1.

\[ \frac{2}{a^6} \]

Which of the following is equivalent to the expression above?

A. \( \frac{3}{\sqrt[3]{a}} \)
B. \( \sqrt[3]{3a} \)
C. \( \frac{a}{3} \)
D. \( \frac{2}{a^6} \)

**Difficulty:** Easy

**Category:** Passport to Advanced Math / Exponents

**Strategic Advice:** A variable with a fraction exponent can be written as a radical expression by writing the numerator of the fraction as the power of the radicand and the denominator as the degree (also called the index) of the root. For example:

\[ x^{\frac{2}{3}} = \sqrt[3]{x^2} \]

**Getting to the Answer:** Start by reducing the fraction in the exponent: \( \frac{2}{6} = \frac{1}{3} \). The variable \( a \) is being raised to the \( \frac{1}{3} \) power, so rewrite the term as a radical expression with a 3 as the degree of the root and 1 as the power to which \( a \) is being raised.

\[ a^{\frac{2}{6}} = a^\frac{1}{3} = \sqrt[3]{a^1} = \sqrt[3]{a} \]

2.

A nutritionist is studying the effects of nutritional supplements on athletes. She uses the function \( P_t(a) \) to represent the results of her study, where \( a \) represents the number of athletes who participated in the study, and \( P_t \) represents the number of athletes who experienced increased performance while using the supplements over a given period of time. Which of the following lists could represent a portion of the domain for the nutritionist's function?

A. \{ \ldots, -100, -75, -50, -25, 0, 25, 50, 75, 100 \ldots \}
B. \{ -100, -75, -50, -25, 0, 25, 50, 75, 100 \}
C. \{ 0, 2.5, 5, 7.5, 10, 12.5, 15 \ldots \}
D. \{ 0, 15, 30, 45, 60, 75 \ldots \}

**Difficulty:** Medium

**Category:** Passport to Advanced Math / Functions

**Strategic Advice:** The domain of a function represents the possible values of \( x \), or the input values. In this function, \( x \) is represented by \( a \), which is the number of athletes who participated in the study.

**Getting to the Answer:** This is a real-world scenario, so you cannot simply use rules of functions to determine the domain. Because there cannot be a negative number of athletes or a fraction of an athlete, the list in (D) is the only one that could represent a portion of the function's domain.

3.

Which of the following expressions has the same value as \( \sqrt{0.25} \times \sqrt{2} \)?

A. \( \frac{\sqrt{2}}{4} \)
B. \( \frac{1}{2} \)
C. \( \frac{\sqrt{2}}{2} \)
D. \( \frac{\sqrt{2}}{2} \)
**Difficulty:** Medium

**Category:** Passport to Advanced Math / Exponents

**Strategic Advice:** Use the rules for radicals to simplify the product. Don’t actually try to find the value of each answer choice.

**Getting to the Answer:** When two radical expressions with the same degree root are multiplied, you can multiply the numbers under the radicals, leaving the product inside. The root stays the same. Writing 0.25 as \( \frac{1}{4} \) may make finding the product easier:

\[
\sqrt{0.25} \times \sqrt{2} = \frac{1}{\sqrt{4}} \times \sqrt{2} = \frac{1}{2} \times \sqrt{2} = \frac{\sqrt{2}}{2}.
\]

It’s not proper to leave a radical in the denominator (and this is not one of the answer choices), so rewrite the expression by multiplying top and bottom by \( \sqrt{2} \) to get

\[
\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}.
\]

4.

Given the polynomial \( 6x^4 + 2x^2 - 8x - c \), where \( c \) is a constant, for what value of \( c \) will \( \frac{6x^4 + 2x^2 - 8x - c}{x+2} \) have no remainder?

- \( A \) -120
- \( B \) -60
- \( C \) 60
- \( D \) 120

**Difficulty:** Hard

**Category:** Passport to Advanced Math / Exponents

**Strategic Advice:** Don’t bother with trial and error; it will take far too long. Use polynomial long division and your reasoning skills instead.

