

9.3

Surface Area of Pyramids and Cones

Goal

Find the surface areas of pyramids and cones.

Key Words

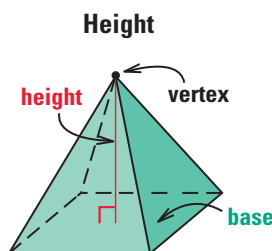
- pyramid
- height of a pyramid
- slant height of a pyramid
- cone
- height of a cone
- slant height of a cone

The Rainforest Pyramid on Galveston Island in Texas is one example of how pyramids are used in architecture.

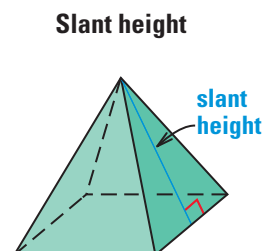
The base of a **pyramid** is a polygon and the lateral faces are triangles with a common vertex.



This greenhouse is home to plants, butterflies, and bats.

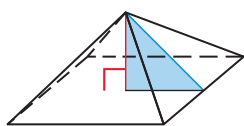


The **height of a pyramid** is the perpendicular distance between the vertex and base.



The **slant height of a pyramid**, represented by the letter l , is the height of any of its lateral faces.

Visualize It!



For a square pyramid, the **slant height** is the length of the hypotenuse of a right triangle formed by the height of the pyramid and half of the length of the base.

EXAMPLE 1 Find the Slant Height

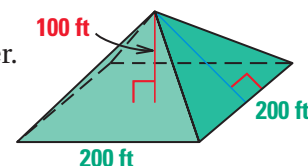
Find the slant height of the Rainforest Pyramid. Round your answer to the nearest whole number.

Solution

To find the slant height, use the right triangle formed by the height and half of the base.

$$\begin{aligned}
 (\text{slant height})^2 &= (\text{height})^2 + \left(\frac{1}{2} \text{ side}\right)^2 \\
 &= 100^2 + \left(\frac{1}{2} \cdot 200\right)^2 \\
 &= 10,000 + 10,000 \\
 \text{slant height} &= \sqrt{20,000} \\
 &\approx 141.42
 \end{aligned}$$

ANSWER ▶ The slant height is about 141 feet.



Use the Pythagorean Theorem.

Substitute 100 for height and 200 for base side length.

Simplify.

Take the positive square root.

Use a calculator.

Surface Area of a Pyramid The diagrams show the surface area of a pyramid with a square base.

$$\begin{aligned}
 \text{Surface area} &= \text{area of base} + \text{area of lateral faces} \\
 &= B + \frac{1}{2}Pl \\
 &= \text{[Diagram of a pyramid with base side } s \text{ and slant height } \ell] + \text{[Diagram of a square base } B \text{ and four triangular lateral faces with side } s \text{ and slant height } \ell] \\
 &= \text{[Diagram of a square base } B \text{ and four triangular lateral faces with side } s \text{ and slant height } \ell] + \text{[Diagram of a square base } B \text{ and four triangular lateral faces with side } s \text{ and slant height } \ell]
 \end{aligned}$$

ℓ is the height of a triangular side.

$P = 4s$

Student Help

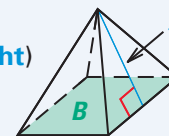
STUDY TIP

A *regular pyramid* has a regular polygon for a base. The slant height, ℓ , is the same on all of the lateral faces of a regular pyramid.

SURFACE AREA OF A PYRAMID

Words Surface area = (area of base) + $\frac{1}{2}$ (perimeter of base)(slant height)

Symbols $S = B + \frac{1}{2}Pl$



EXAMPLE 2 Find Surface Area of a Pyramid

Find the surface area of the pyramid.

Solution

- 1 Find the area of the base.

$$B = 6 \times 6 = 36$$

- 2 Find the perimeter of the base.

$$P = 6 + 6 + 6 + 6 = 24$$

- 3 Find the slant height.

$$\begin{aligned}
 (\text{slant height})^2 &= (\text{height})^2 + \left(\frac{1}{2}\text{side}\right)^2 \\
 &= 4^2 + 3^2 \\
 &= 16 + 9 \\
 &= 25
 \end{aligned}$$

$$\text{slant height} = \sqrt{25} = 5$$

Use the Pythagorean Theorem.

Substitute. Half of 6 is 3.

