}\pi r^2 h \qquad\qquad \text{Write the formula for volume of a cone.}$$

$$= \frac{1}{3}\pi (8^2)(12) \qquad \text{Substitute 8 for } r \text{ and 12 for } h.$$

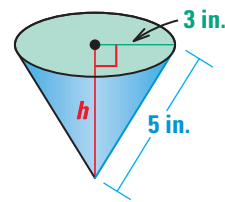$$\approx 804 \qquad\qquad\qquad \text{Multiply.}$$

ANSWER ▶ The volume is about 804 cubic centimeters.

Using Algebra

EXAMPLE **3** **Find the Volume of a Cone**

What is the volume of the cone shown at the right?

3 in.

h

5 in.

Solution

You are given the slant height of the cone. You need to find the height of the cone before you can find the volume.

❶ Find the height.

$$(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2 \qquad \text{Use the Pythagorean Theorem.}$$

$$3^2 + h^2 = 5^2 \qquad \text{Substitute.}$$

$$9 + h^2 = 25 \qquad \text{Simplify.}$$

$$h^2 = 25 - 9 \qquad \text{Subtract 9 from each side.}$$

$$h^2 = 16 \qquad \text{Simplify.}$$

$$h = \sqrt{16} \qquad \text{Take the positive square root.}$$

$$h = 4 \qquad \text{Simplify.}$$

❷ Find the volume.

$$V = \frac{1}{3}\pi r^2 h \qquad \text{Write the formula for volume.}$$

$$= \frac{1}{3}\pi(3^2)(4) \qquad \text{Substitute 3 for } r \text{ and 4 for } h.$$
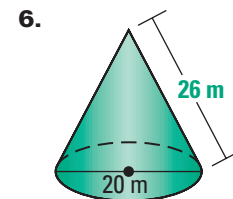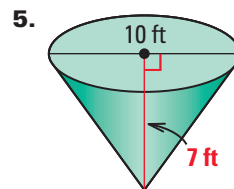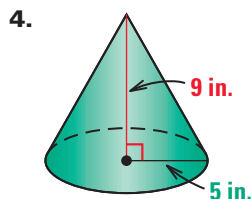
$$\approx 38 \qquad \text{Multiply.}$$

ANSWER ▶ The volume is about 38 cubic inches.

Checkpoint ✓ *Find the Volume of a Cone*

Find the volume of the cone. Round your answer to the nearest whole number.

4.

9 in.

5 in.

5.

10 ft

7 ft

6.

26 m

20 m

7. Find the volume of a cone with a height of 6 inches and a diameter of 8 inches.

8. Find the volume of a cone with a slant height of 17 feet and a diameter of 16 feet.

9.5 Exercises

Guided Practice

Vocabulary Check **Match the solid with its volume formula. Use each formula once.**

A. Pyramid   **B.** Cone   **C.** Prism   **D.** Cylinder
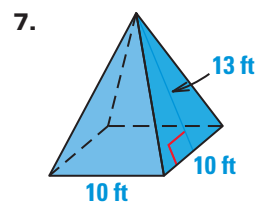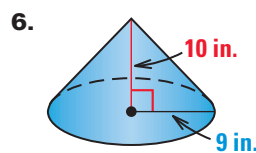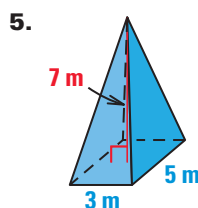
1. $V = \pi r^2 h$ **2.** $V = Bh$ **3.** $V = \frac{1}{3}\pi r^2 h$ **4.** $V = \frac{1}{3}Bh$

Skill Check **Find the volume of the solid. If necessary, round your answer to the nearest whole number.**

5.

7 m

5 m

3 m

6.
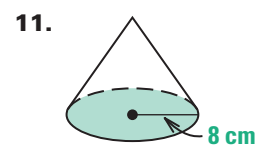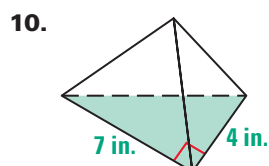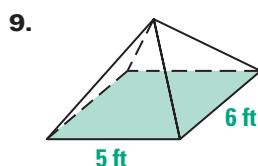
10 in.

9 in.

7.

13 ft

10 ft

10 ft

8. Find the volume of a cone with a slant height of 15 inches and a radius of 9 inches. Leave your answer in terms of π.
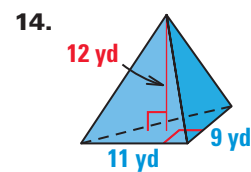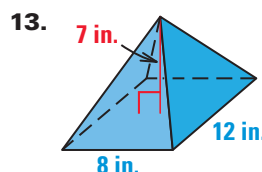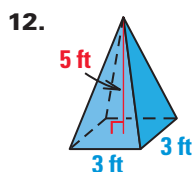
Practice and Applications

Extra Practice
See p. 692.

Find Base Areas **Find the area of the base of the solid.**

9.

6 ft

5 ft

10.

7 in.   4 in.

11.

8 cm

Volume of a Pyramid **Find the volume of the pyramid.**

12.

