

Continuity

Warm-up: Are the following functions continuous?

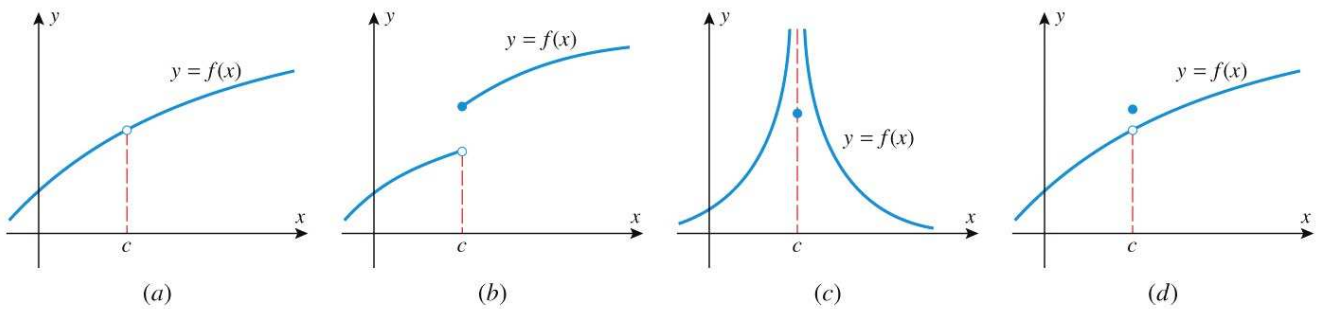


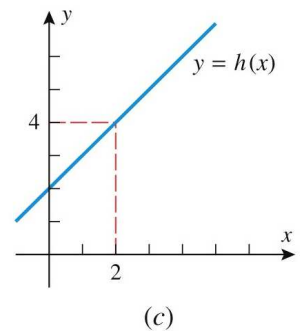
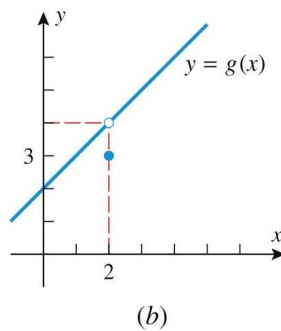
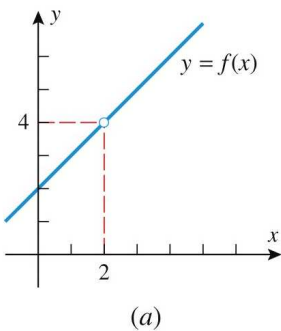
Figure 1.5.1
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Example 1: Determine if the following functions are continuous.

a) $f(x) = \frac{x^2 - 4}{x - 2}$

b) $g(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x \neq 2 \\ 3, & x = 2 \end{cases}$

c) $h(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x \neq 2 \\ 4, & x = 2 \end{cases}$



Definition of Continuity

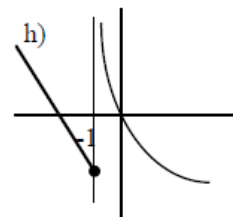
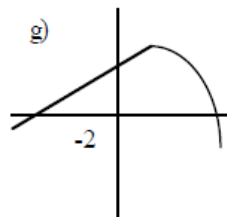
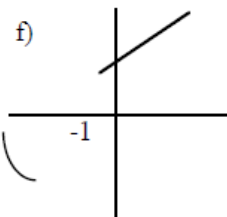
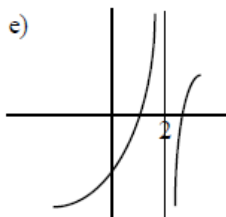
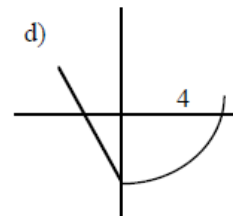
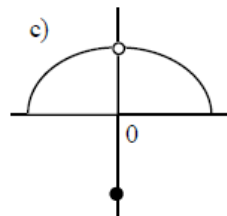
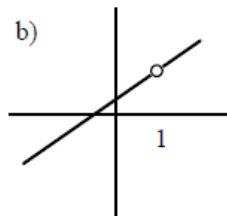
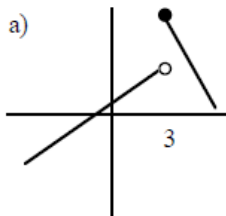
A function is continuous at $x = c$ if:

1. _____
2. _____
3. _____

Continuity

Example 2: Apply the definition of continuity to Example 1 and confirm your answers.

Practice Problem 1: Determine if each function is continuous. If it is not, give the condition of the definition of continuity that is broken.



Facts About Continuity

1. _____
2. _____

3. _____

4. _____

Continuity

Example 3: Find any points of discontinuity of the following functions.

a) $f(x) = -3x^2 - 5x + 1$ b) $f(x) = \frac{x-2}{x^2-4}$ c) $f(x) = \sqrt[3]{x^2+2x-1}$ d) $f(x) = \sqrt{x^2-x-6}$

Example 4: Use the definition of continuity to determine if the following functions are continuous.

a) $f(x) = \begin{cases} x^2 - 3, & x \geq 1 \\ 1 - x, & x < 1 \end{cases}$ b) $f(x) = \begin{cases} x^2 + 3x - 2, & x \geq -2 \\ -x^2, & x < -2 \end{cases}$ c) $f(x) = \begin{cases} \frac{x-4}{x^2-16}, & x \neq 4 \\ \frac{1}{3x-4}, & x = 4 \end{cases}$

Example 5: Find the value of the constant k that will make the function continuous.

a) $f(x) = \begin{cases} 3x + 2, & x \geq 1 \\ 2k - x, & x < 1 \end{cases}$ b) $f(x) = \begin{cases} kx^2, & x \geq 2 \\ kx - 6, & x < 2 \end{cases}$ c) $f(x) = \begin{cases} k^2 - 12x, & x \geq 1 \\ kx, & x < 1 \end{cases}$