2.3 Complementary and Supplementary Angles

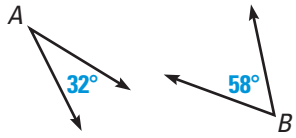
Goal

Find measures of complementary and supplementary angles.

Key Words

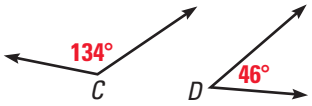
- complementary angles
- supplementary angles
- adjacent angles
- theorem

Two angles are **complementary angles** if the sum of their measures is 90°. Each angle is the **complement** of the other.



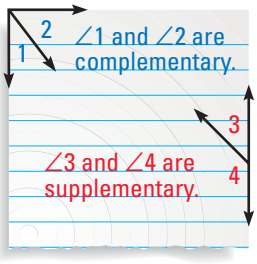
∠A and ∠B are complementary angles.
$m\angle A + m\angle B = 32° + 58° = 90°$

Two angles are **supplementary angles** if the sum of their measures is 180°. Each angle is the **supplement** of the other.



∠C and ∠D are supplementary angles.
$m\angle C + m\angle D = 134° + 46° = 180°$

Visualize It!

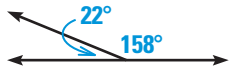


Complementary angles make up the Corner of a piece of paper. Supplementary angles make up the Side of a piece of paper.

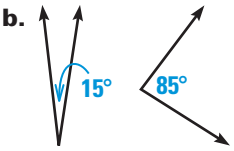
EXAMPLE 1 Identify Complements and Supplements

Determine whether the angles are *complementary*, *supplementary*, or *neither*.

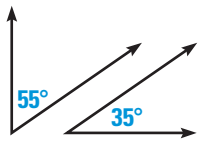
a.



b.



c.



Solution

a. Because 22° + 158° = 180°, the angles are supplementary.

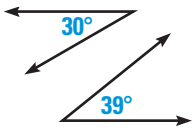
b. Because 15° + 85° = 100°, the angles are neither complementary nor supplementary.

c. Because 55° + 35° = 90°, the angles are complementary.

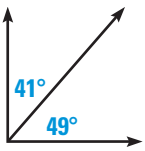
Checkpoint Identify Complements and Supplements

Determine whether the angles are *complementary*, *supplementary*, or *neither*.

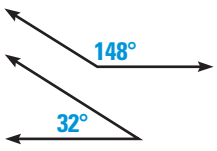
1.



2.



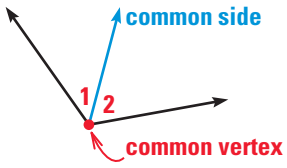
3.



Student Help

STUDY TIP
You can use numbers to refer to angles. Make sure that you do not confuse angle names with angle measures.

Two angles are **adjacent angles** if they share a common vertex and side, but have no common interior points.

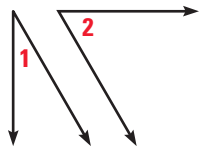


∠1 and ∠2 are adjacent angles.

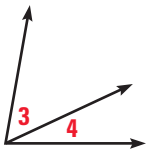
EXAMPLE 2 Identify Adjacent Angles

Tell whether the numbered angles are *adjacent* or *nonadjacent*.

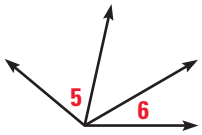
a.



b.



c.



Solution

a. Because the angles do not share a common vertex or side, ∠1 and ∠2 are nonadjacent.

b. Because the angles share a common vertex and side, and they do not have any common interior points, ∠3 and ∠4 are adjacent.

c. Although ∠5 and ∠6 share a common vertex, they do not share a common side. Therefore, ∠5 and ∠6 are nonadjacent.

EXAMPLE 3 Measures of Complements and Supplements

a. $\angle A$ is a complement of $\angle C$, and $m\angle A = 47°$. Find $m\angle C$.

b. $\angle P$ is a supplement of $\angle R$, and $m\angle R = 36°$. Find $m\angle P$.