5 ft

3 ft

3 ft

13.

7 in.

12 in.

8 in.

14.

12 yd

9 yd

11 yd

Homework Help

Example 1: Exs. 12–16, 20–25
**Example 2:** Exs. 17–19, 21, 26
Example 3: Exs. 30–32

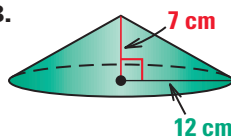**Volume of a Pyramid** **Find the volume.**

15. Find the volume of a pyramid with a base area of 48 square feet and a height of 5 feet.

16. Find the volume of a pyramid with a height of 3 inches and a square base with side lengths of 4 inches.

Volume of a Cone Find the volume of the cone. Round your answer to the nearest whole number.
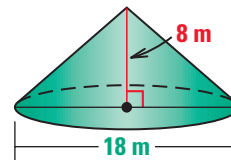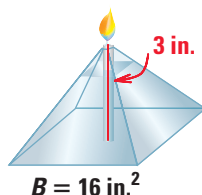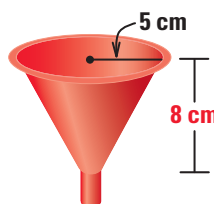
17.

10 yd

3 yd

18.

7 cm

12 cm

19.

8 m

18 m
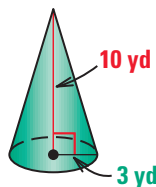
Finding Volume Find the volume of the object. Round your answer to the nearest whole number.

20.

3 in.

$B = 16$ in.^{2}
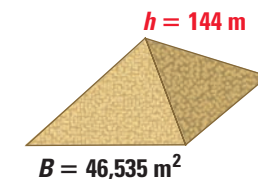
21.

5 cm

8 cm

22.
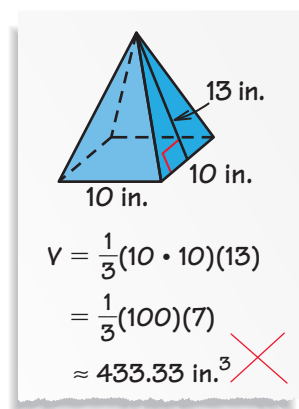
$h = 144$ m

$B = 46{,}535$ m^2

Logical Reasoning In Exercises 23 and 24, use a pyramid that has a height of 8 feet and a square base with a side length of 6 feet.
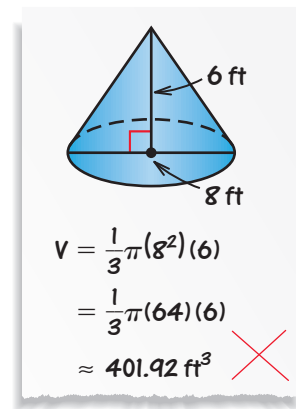
23. How does the volume of the pyramid change if the base stays the same and the height is doubled?

24. How does the volume of the pyramid change if the height stays the same and the side length of the base is doubled?

**Error Analysis** In Exercises 25 and 26, explain the student's error and correct it.

25.

13 in.

10 in.

10 in.

$$V = \frac{1}{3}(10 \cdot 10)(13)$$

$$= \frac{1}{3}(100)(7)$$

$$\approx 433.33 \text{ in.}^3 \quad \times$$

26.

6 ft

8 ft

$$V = \frac{1}{3}\pi(8^2)(6)$$

$$= \frac{1}{3}\pi(64)(6)$$

$$\approx 401.92 \text{ ft}^3 \quad \times$$

**Finding Dimensions** Find the missing dimension.

27. A pyramid has a volume of 20 cubic inches and the area of the base is 15 square inches. What is the height of the pyramid?

28. A cone has a volume of 8π cubic feet and a height of 6 feet. What is the radius of the base?

29. A pyramid with a square base has a volume of 120 cubic meters and a height of 10 meters. What is a side length of the base?

Finding Volume with Slant Height Find the volume of the solid.
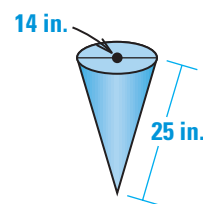If necessary, round your answer to the nearest whole number.

30.
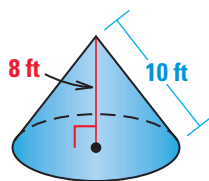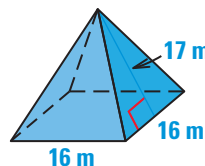
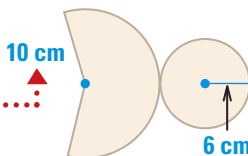8 ft   10 ft

31.

17 m

16 m

16 m

32. 14 in.

25 in.

Using Nets In Exercises 33–35, use the net to sketch the solid.
Then find the volume of the solid. Round your answer to the nearest
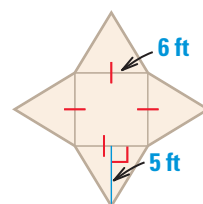whole number.

33.

10 cm

6 cm

**34.**   4 m

16 m

35.