**Getting to the Answer:** Use long division to divide the two expressions. Don’t forget to fill in 0 as a placeholder for the missing \( x^3 \) term.

\[
\begin{array}{c|ccccc}
\multicolumn{1}{r|}{\phantom{6x^4}} & 6x^4 & + & 0x^3 & + & 2x^2 & - & 8x & - & c \\
\cline{2-10}
\multicolumn{1}{r|}{x+2} & 6x^4 & + & 12x^3 & & & & & & -c \\
\multicolumn{1}{r|}{\phantom{6x^4}} & -6x^4 & - & 12x^3 & & & & & & \\
\multicolumn{1}{r|}{\phantom{6x^4}} & & & & -12x^3 & + & 2x^2 & - & 8x & - & c \\
\multicolumn{1}{r|}{\phantom{6x^4}} & & & & -12x^3 & - & 24x^2 & & & & \\
\multicolumn{1}{r|}{\phantom{6x^4}} & & & & & & & 26x^2 & - & 8x & - & c \\
\multicolumn{1}{r|}{\phantom{6x^4}} & & & & & & & -26x^2 & - & 52x & & \\
\multicolumn{1}{r|}{\phantom{6x^4}} & & & & & & & & & 60x & - & c \\
\multicolumn{1}{r|}{\phantom{6x^4}} & & & & & & & & & -60x & - & 120 \\
\multicolumn{1}{r|}{\phantom{6x^4}} & & & & & & & & & -c & + & 120 \\
\end{array}
\]

To make sure there is no remainder, \( c \) would have to be 120.
Which of the following piecewise functions could have been used to generate the graph above?

A. \( g(x) = \begin{cases} \frac{3}{2}x - 4, & \text{if } x < 0 \\ \sqrt{x - 1}, & \text{if } x \geq 0 \end{cases} \)

B. \( g(x) = \begin{cases} \frac{3}{2}x - 4, & \text{if } x < 0 \\ \sqrt{x - 1}, & \text{if } x \geq 0 \end{cases} \)

C. \( g(x) = \begin{cases} \frac{3}{2}x - 4, & \text{if } x < 0 \\ \sqrt{x + 1}, & \text{if } x > 0 \end{cases} \)

D. \( g(x) = \begin{cases} \frac{2}{3}x - 4, & \text{if } x < 0 \\ \sqrt{x + 1}, & \text{if } x \geq 0 \end{cases} \)

Difficulty: Hard

Category: Passport to Advanced Math / Functions

Strategic Advice: Graphing piecewise functions can be tricky. Try describing the graph in words first and then find the matching function. Use words like "to the left of" (which translates as less than) and "to the right of" (which translates as greater than).

Getting to the Answer: First, notice that both pieces of the graph either start or stop at 0, but one has a closed dot and the other has an open dot. This means you can eliminate C right away because the inequality symbol in both equations would lead to open dots on the graph. To choose among the remaining answers, think about parent functions and transformations. To the left of \( x = 0 \), the graph is a line with a slope of \(-\frac{3}{2}\) and a \( y \)-intercept of \(-4\), so you can eliminate D because the slope of the line is incorrect. Now, look to the right of \( x = 0 \)—the graph is a square root function that has been moved down 1 unit, so its equation is \( y = \sqrt{x} - 1 \). This means (A) is correct. (The square root portion of C would have been moved to the left 1 unit rather than down 1.)

6.

\[ 18 - \frac{(3x)^2}{2} = 15 \]

What value of \( x \) satisfies the equation above?
7. 

If \( g(x) = 2x^3 - 5x^2 + 4x + 6 \), and \( P \) is the point on the graph of \( g(x) \) that has an \( x \)-coordinate of 1, then what is the \( y \)-coordinate of the corresponding point on the graph of \( g(x - 3) + 4 \)?

**Difficulty:** Hard

**Category:** Passport to Advanced Math / Functions

**Strategic Advice:** This question is, for the most part, conceptual. Start by finding the \( y \)-coordinate of \( P \) in the original equation. Then, perform the transformation on the coordinates (rather than the function) to save yourself valuable time.

