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Derivative Rules – Implicit, Ln, e
AP Readiness Session 2

Answers to examples posted on my website

Derivative Rules	**Examples**: For each function, find $f'(x)$ or $\frac{dy}{dx}$
Implicit Differentiation: Differentiate each variable independently with respect to x. Every derivative of y gets multiplied by $\frac{dy}{dx}$ Group all terms with $\frac{dy}{dx}$ on one side with all other terms on the other side. Factor out $\frac{dy}{dx}$ and divide by the result	$x^2y^2-2x=4-4y$ Find $\frac{dy}{dx}$ $x^2(2y)\left(\frac{dy}{dx}\right)+y^2(2x)-2=-4\left(\frac{dy}{dx}\right)$ $x^2(2y)\left(\frac{dy}{dx}\right)+4\left(\frac{dy}{dx}\right)=2-y^2(2x)$ $\left(\frac{dy}{dx}\right)(x^2(2y)+4)=2-y^2(2x)$ $\frac{dy}{dx}=\frac{2-y^2(2x)}{x^2(2y)+4}=\frac{2-2xy^2}{2x^2y+4}=\frac{2(1-xy^2)}{2(x^2y+2)}=\frac{1-xy^2}{x^2y+2}$
Derivative of natural log: $\frac{d}{dx}(\ln(u))=\frac{1}{u}\frac{du}{dx}$ (remember the chain rule)	$f(x)=\ln(3x^2-5x+8)$ Find $f'(x)$ $f'(x)=\frac{6x-5}{3x^2-5x+8}$
Derivative of e^x: $\frac{d}{dx}(e^u)=e^u\frac{du}{dx}$ Remember to use product or quotient rules if needed	$f(x)=(e^{3x})(\cos(2x))$ Find $f'(x)$ $f'(x)=(e^{3x})(-\sin(2x)(2)+\cos(2x)(e^{3x})(3)$ $f'(x)=(e^{3x})(-2\sin(2x))+3\cos(2x)(e^{3x})$ $f'(x)=e^{3x}[-2\sin(2x))+3\cos(2x)]$
Derivative of log with other bases and exponential function with other bases $\frac{d}{dx}(\log_b(u))=\frac{1}{u}\left(\frac{1}{\ln b}\right)\frac{du}{dx}$ $\frac{d}{dx}(b^u)=b^u(\ln b)\frac{du}{dx}$	$f(x)=\log_3(\tan x)$ $f'(x)=\left(\frac{1}{\tan x}\right)\left(\frac{1}{\ln 3}\right)(\sec^2 x)$ $f(x)=\frac{5^x}{x^2}$ $f'(x)=\frac{(x^2)(5^x)(\ln 5)-5^x(2x)}{(x^2)^2}=\frac{(x)(5^x)[x\ln 5-2)]}{x^4}$ $f'(x)=\frac{(5^x)[x\ln 5-2)]}{x^3}$

Using Log Rules to simplify derivatives:

$$\log(ab) = \log a + \log b$$

$$\log\left(\frac{a}{b}\right) = \log a - \log b$$

$$\log\left(a^b\right) = b\log a$$

Logarithmic Differentiation:

1. Take Ln of both sides
2. Simplify using ln rules
3. Differentiate implicitly
   $\ln y$ becomes $\frac{1}{y}\frac{dy}{dx}$
4. Multiply both sides by y to get $\frac{dy}{dx}$

$$f(x) = \ln\left(\frac{(x^4 - 3x^3 + 2)\sqrt{3x^2 - 2x}}{(2x-3)(\sin x)}\right)$$ Find $f'(x)$

$$f(x) = \ln(x^4 - 3x^3 + 2) + \frac{1}{2}\ln(3x^2 - 2x) - \ln(2x-3) - \ln(\sin x)$$

$$f'(x) = \frac{4x^3 - 9x^2}{x^4 - 3x^3 + 2} + \frac{6x-2}{2(3x^2 - 2x)} - \frac{2}{2x-3} - \frac{\cos x}{\sin x}$$

$$f'(x) = \frac{4x^3 - 9x^2}{x^4 - 3x^3 + 2} + \frac{3x-1}{3x^2 - 2x} - \frac{2}{2x-3} - \cot x$$

$$y = (\tan x)^{x^2}$$ find $\frac{dy}{dx}$

$$\ln y = x^2 \ln(\tan x)$$

$$\frac{1}{y}\frac{dy}{dx} = x^2\left(\frac{\sec^2 x}{\tan x}\right) + \ln(\tan x)(2x)$$

$$\frac{dy}{dx} = \left(x^2\left(\frac{\sec^2 x}{\tan x}\right) + \ln(\tan x)(2x)\right)(y)$$

$$\frac{dy}{dx} = \left(x^2\left(\frac{\sec^2 x}{\tan x}\right) + \ln(\tan x)(2x)\right)(\tan x)^{x^2}$$