Simplify powers.

Simplify.

Take positive square root.

- 4 Substitute values into the formula for surface area of a pyramid.

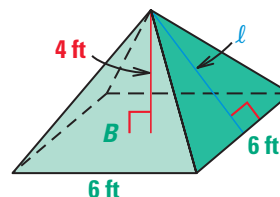
$$\begin{aligned}
 S &= B + \frac{1}{2}Pl \\
 &= 36 + \frac{1}{2}(24)(5) \\
 &= 96
 \end{aligned}$$

Write the formula for surface area.

Substitute.

Simplify.

ANSWER ▶ The surface area of the pyramid is 96 square feet.



Student Help

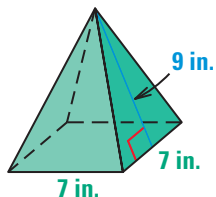
SKILLS REVIEW

For help with area and perimeter, see p. 674.

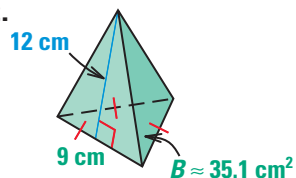
Checkpoint**Find Surface Area of a Pyramid**

Find the surface area of the pyramid.

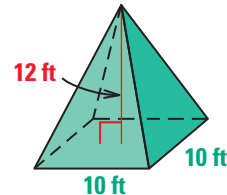
1.



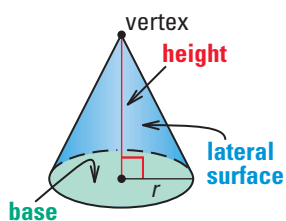
2.



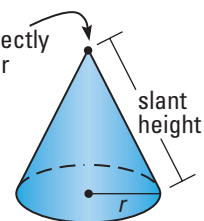
3.



Surface Area of a Cone A **cone** has a circular base and a vertex that is not in the same plane as the base. In a right cone, the height meets the base at its center. In this lesson, only right cones are shown.



The vertex is directly above the center of the circle.



The **height of a cone** is the perpendicular distance between the vertex and the base.

The **slant height of a cone** is the distance between the vertex and a point on the base edge.

Student Help**LOOK BACK**

To review how to find the area of a sector, see p. 454.

The diagrams show the surface area of a cone.

Surface area

=

area of base

+

area of sector

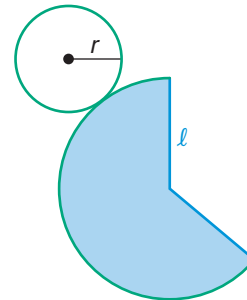
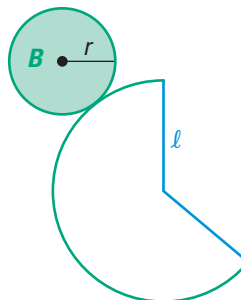
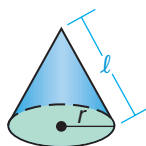
=

 B

+

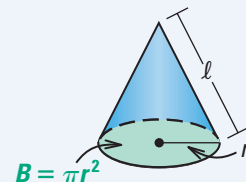
 $\pi r \ell$

=

**SURFACE AREA OF A CONE**

Words Surface area = (area of base) + (area of sector)
 = (area of base) + π (radius of base)(slant height)

Symbols $S = B + \pi r \ell$
 $= \pi r^2 + \pi r \ell$

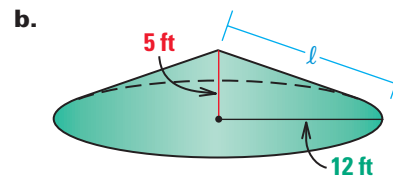
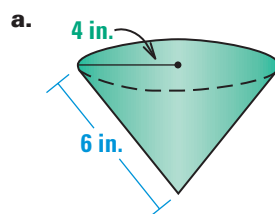


**MORE EXAMPLES**

More examples at
classzone.com

EXAMPLE 3 Find Surface Area of a Cone

Find the surface area of the cone to the nearest whole number.

**Solution**

- a. The radius of the base is 4 inches and the slant height is 6 inches.

$$S = \pi r^2 + \pi r \ell$$

Write the formula for surface area of a cone.

$$= \pi(4)^2 + \pi(4)(6)$$

Substitute 4 for r and 6 for ℓ .

$$= 40\pi$$

Simplify $16\pi + 24\pi$.

$$\approx 126$$

Multiply.

