11.3 Arcs and Central Angles

Goal

Use properties of arcs of circles.

Key Words

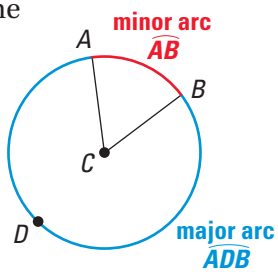
- minor arc
- major arc
- semicircle
- congruent circles
- congruent arcs
- arc length

Any two points A and B on a circle C determine a *minor arc* and a *major arc* (unless the points lie on a diameter).

If the measure of $\angle ACB$ is less than 180°, then A, B, and all the points on $\odot C$ that lie in the interior of $\angle ACB$ form a **minor arc**.

Points A, B, and all the points on $\odot C$ that do not lie on $\overset{\frown}{AB}$ form a **major arc**.

You name an arc by its endpoints. Use one other point on a major arc as part of its name to distinguish it from the minor arc.



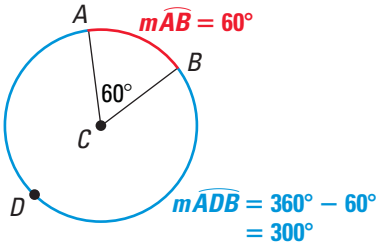
The *measures* of a minor arc and a major arc depend on the central angle of the minor arc.

Student Help

LOOK BACK
For the definition of a central angle, see p. 454.

The **measure of a minor arc** is the measure of its central angle.

The **measure of a major arc** is the difference of 360° and the measure of the related minor arc.

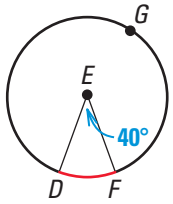


A **semicircle** is an arc whose central angle measures 180°. A semicircle is named by three points. Its measure is 180°.

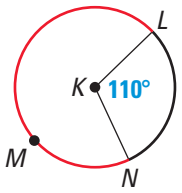
EXAMPLE 1 Name and Find Measures of Arcs

Name the red arc and identify the type of arc. Then find its measure.

a.



b.

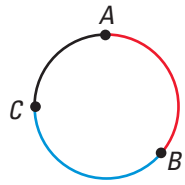


Solution

a. $\overset{\frown}{DF}$ is a minor arc. Its measure is 40°.

b. $\overset{\frown}{LMN}$ is a major arc. Its measure is $360° - 110° = 250°$.

Visualize It!



Arcs of a circle are *adjacent* if they intersect *only* at their endpoints.

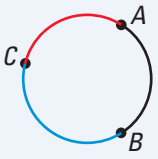
$\overset{\frown}{AB}$ and $\overset{\frown}{BC}$ are adjacent.

POSTULATE 16

Arc Addition Postulate

Words The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

Symbols $m\overset{\frown}{ACB} = m\overset{\frown}{AC} + m\overset{\frown}{CB}$

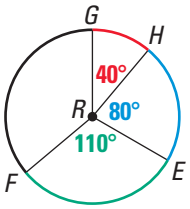


EXAMPLE 2 Find Measures of Arcs

Find the measure of $\overset{\frown}{GEF}$.

Solution

$$m\overset{\frown}{GEF} = m\overset{\frown}{GH} + m\overset{\frown}{HE} + m\overset{\frown}{EF}$$
$$= 40° + 80° + 110°$$
$$= 230°$$

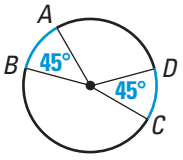


Two circles are **congruent circles** if they have the same radius. Two arcs of the same circle or of congruent circles are **congruent arcs** if they have the same measure.

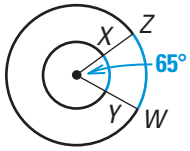
EXAMPLE 3 Identify Congruent Arcs

Find the measures of the blue arcs. Are the arcs congruent?

a.



b.



Solution

a. Notice that $\overset{\frown}{AB}$ and $\overset{\frown}{DC}$ are in the same circle. Because $m\overset{\frown}{AB} = m\overset{\frown}{DC} = 45°$, $\overset{\frown}{AB} \cong \overset{\frown}{DC}$.

b. Notice that $\overset{\frown}{XY}$ and $\overset{\frown}{ZW}$ are not in the same circle or in congruent circles. Therefore, although $m\overset{\frown}{XY} = m\overset{\frown}{ZW} = 65°$, $\overset{\frown}{XY} \ncong \overset{\frown}{ZW}$.