Solution

a. $\angle A$ and $\angle C$ are complements, so their sum is 90°.

$$m\angle A + m\angle C = 90°$$
$$47° + m\angle C = 90°$$
$$47° + m\angle C - 47° = 90° - 47°$$
$$m\angle C = 43°$$

b. $\angle P$ and $\angle R$ are supplements, so their sum is 180°.

$$m\angle P + m\angle R = 180°$$
$$m\angle P + 36° = 180°$$
$$m\angle P + 36° - 36° = 180° - 36°$$
$$m\angle P = 144°$$

Checkpoint ✓ **Measures of Complements and Supplements**

4. $\angle B$ is a complement of $\angle D$, and $m\angle D = 79°$. Find $m\angle B$.

5. $\angle G$ is a supplement of $\angle H$, and $m\angle G = 115°$. Find $m\angle H$.

A **theorem** is a true statement that follows from other true statements. The two theorems that follow are about complementary and supplementary angles.

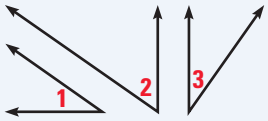
Student Help

VISUAL STRATEGY
Draw examples of these theorems with specific measures, as shown on p. 52.

THEOREMS 2.1 and 2.2

2.1 Congruent Complements Theorem

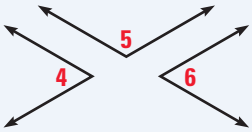
Words If two angles are complementary to the same angle, then they are congruent.



Symbols If $m\angle 1 + m\angle 2 = 90°$ and $m\angle 2 + m\angle 3 = 90°$, then $\angle 1 \cong \angle 3$.

2.2 Congruent Supplements Theorem

Words If two angles are supplementary to the same angle, then they are congruent.

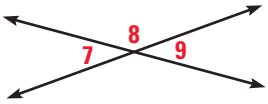


Symbols If $m\angle 4 + m\angle 5 = 180°$ and $m\angle 5 + m\angle 6 = 180°$, then $\angle 4 \cong \angle 6$.

You can use theorems in your reasoning about geometry, as shown in Example 4.

EXAMPLE 4 Use a Theorem

$\angle 7$ and $\angle 8$ are supplementary, and $\angle 8$ and $\angle 9$ are supplementary. Name a pair of congruent angles. Explain your reasoning.



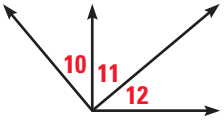
Solution

$\angle 7$ and $\angle 9$ are both supplementary to $\angle 8$. So, by the Congruent Supplements Theorem, $\angle 7 \cong \angle 9$.

Checkpoint Use a Theorem

6. In the diagram, $m\angle 10 + m\angle 11 = 90°$, and $m\angle 11 + m\angle 12 = 90°$.

Name a pair of congruent angles. Explain your reasoning.



2.3 Exercises

Guided Practice

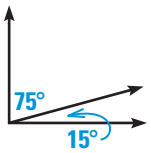
Vocabulary Check

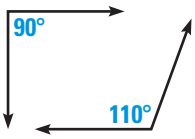
1. Explain the difference between *complementary angles* and *supplementary angles*.

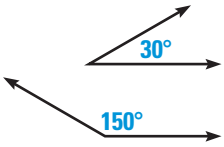
2. Complete the statement: Two angles are __?__ if they share a common vertex and a common side, but have no common interior points.

Skill Check

In Exercises 3–5, determine whether the angles are *complementary*, *supplementary*, or *neither*. Also tell whether the angles are *adjacent* or *nonadjacent*.

3. 

4. 

5. 

6. $\angle A$ is a complement of $\angle B$, and $m\angle A = 10°$. Find $m\angle B$.

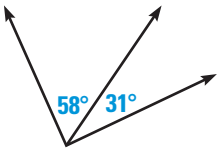
7. $\angle C$ is a supplement of $\angle D$, and $m\angle D = 109°$. Find $m\angle C$.

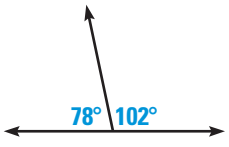
Practice and Applications

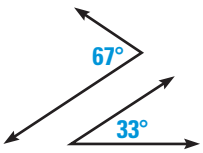
Extra Practice

See p. 677.