ANSWER ▶ The surface area is about 126 square inches.

- b. **First** find the slant height.

$$(\text{slant height})^2 = r^2 + h^2$$

Use the Pythagorean Theorem.

$$= (12)^2 + (5)^2$$

Substitute 12 for r and 5 for h .

$$= 169$$

Simplify $144 + 25$.

$$\text{slant height} = \sqrt{169}$$

Find the positive square root.

$$= 13$$

Simplify.

Next substitute 12 for r and 13 for ℓ in the formula for surface area.

$$S = \pi r^2 + \pi r \ell$$

Write the formula for surface area.

$$= \pi(12)^2 + \pi(12)(13)$$

Substitute.

$$= 300\pi$$

Simplify $144\pi + 156\pi$.

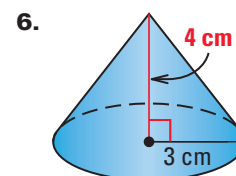
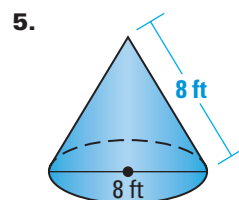
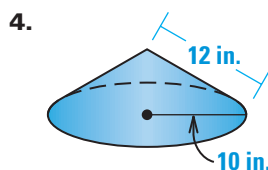
$$\approx 942$$

Multiply.

ANSWER ▶ The surface area is about 942 square feet.

**Checkpoint** Find Surface Area of Cones

Find the surface area of the cone to the nearest whole number.



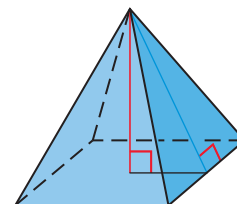
9.3 Exercises

Guided Practice

Vocabulary Check

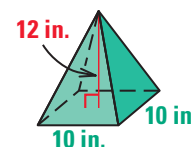
Complete the statement using *height* or *slant height*.

1. The red line segment is the ? of the pyramid.
2. The blue line segment is the ? of the pyramid.
3. The height of the lateral faces of the pyramid is the ?.
4. The ? of a pyramid is the perpendicular distance between the vertex and base.

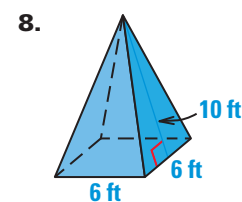
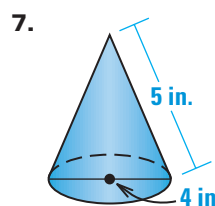
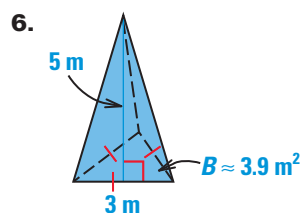


Skill Check

5. Find the slant height of the pyramid shown at the right.



Find the surface area of the solid.

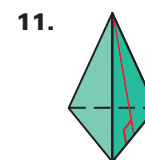
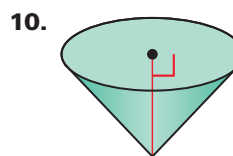


Practice and Applications

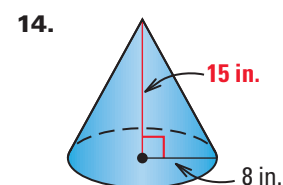
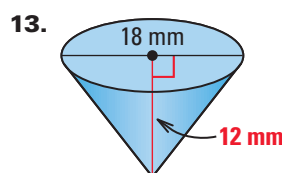
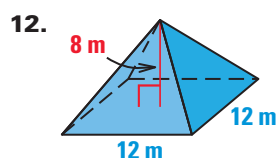
Extra Practice

See p. 691.

Recognizing Slant Height Tell whether the red line segment is the *height* or the *slant height*.



Finding Slant Height Find the slant height of the solid.



Homework Help

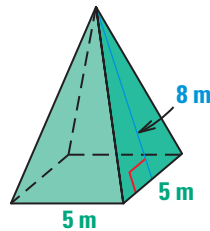
Example 1: Exs. 9–14

Example 2: Exs. 15–20

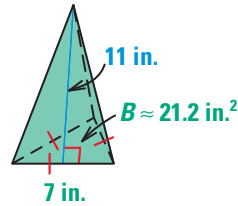
Example 3: Exs. 23–28

Surface Area of a Pyramid In Exercises 15–20, find the surface area of the pyramid.