Checkpoint Identify Congruent Arcs

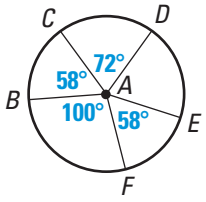
Find the measures of the arcs. Are the arcs congruent?

1. $\overset{\frown}{BC}$ and $\overset{\frown}{EF}$

2. $\overset{\frown}{BC}$ and $\overset{\frown}{CD}$

3. $\overset{\frown}{CD}$ and $\overset{\frown}{DE}$

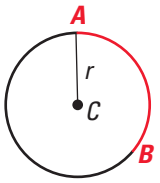
4. $\overset{\frown}{BFE}$ and $\overset{\frown}{CBF}$



Student Help

SKILLS REVIEW
To review finding circumference of a circle, see p. 674.

Arc Length An **arc length** is a portion of the circumference of a circle. You can write a proportion to find *arc length*.



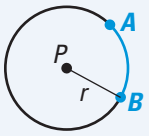
$$\frac{\text{arc length of } \overset{\frown}{AB}}{2\pi r} = \frac{m\overset{\frown}{AB}}{360^\circ}$$

arc length — central angle — full circumference — full circle

ARC LENGTH

Words In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.

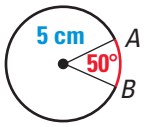
Symbols Arc length of $\overset{\frown}{AB} = \frac{m\overset{\frown}{AB}}{360^\circ} \cdot 2\pi r$



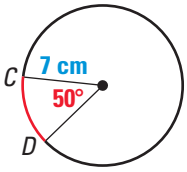
EXAMPLE 4 Find Arc Lengths

Find the length of the red arc.

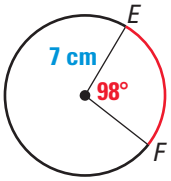
a.



b.



c.



Student Help

STUDY TIP
You can substitute 3.14 as an approximation of π or use a calculator.

Solution

a. Arc length of $\overset{\frown}{AB} = \frac{50^\circ}{360^\circ} \cdot 2\pi(5) \approx 4.36$ centimeters

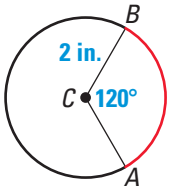
b. Arc length of $\overset{\frown}{CD} = \frac{50^\circ}{360^\circ} \cdot 2\pi(7) \approx 6.11$ centimeters

c. Arc length of $\overset{\frown}{EF} = \frac{98^\circ}{360^\circ} \cdot 2\pi(7) \approx 11.97$ centimeters

Checkpoint Find Arc Lengths

Find the length of the red arc. Round your answer to the nearest hundredth.

5.



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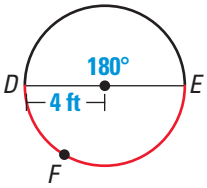

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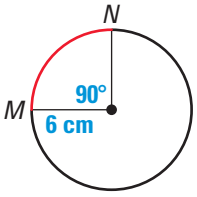
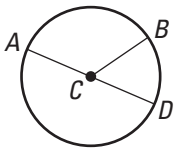


11.3 Exercises

Guided Practice

Vocabulary Check

1. In the diagram at the right, identify a *major arc*, a *minor arc*, and a *semicircle*.



2. Draw a circle with a pair of congruent arcs.

3. What is the difference between *arc measure* and *arc length*?

Skill Check

Find the measure in $\odot T$.

4. $m\overset{\frown}{RS}$

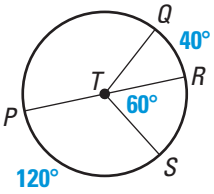
5. $m\overset{\frown}{RPS}$

6. $m\overset{\frown}{PQR}$

7. $m\overset{\frown}{QS}$

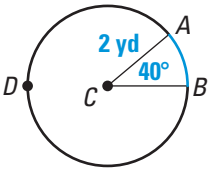
8. $m\overset{\frown}{QSP}$

9. $m\angle QTR$

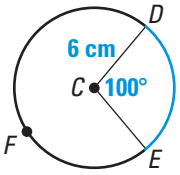


Find the blue arc length. Round your answer to the nearest hundredth.

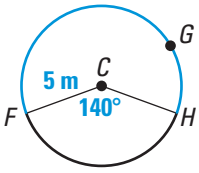
10. Length of $\overset{\frown}{AB}$



11. Length of $\overset{\frown}{DE}$



12. Length of $\overset{\frown}{FGH}$



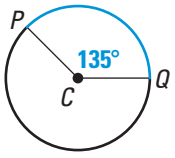
Practice and Applications

Extra Practice

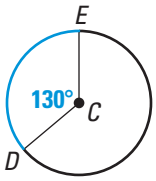
See p. 695.

