

24. D The maximum acceleration will occur when its derivative changes from positive to negative or at an endpoint of the interval.  $a(t) = v'(t) = 3t^2 - 6t + 12 = 3(t^2 - 2t + 4)$  which is always positive. Thus the acceleration is always increasing. The maximum must occur at  $t = 3$  where  $a(3) = 21$
25. D The area is given by  $\int_0^2 x^2 - (-x) dx = \left( \frac{1}{3}x^3 + \frac{1}{2}x^2 \right) \Big|_0^2 = \frac{8}{3} + 2 = \frac{14}{3}$ .
26. A Any value of  $k$  less than  $1/2$  will require the function to assume the value of  $1/2$  at least twice because of the Intermediate Value Theorem on the intervals  $[0, 1]$  and  $[1, 2]$ . Hence  $k = 0$  is the only option.
27. A  $\frac{1}{2} \int_0^2 x^2 \sqrt{x^3 + 1} dx = \frac{1}{2} \int_0^2 (x^3 + 1)^{\frac{1}{2}} \left( \frac{1}{3} \cdot 3x^2 \right) dx = \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{2}{3} (x^3 + 1)^{\frac{3}{2}} \Big|_0^2 = \frac{1}{9} \left( 9^{\frac{3}{2}} - 1^{\frac{3}{2}} \right) = \frac{26}{9}$
28. E  $f'(x) = \sec^2(2x) \cdot \frac{d}{dx}(2x) = 2 \sec^2(2x)$ ;  $f'\left(\frac{\pi}{6}\right) = 2 \sec^2\left(\frac{\pi}{3}\right) = 2(4) = 8$

## 1998 Calculus AB Solutions: Part B

76. A From the graph it is clear that  $f$  is not continuous at  $x = a$ . All others are true.
77. C Parallel tangents will occur when the slopes of  $f$  and  $g$  are equal.  $f'(x) = 6e^{2x}$  and  $g'(x) = 18x^2$ . The graphs of these derivatives reveal that they are equal only at  $x = -0.391$ .
78. B  $A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ . However,  $C = 2\pi r$  and  $\frac{dr}{dt} = -0.1$ . Thus  $\frac{dA}{dt} = -0.1C$ .
79. A The graph of the derivative would have to change from positive to negative. This is only true for the graph of  $f'$ .
80. B Look at the graph of  $f'(x)$  on the interval  $(0, 10)$  and count the number of  $x$ -intercepts in the interval.
81. D Only II is false since the graph of the absolute value function has a sharp corner at  $x = 0$ .
82. E Since  $F$  is an antiderivative of  $f$ ,  $\int_1^3 f(2x) dx = \frac{1}{2} F(2x) \Big|_1^3 = \frac{1}{2} (F(6) - F(2))$
83. B  $\lim_{x \rightarrow a} \frac{x^2 - a^2}{x^4 - a^4} = \lim_{x \rightarrow a} \frac{x^2 - a^2}{(x^2 - a^2)(x^2 + a^2)} = \lim_{x \rightarrow a} \frac{1}{x^2 + a^2} = \frac{1}{2a^2}$
84. A A known solution to this differential equation is  $y(t) = y(0)e^{kt}$ . Use the fact that the population is  $2y(0)$  when  $t = 10$ . Then  $2y(0) = y(0)e^{k(10)} \Rightarrow e^{10k} = 2 \Rightarrow k = (0.1) \ln 2 = 0.069$
85. C There are 3 trapezoids.  $\frac{1}{2} \cdot 3(f(2) + f(5)) + \frac{1}{2} \cdot 2(f(5) + f(7)) + \frac{1}{2} \cdot 1(f(7) + f(8))$
86. C Each cross section is a semicircle with a diameter of  $y$ . The volume would be given by  $\int_0^8 \frac{1}{2} \pi \left(\frac{y}{2}\right)^2 dx = \frac{\pi}{8} \int_0^8 \left(\frac{8-x}{2}\right)^2 dx = 16.755$
87. D Find the  $x$  for which  $f'(x) = 1$ .  $f'(x) = 4x^3 + 4x = 1$  only for  $x = 0.237$ . Then  $f(0.237) = 0.115$ . So the equation is  $y - 0.115 = x - 0.237$ . This is equivalent to option (D).

88. C  $F(9) - F(1) = \int_1^9 \frac{(\ln t)^3}{t} dt = 5.827$  using a calculator. Since  $F(1) = 0$ ,  $F(9) = 5.827$ .

Or solve the differential equation with an initial condition by finding an antiderivative for  $\frac{(\ln x)^3}{x}$ . This is of the form  $u^3 du$  where  $u = \ln x$ . Hence  $F(x) = \frac{1}{4}(\ln x)^4 + C$  and since  $F(1) = 0$ ,  $C = 0$ . Therefore  $F(9) = \frac{1}{4}(\ln 9)^4 = 5.827$

89. B The graph of  $y = x^2 - 4$  is a parabola that changes from positive to negative at  $x = -2$  and from negative to positive at  $x = 2$ . Since  $g$  is always negative,  $f'$  changes sign opposite to the way  $y = x^2 - 4$  does. Thus  $f$  has a relative minimum at  $x = -2$  and a relative maximum at  $x = 2$ .

90. D The area of a triangle is given by  $A = \frac{1}{2}bh$ . Taking the derivative with respect to  $t$  of both sides of the equation yields  $\frac{dA}{dt} = \frac{1}{2}\left(\frac{db}{dt} \cdot h + b \cdot \frac{dh}{dt}\right)$ . Substitute the given rates to get  $\frac{dA}{dt} = \frac{1}{2}(3h - 3b) = \frac{3}{2}(h - b)$ . The area will be decreasing whenever  $\frac{dA}{dt} < 0$ . This is true whenever  $b > h$ .

91. E I. True. Apply the Intermediate Value Theorem to each of the intervals  $[2, 5]$  and  $[5, 9]$ .

II. True. Apply the Mean Value Theorem to the interval  $[2, 9]$ .

III. True. Apply the Intermediate Value Theorem to the interval  $[2, 5]$ .

92. D  $\int_k^{\pi/2} \cos x dx = 0.1 \Rightarrow \sin\left(\frac{\pi}{2}\right) - \sin k = 0.1 \Rightarrow \sin k = 0.9$ . Therefore  $k = \sin^{-1}(0.9) = 1.120$ .