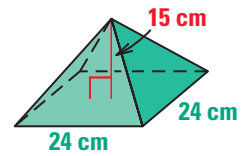
15.



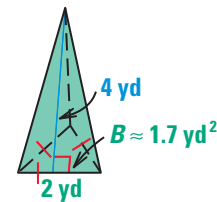
16.



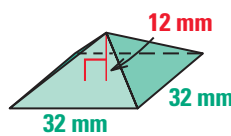
17.



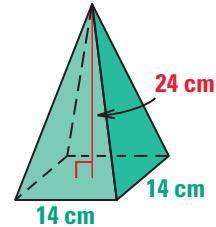
18.



19.

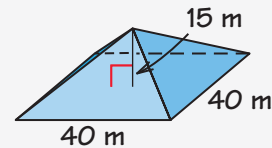


20.



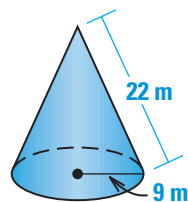
21. **Logical Reasoning** Explain why the slant height of a pyramid must be greater than the height of the pyramid.
22. **Error Analysis** Jamie is trying to find the surface area of the pyramid below. His solution is shown. What did he do wrong in his calculations?

$$\begin{aligned}
 S &= 40^2 + \frac{1}{2}(160)(15) \\
 &= 1600 + 1200 \\
 &= 2800 \text{ m}^2
 \end{aligned}$$

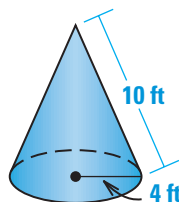


Surface Area of a Cone Find the surface area of the cone. Round your answer to the nearest whole number.

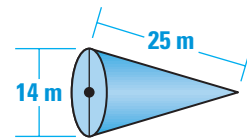
23.



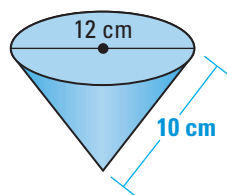
24.



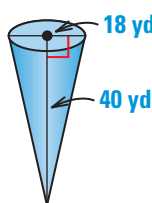
25.



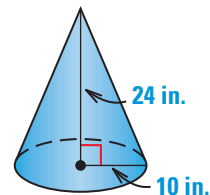
26.



27.



28.



Finding Lateral Area Find the lateral area of the object.

29.



$$P = 28 \text{ cm}$$

$$\ell = 14 \text{ cm}$$

30.



$$r = 4.3 \text{ in.}$$

$$\ell = 22.3 \text{ in.}$$

31.



$$d = 8 \text{ in.}$$

$$\ell = 14 \text{ in.}$$


Student Help
CLASSZONE.COM
HOMEWORK HELP

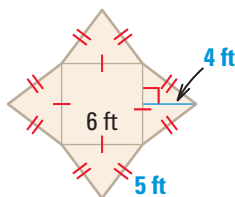
Extra help with problem solving in Exs. 32–35 is at classzone.com

Visualize It! Sketch the described solid and find its surface area. If necessary, round your answer to the nearest whole number.

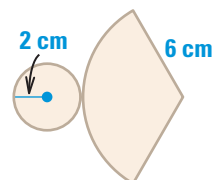
32. A pyramid has a square base with congruent edges of 12 meters and a height of 8 meters.
33. A pyramid has a triangular base with congruent edges of 8 feet and a slant height of 13 feet. The base area is 27.7 square feet.
34. A cone has a diameter of 6 yards and a slant height of 7 yards.
35. A cone has a radius of 10 inches and a height of 14 inches.

Using Nets Name the solid that can be folded from the net. Then find its surface area.

36.



37.



Link to
Careers



VETERINARIANS provide treatment for sick and injured animals. They also offer preventive care, such as vaccinations, for healthy animals.


Career Links
CLASSZONE.COM

Veterinary Medicine A cone-shaped collar, called an Elizabethan collar, is used to prevent pets from aggravating a healing wound.