Naming Arcs **Name the blue minor arc and find its measure.**

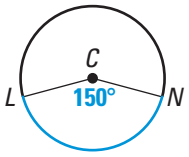
13.



14.

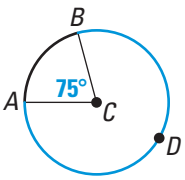


15.



Naming Arcs **Name the blue major arc and find its measure.**

16.



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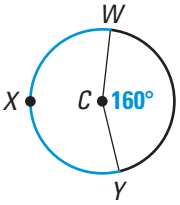
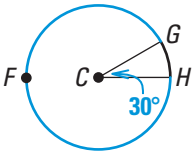


18.



Homework Help

Example 1: Exs. 13–39
Example 2: Exs. 30–42
Example 3: Exs. 43–46
Example 4: Exs. 47–54

Types of Arcs Determine whether the arc is a *minor arc*, a *major arc*, or a *semicircle* of $\odot R$. $\overline{PT}$ and $\overline{QU}$ are diameters.

19. $\overset{\frown}{PQ}$

20. $\overset{\frown}{SU}$

21. $\overset{\frown}{PQT}$

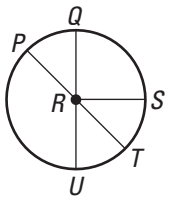
22. $\overset{\frown}{QT}$

23. $\overset{\frown}{TUQ}$

24. $\overset{\frown}{TUP}$

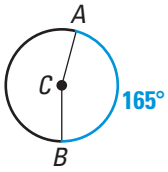
25. $\overset{\frown}{QUT}$

26. $\overset{\frown}{PUQ}$

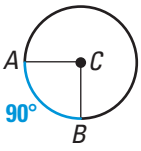


Finding the Central Angle Find the measure of $\angle ACB$.

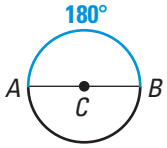
27.



28.



29.



Student Help

VISUAL STRATEGY
In Exs. 30–39, copy the diagram and add information to it as you solve the exercises, as shown on p. 588.

Measuring Arcs and Central Angles $\overline{KN}$ and $\overline{JL}$ are diameters. Find the measure.

30. $m\overset{\frown}{KL}$

31. $m\overset{\frown}{MN}$

32. $m\overset{\frown}{LNK}$

33. $m\overset{\frown}{MKN}$

34. $m\overset{\frown}{NJK}$

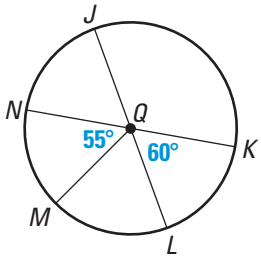
35. $m\angle MQL$

36. $m\overset{\frown}{ML}$

37. $m\angle JQN$

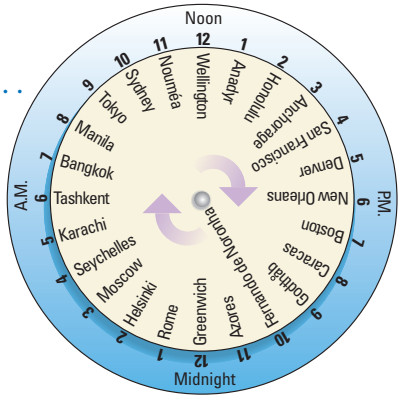
38. $m\overset{\frown}{JM}$

39. $m\overset{\frown}{LN}$



Time Zone Wheel **In Exercises 40–42, use the following information.**
To find the time in Tokyo when it is 4 P.M. in San Francisco, rotate the small wheel until 4 P.M. and San Francisco line up as shown. Then look at Tokyo to see that it is 9 A.M. there.

When it is 9 A.M. in Tokyo . . .



. . . it is 4 P.M. in San Francisco

40. What is the arc measure for each time zone on the wheel?

41. What is the measure of the minor arc from the Tokyo zone to the Anchorage zone?

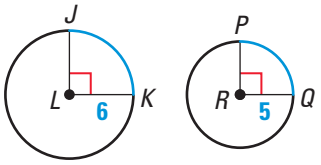
42. If two cities differ by 180° on the wheel, then it is 3:00 P.M. in one city when it is __?__ in the other city.

Student Help
CLASSZONE.COM

HOMEWORK HELP
Extra help with problem solving in Exs. 47–52 is at classzone.com

Naming Congruent Arcs Are the blue arcs congruent? Explain.

43.