6 ft

5 ft
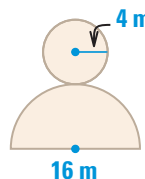
Popcorn A movie theater serves a small size of popcorn in a conical
container and a large size of popcorn in a cylindrical container.
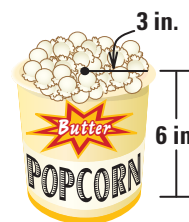
36. What is the volume of the small
container? What is the volume of
the large container?

37. How many small containers of
popcorn do you have to buy to
equal the amount of popcorn in
a large container?

38. Which container gives you
more popcorn for your money?
Explain your reasoning.
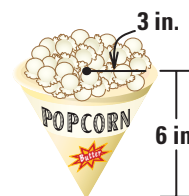
3 in.

POPCORN

6 in.

$2.00

3 in.

Butter

POPCORN

6 in.

$4.00

Pet Feeder In Exercises 39–41, use the diagram of the automatic
pet feeder.

39. Calculate the amount of food that
can be placed in the feeder. (*Hint*:
Add the volume of the cylinder and
the volume of the cone.)

40. If a cat eats 1 cup of food each day,
how much food does the cat eat
in five days? Express your answer in
cubic inches. (1 cup ≈ 14.4 in.^{3})

41. Will the feeder hold enough food
for the five days described in
Exercise 40? Explain.

2.5 in.

7.5 in.

4 in.

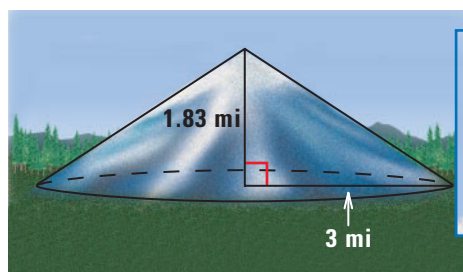**9.5** *Volume of Pyramids and Cones*   **515**
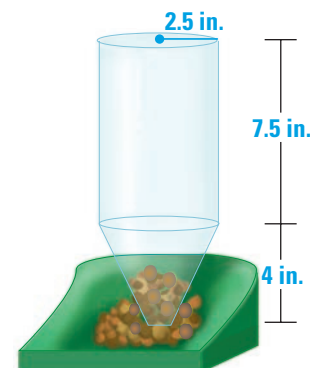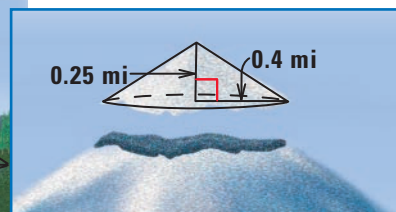
Link to
Career

GEOLOGISTS collect and interpret data about volcanoes to help predict when a volcano will erupt.

Volcanoes In Exercises 42–44, use the information below.

Before 1980, Mount St. Helens was a cone-shaped volcano.

**In 1980, Mount St. Helens erupted, destroying the cone-shaped tip.**

42. What was the volume of Mount St. Helens before 1980?

43. What was the volume of the cone-shaped tip that was destroyed?

44. What is the volume of Mount St. Helens today? (*Hint*: Subtract the volume of the tip from the volume before 1980.)

Standardized Test Practice

45. Multiple Choice A pyramid has a height of 9 yards and a volume of 96 cubic yards. Which of these are possible dimensions for its rectangular base?

   A 4 yards by 8 yards   **B** 2 yards by 8 yards

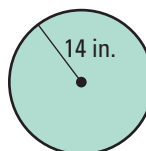   **C** 6 yards by 7 yards   **D** 5 yards by 8 yards

Mixed Review

Finding Circumference and Area Find the circumference and the area of the circle. Round your answer to the nearest whole number. (*Lesson 8.7*)
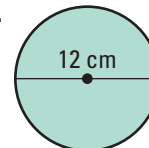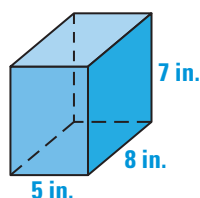
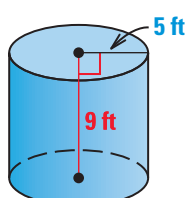**46.**   **47.**   **48.**

6 m   14 in.   12 cm
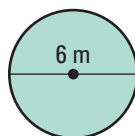
Surface Area Find the surface area of the figure. If necessary, round your answer to the nearest whole number. (*Lessons 9.2, 9.3*)
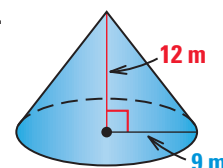
49.   **50.**   **51.**

7 in.   8 in.   5 in.   5 ft   9 ft   12 m   9 m

Algebra Skills

Slope Plot the points and draw a line through them. Then tell if the slope is *positive, negative, zero,* or *undefined*. (*Skills Review, p. 665*)

52. (0, 0) and (4, 3)   **53.** (5, 5) and (1, −2)   **54.** (−3, 2) and (6, 2)

55. (−2, −2) and (0, 4)   **56.** (−4, −1) and (−4, 7)   **57.** (−3, 5) and (4, −1)