

Chapter 9 FRQ Classwork

1.

The Maclaurin series for a function f is given by $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{n} x^n = x - \frac{3}{2}x^2 + 3x^3 - \dots + \frac{(-3)^{n-1}}{n} x^n + \dots$ and converges to $f(x)$ for $|x| < R$, where R is the radius of convergence of the Maclaurin series.

- Use the ratio test to find R .
- Write the first four nonzero terms of the Maclaurin series for f' , the derivative of f . Express f' as a rational function for $|x| < R$.
- Write the first four nonzero terms of the Maclaurin series for e^x . Use the Maclaurin series for e^x to write the third-degree Taylor polynomial for $g(x) = e^x f(x)$ about $x = 0$.

2.

The Taylor series for a function f about $x = 1$ is given by $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^n}{n} (x-1)^n$ and converges to $f(x)$ for $|x-1| < R$, where R is the radius of convergence of the Taylor series.

- Find the value of R .
- Find the first three nonzero terms and the general term of the Taylor series for f' , the derivative of f , about $x = 1$.
- The Taylor series for f' about $x = 1$, found in part (b), is a geometric series. Find the function f' to which the series converges for $|x-1| < R$. Use this function to determine f for $|x-1| < R$.

3.

A function f has derivatives of all orders at $x = 0$. Let $P_n(x)$ denote the n th-degree Taylor polynomial for f about $x = 0$.

- It is known that $f(0) = -4$ and that $P_1\left(\frac{1}{2}\right) = -3$. Show that $f'(0) = 2$.
- It is known that $f''(0) = -\frac{2}{3}$ and $f'''(0) = \frac{1}{3}$. Find $P_3(x)$.
- The function h has first derivative given by $h'(x) = f(2x)$. It is known that $h(0) = 7$. Find the third-degree Taylor polynomial for h about $x = 0$.

4.

Let $f(x) = \ln(1+x^3)$.

- The Maclaurin series for $\ln(1+x)$ is $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n+1} \cdot \frac{x^n}{n} + \dots$. Use the series to write the first four nonzero terms and the general term of the Maclaurin series for f .
- The radius of convergence of the Maclaurin series for f is 1. Determine the interval of convergence. Show the work that leads to your answer.
- Write the first four nonzero terms of the Maclaurin series for $f'(t^2)$. If $g(x) = \int_0^x f'(t^2) dt$, use the first two nonzero terms of the Maclaurin series for g to approximate $g(1)$.
- The Maclaurin series for g , evaluated at $x = 1$, is a convergent alternating series with individual terms that decrease in absolute value to 0. Show that your approximation in part (c) must differ from $g(1)$ by less than $\frac{1}{5}$.