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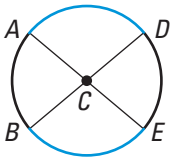

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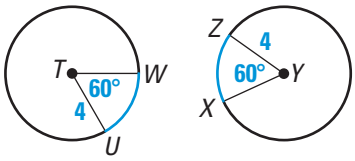
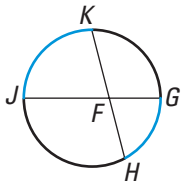
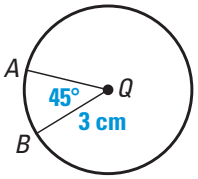


46.



Finding Arc Length Find the length of $\overset{\frown}{AB}$. Round your answer to the nearest hundredth.

47.



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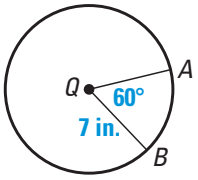

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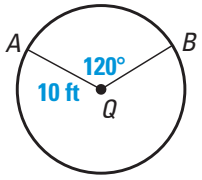
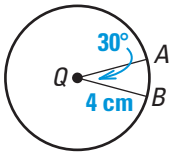
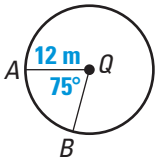


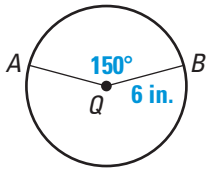
50.



51.



52.



53. **You be the Judge** A friend tells you two arcs from different circles have the same arc length if their central angles are equal. Is your friend correct? Explain your reasoning.

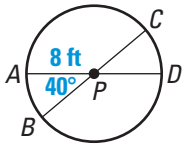
54. **Challenge** Engineers reduced the lean of the Leaning Tower of Pisa. If they moved it back 0.46°, what was the arc length of the move? Round your answer to the nearest whole number.

Standardized Test Practice

55. Multiple Choice What is the length of $\overset{\frown}{AC}$ in $\odot P$ shown below?

Ⓐ 5.6 ft Ⓑ 16.8 ft

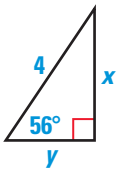
Ⓒ 19.5 ft Ⓓ 25.1 ft



Mixed Review

Finding Leg Lengths **Find the lengths of the legs of the triangle. Round your answers to the nearest tenth.** *(Lesson 10.5)*

56.



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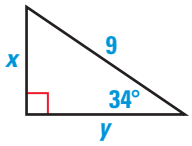
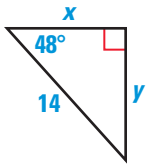


58.



Algebra Skills

Simplifying Ratios **Simplify the ratio.** *(Skills Review, p. 660)*

59. $\frac{2 \text{ km}}{400 \text{ km}}$ **60.** $\frac{5 \text{ ft}}{72 \text{ in.}}$ **61.** $\frac{3 \text{ yards}}{27 \text{ ft}}$ **62.** $\frac{4 \text{ ounces}}{8 \text{ pounds}}$

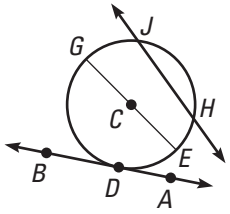
Quiz 1

Tell whether the given line, segment, or point is best described as a *chord*, a *secant*, a *tangent*, a *diameter*, a *radius*, or a *point of tangency*. *(Lesson 11.1)*

1. $\overleftrightarrow{AB}$ **2.** $\overleftrightarrow{JH}$

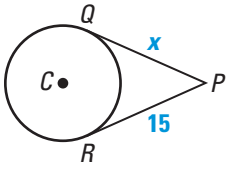
3. $\overline{GE}$ **4.** $\overline{JH}$

5. $\overline{CE}$ **6.** D

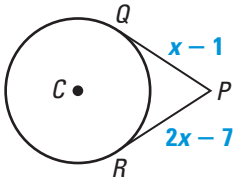


$\overline{PQ}$ and $\overline{PR}$ are tangent to $\odot C$. Find the value of x. *(Lesson 11.2)*

7.



8.



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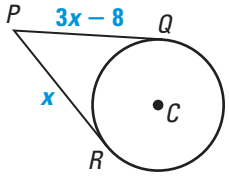
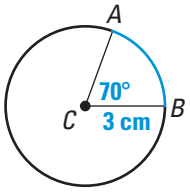
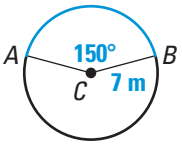


Find the length of $\overset{\frown}{AB}$. Round your answer to the nearest hundredth. *(Lesson 11.3)*

10.



11.



12.