**Getting to the Answer:** Substitute 1 for \( x \) in the original equation. Graphically, the resulting value of \( g(1) \) is the \( y \)-coordinate of the point.

\[
g(x) = 2x^3 - 5x^2 + 4x + 6 \\
g(1) = 2(1)^3 - 5(1)^2 + 4(1) + 6 \\
= 2 - 5 + 4 + 6 \\
= 7
\]

The point on the graph of \( g(x) \) is \((1, 7)\). Now, the question asks for the \( y \)-coordinate of the corresponding point on the transformed graph. When performing transformations, the operations grouped with the \( x \) are performed on the \( x \)-coordinate, and the operations not grouped with the \( x \) are performed on the \( y \)-coordinate. So, add 4 to 7 to find that the \( y \)-coordinate of the point on the transformed graph is 11.

8. **Calculator**

\[
\frac{-9}{2} x^{10} - \frac{3}{2} x^{9} + \frac{15}{2} x^{8}
\]

Which of the following is equivalent to the expression above?

A. \( \frac{-3}{2} x^8(3x^2 + x - 5) \)

B. \( -\frac{1}{2} x^8(9x^2 + 3x - 5) \)

C. \( \frac{3}{2} x^8(-3x^2 + x + 5) \)

D. \( 3x^8(-3x^2 - x + 5) \)
9. Calculator

The graph above shows a delivery truck's distance from the company's warehouse over a two-hour period, during which time the delivery people made two deliveries and then returned to the warehouse. Based on the graph, which of the following statements could be true?

- Each delivery took 30 minutes to complete, not including driving time.
- The location of the second delivery was about 70 miles from the warehouse.
- The truck traveled about 18 miles from the time it left the warehouse until it returned.
- The second delivery was about 18 miles farther from the warehouse than the first delivery.

**Difficulty:** Medium

**Category:** Passport to Advanced Math / Functions

**Strategic Advice:** Pay careful attention to the axis labels as you read the answer choices. Time is graphed on the x-axis, and distance is graphed on the y-axis.

**Getting to the Answer:** Compare each answer choice to the graph, eliminating false statements as you go.

Choice (A): The truck is stopped when it is making a delivery. This means its distance is not changing, and the graph should be flat. Both flat sections of the graph span 30 minutes (20 to 50 and 70 to 100), so each delivery took 30 minutes. Choice (A) is correct. If you're confident in your answer, move on to the next question. If not, you can quickly check the other answer choices to be sure.

Choice (B): The second delivery starts at (70, 18), which means it was about 18 miles away from the warehouse, not 70.

Choice (C): When the truck arrived at the first delivery, it was about 8 miles from the warehouse, and when it was at the second delivery, it was about 18 miles from the warehouse. Then, it had to travel 18 miles back to the warehouse, so it traveled a total of 36 miles, not 18.

Choice (D): The second delivery took place 18 miles from the warehouse, and the first delivery took place 8 miles from the warehouse, which means the second delivery was about 10 miles farther from the warehouse, not 18.
10. Calculator

\[
\left(5x^4 - \frac{1}{4}x^3 + 3x\right) + \frac{1}{2}x
\]

What is the result of dividing the two expressions above?

A. \[\frac{5}{2}x^3 - \frac{1}{8}x^2 + \frac{3}{2}\]

B. \[\frac{5}{2}x^3 - 2x^2 + \frac{3}{2}x\]

C. \[10x^3 - \frac{1}{2}x^2 + 6\]

D. \[10x^3 - \frac{1}{8}x^2 + 6x\]

**Difficulty:** Medium

**Category:** Passport to Advanced Math / Exponents

**Strategic Advice:** Division and factoring are interchangeable, so think of factoring out the \(x\). Then, instead of dividing by \(\frac{1}{2}\), you can multiply by its reciprocal, 2. Using these two strategies will make solving a question like this considerably easier.

**Getting to the Answer:** First, divide (factor) out the \(x\) by subtracting 1 from each exponent: The result is \(5x^4 - \frac{1}{4}x^3 + 3x\) + \(\frac{1}{2}x = 5x^3 - \frac{1}{4}x^2 + 3\). Now, multiply each term by 2 to get this:

\[
5x^3 - \frac{1}{4}x^2 + 3 + \frac{1}{2} = 2\left(5x^3 - \frac{1}{4}x^2 + 3\right)
\]

\[
= 10x^3 - \frac{1}{2}x^2 + 6
\]