Identifying Angles **Determine whether the angles are *complementary*, *supplementary*, or *neither*. Also tell whether the angles are *adjacent* or *nonadjacent*.**

8. 

9. 

10. 

Identifying Angles **Determine whether the two angles shown on the clock faces are *complementary*, *supplementary*, or *neither*.**

11. 

12. 

13. 

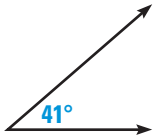
14. 

Homework Help

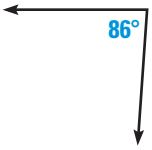
Example 1: Exs. 8–14, 30–32
Example 2: Exs. 8–10
Example 3: Exs. 15–28 33, 34
Example 4: Exs. 38–42

Finding Complements **Find the measure of a complement of the angle given.**

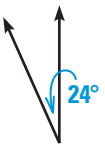
15.



16.



17.

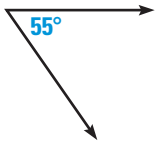


18. $\angle K$ is a complement of $\angle L$, and $m\angle K = 74°$. Find $m\angle L$.

19. $\angle P$ is a complement of $\angle Q$, and $m\angle P = 9°$. Find $m\angle Q$.

Finding Supplements **Find the measure of a supplement of the angle given.**

20.



21.

22.

23. $\angle A$ is a supplement of $\angle B$, and $m\angle A = 96°$. Find $m\angle B$.

24. $\angle P$ is a supplement of $\angle Q$, and $m\angle P = 7°$. Find $m\angle Q$.

Finding Complements and Supplements **Find the measures of a complement and a supplement of the angle.**

25. $m\angle A = 39°$ **26.** $m\angle B = 89°$ **27.** $m\angle C = 54°$

28. Bridges The Alamillo Bridge in Seville, Spain, was designed by Santiago Calatrava. In the bridge, $m\angle 1 = 58°$, and $m\angle 2 = 24°$. Find the measures of the supplements of both $\angle 1$ and $\angle 2$.

Link to Careers

ARCHITECT Santiago Calatrava, a Spanish born architect, has developed designs for bridges, train stations, stadiums, and art museums.

Career Links CLASSZONE.COM

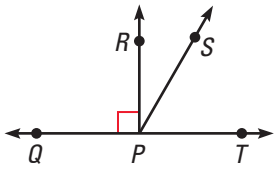
Naming Angles **In the diagram, $\angle QPR$ is a right angle.**

29. Name a straight angle.

30. Name two congruent supplementary angles.

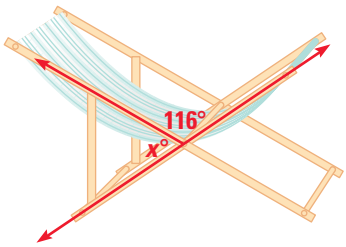
31. Name two supplementary angles that are not congruent.

32. Name two complementary angles.

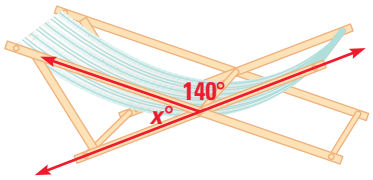


Beach Chairs **Adjustable beach chairs form angles that are supplementary. Find the value of *x*.**

33.



34.

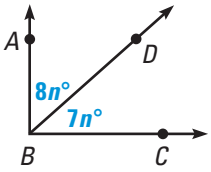


Student Help
CLASSZONE.COM

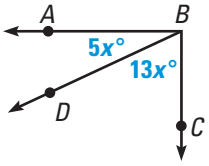
HOMEWORK HELP
Extra help with problem solving in Exs. 35–37 is at classzone.com

Using Algebra **$\angle ABD$ and $\angle DBC$ are complementary angles. Find the value of the variable.**

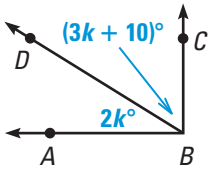
35.



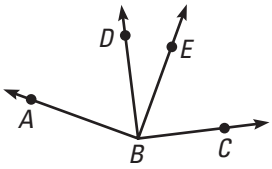
36.