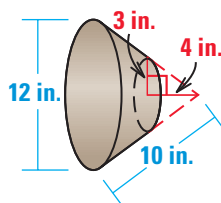


Diagram of a collar



38. Find the lateral area of the entire cone shown above.
39. Find the lateral area of the small cone that has a radius of 3 inches and a height of 4 inches.
40. Use your answers to Exercises 38 and 39 to find the amount of material needed to make the Elizabethan collar shown.

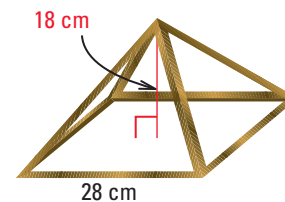
Link to Lamp Design



LAMP SHADES Many lampshades are shaped like cones or pyramids. This lamp was designed by architect and designer Frank Lloyd Wright.

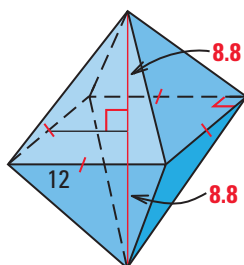
Lampshades In Exercises 41 and 42, refer to the lampshade with a square base shown at the right.

41. Use the Pythagorean Theorem to find the slant height of the lampshade. Round your answer to the nearest whole number.
42. Estimate the amount of glass needed to make the lampshade by calculating the lateral area of the pyramid.

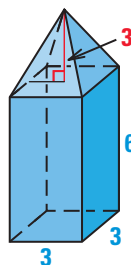


Challenge Find the surface area of the combined solids. (*Hint: Find the surface area of each solid and add them together. In each calculation, remember to omit the surface where the solids connect.*)

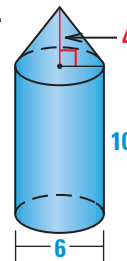
43.



44.



45.



Standardized Test Practice

46. **Multi-Step Problem** Complete parts (a)–(e).

- a. Find the surface area of each cone described in the table. Round your answer to the nearest whole number.

	Radius	Slant Height	Surface Area
Cone A	3 ft	6 ft	?
Cone B	3 ft	8 ft	?
Cone C	3 ft	10 ft	?

- b. What measurement stayed the same in the cones in part (a)?
- c. Find the surface area of each cone described in the table. Round your answer to the nearest whole number.

	Radius	Slant Height	Surface Area
Cone D	2 ft	8 ft	?
Cone E	4 ft	8 ft	?
Cone F	6 ft	8 ft	?

- d. What measurement stayed the same in the cones in part (c)?
- e. Compare the measurements you found in the two tables. Which measurement has a greater influence on surface area? Why?

Mixed Review

Evaluating Expressions Evaluate the expression for the given value of the variable. (*Skills Review, p. 670*)

47. $3x^2$ when $x = 6$

48. $x^2 + 6$ when $x = 4$

49. $2x^2 - 3$ when $x = 3$

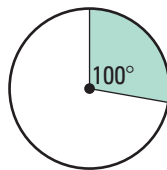
50. $4x^2 - 10$ when $x = 2$

51. $x^2 + 4x$ when $x = 6$

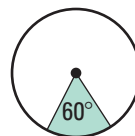
52. $3x^2 + 5x$ when $x = 5$

Finding Areas of Sectors Find the area of the green sector given the area of the circle. Round your answer to the nearest whole number. (*Lesson 8.7*)

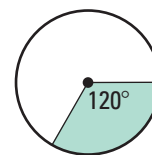
53. $A = 180 \text{ m}^2$



54. $A = 114 \text{ ft}^2$



55. $A = 258 \text{ cm}^2$



Algebra Skills

Simplifying Expressions Simplify. (*Skills Review, p. 670*)

56. $-2x + 3 + 6x$

57. $4y + 5 - 4y - 1$

58. $4 - (2x - 1) + x$

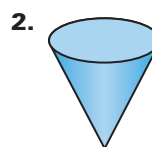
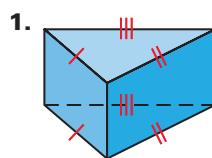
59. $7x - 2 - (3x + 2)$

60. $x - 9x + 6x$

61. $10c - (5 - 3c)$

Quiz 1

Identify the shape of the base(s) of the solid and name the solid. Then tell if the solid is a polyhedron. If so, count the number of faces of the polyhedron. (*Lesson 9.1*)



Find the surface area of the figure. If necessary, round your answer to the nearest whole number. (*Lessons 9.2, 9.3*)

