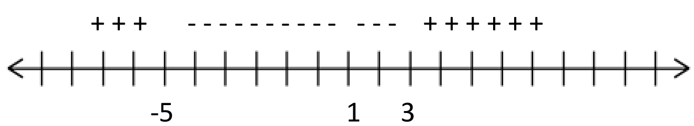
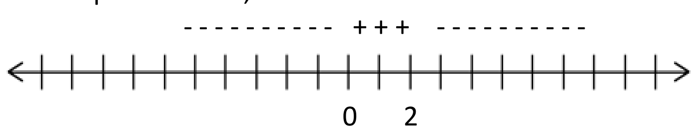
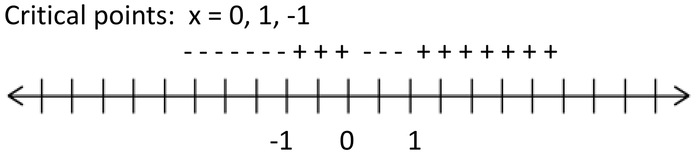
Alan Tupaj Vista Murrieta High School Website: www.vmhs.net (Click on “Teachers” then “Alan Tupaj”)	Relative Extrema AP Readiness Session 4 Answers to examples posted on my website

Critical Points: $f'(x) = 0$ or $f'(x)$ is undefined

Relative Minimum point: Critical point with a sign change from negative to positive

Relative Maximum point: Critical point with a sign change from positive to negative

Find the x-coordinate of each critical point
Classify each as a relative maximum, relative minimum, or neither.

Relative Extrema Question Type	**Examples**
1. Given derivative in factored form The sign does not change at double roots (roots from squared factors)	1. $f'(x) = (x-1)^2(x-3)(x+5)$ Critical points: x = -5, 1, 2  x = -5: rel max, x = 1: neither, x = 3: rel min
2. Polynomial with factorable derivative A leading coefficient that is negative causes large values of x to have negative derivative values.	2. $f(x) = -2x^3 + 6x^2 - 3$ $f'(x) = -6x^2 + 12x = 0$ $-6x(x-2) = 0$ Critical points: x = 0, 2  x = 0: rel min, x = 2: rel max
3. Polynomial with fractional exponents Factor out the term with the lowest exponent value.	3. $f(x) = x^{\frac{8}{3}} - 4x^{\frac{2}{3}}$ $f'(x) = \frac{8}{3}x^{\frac{5}{3}} - \frac{8}{3}x^{\frac{-1}{3}} = 0$ $\frac{8}{3}x^{\frac{-1}{3}}(x^2-1) = 0$ $\frac{8}{3}x^{\frac{-1}{3}}(x+1)(x-1) = 0$ Critical points: x = 0, 1, -1  x = -1: rel min, x = 0: rel max, x = 1: rel min

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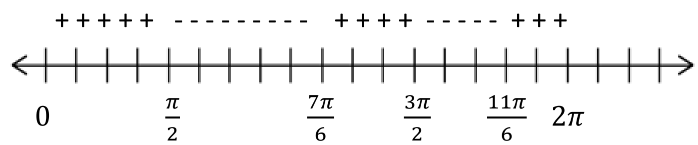
<td>4. Trigonometric functions</td>
<td>4. $f(x) = sin^2x + sinx$ x = [0, 2π] $f'(x) = 2sinxcosx + cosx = 0$

$cosx(2sinx + 1) = 0$ $cosx = 0,\ sinx = \frac{-1}{2}$

Critical points: x = $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$


x = $\frac{\pi}{2}$: rel max, x = $\frac{7\pi}{6}$: rel min, x = $\frac{3\pi}{2}$: rel max, x = $\frac{11\pi}{6}$: rel min</td>
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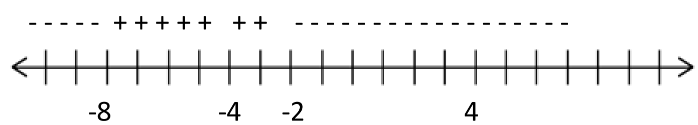
<td>5. Rational functions

Critical points from the denominator are always squared and do not change sign.</td>
<td>5.
$f(x) = \frac{x+5}{x^2-16}$ $f'(x) = \frac{(x^2-16)(1)-(x+5)(2x)}{(x^2-16)^2} = 0$

$\frac{x^2-16-2x^2-10x}{((x+4)(x-4))^2} = 0$ $\frac{-x^2-10x-16}{((x+4)(x-4))^2} = 0$

$\frac{-(x+2)(x+8)}{((x+4)(x-4))^2} = 0$ Critical points: x = -8, -4, -2, 4


x = -8: rel min, x = -4: neither, x = -2: rel max, x = 4: neither</td>
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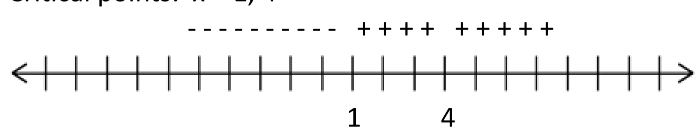
<td>6. Functions with expressions to higher powers.

Factor out the entire expression before simplifying</td>
<td>6. $f(x) = x(x-4)^3$ $f'(x) = x(3)(x-4)^2 + (x-4)^3(1) = 0$

$(x-4)^2(3x + x - 4) = 0$

$(x-4)^2(4x-4) = 0$
Critical points: x = 1, 4

x = 1: rel min, x = 4: neither</td>
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<td>7. Absolute maximum and minimum values

• Find all critical points.
• Substitute all critical points in the given interval and the endpoints into the original function and compare function values.
• Determine the maximum and minimum values.</td>
<td>7. $f(x) = x^4 - 8x^2 + 2$ $f'(x) = 4x^3 - 16x = 0$

Find the absolute maximum and minimum values for $f(x)$ on the interval [-3, 1].

$f'(x) = 4x(x^2-4) = 0$ $f'(x) = 4x(x-2)(x+2) = 0$

Critical points: x = -2, 0, 2 endpoints: -3, 1 (x = 2 not on interval)

$f(-3) = 11,\ f(-2) = -14,\ f(0) = 2,\ f(1) = -5$

Absolute Maximum Value = 11, Absolute Minimum Value = -14</td>
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