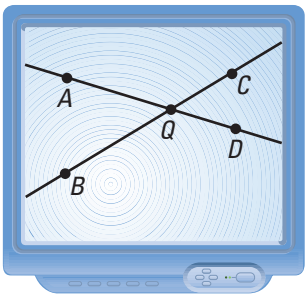
37.



38. Complementary Angles $\angle ABD$ and $\angle DBE$ are complements, and $\angle CBE$ and $\angle DBE$ are complements. Can you show that $\angle ABD \cong \angle CBE$? Explain.



39. Technology Use geometry software to draw two intersecting lines. Measure three of the four angles formed. Drag the points and observe the angle measures. What theorem does this illustrate?



Complements and Supplements **Find the angle measure described.**

40. $\angle 1$ and $\angle 2$ are both supplementary to $\angle 3$, and $m\angle 1 = 43°$. Find the measure of $\angle 2$.

41. $\angle 4$ and $\angle 6$ are both complementary to $\angle 5$, and $m\angle 5 = 85°$. Find the measure of $\angle 4$.

42. $\angle P$ is supplementary to $\angle Q$, $\angle R$ is supplementary to $\angle P$, and $m\angle Q = 60°$. Find the measure of $\angle R$.

43. Challenge $\angle C$ and $\angle D$ are supplementary angles. The measure of $\angle D$ is eight times the measure of $\angle C$. Find $m\angle C$ and $m\angle D$.

Standardized Test Practice

44. Multiple Choice What is the measure of a complement of a 27° angle?

Ⓐ 53° Ⓑ 63° Ⓒ 117° Ⓓ 163°

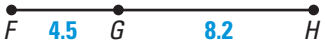
45. Multiple Choice $\angle 1$ and $\angle 2$ are supplementary. Suppose that $m\angle 1 = 60°$ and $m\angle 2 = (2x + 20)°$. What is the value of x?

Ⓕ 5 Ⓖ 10 Ⓗ 50 Ⓙ 100

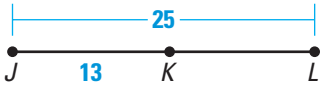
Mixed Review

Segment Addition Postulate **Find the length.** *(Lesson 1.5)*

46. Find FH.



47. Find KL.



Midpoint Formula **Find the coordinates of the midpoint of $\overline{AB}$.** *(Lesson 2.1)*

48. $A(0, 0)$, $B(8, 2)$ **49.** $A(-6, 0)$, $B(2, 4)$ **50.** $A(4, 1)$, $B(10, 3)$

51. $A(-2, 5)$, $B(-2, 7)$ **52.** $A(3, -8)$, $B(-1, 0)$ **53.** $A(-5, -9)$, $B(11, 5)$

Algebra Skills

Evaluating Decimals **Evaluate.** *(Skills Review, p. 655)*

54. $2.58 + 8.04$ **55.** $5.17 - 1.96$ **56.** 1.4×3.1

57. 0.61×0.38 **58.** $11.2 \div 1.4$ **59.** $2 \times 5.4 \times 3.9$

Quiz 1

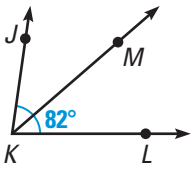
1. In the diagram, K is the midpoint of $\overline{JL}$. Find KL and JL. *(Lesson 2.1)*

Find the coordinates of the midpoint of $\overline{AB}$. *(Lesson 2.1)*

2. $A(1, 3)$, $B(7, -1)$ **3.** $A(-4, -2)$, $B(6, 4)$ **4.** $A(-5, 3)$, $B(3, -3)$

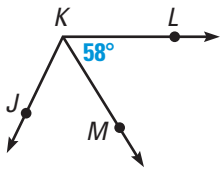
In Exercises 5–7, $\overrightarrow{KM}$ bisects $\angle JKL$. Find the angle measure. *(Lesson 2.2)*

5. Find $m\angle JKM$.



6. Find $m\angle JKL$.

7. Find $m\angle JKL$.



8. $\angle F$ is a supplement of $\angle G$, and $m\angle F = 101°$. Find $m\angle G$. *(Lesson 2.3)*

9. The measure of $\angle D$ is 83°. Find the measure of a complement and a supplement of $\angle D$. *(Lesson 2.3)*