Any two points $A$ and $B$ on a circle $C$ determine a \textit{minor arc} and a \textit{major arc} (unless the points lie on a diameter).

If the measure of $\angle ACB$ is less than $180^\circ$, then $A$, $B$, and all the points on $\odot C$ that lie in the interior of $\angle ACB$ form a \textit{minor arc}.

Points $A$, $B$, and all the points on $\odot C$ that do not lie on $\overline{AB}$ form a \textit{major arc}.

You name an arc by its endpoints. Use one other point on a major arc as part of its name to distinguish it from the minor arc.

The \textit{measures} of a minor arc and a major arc depend on the central angle of the minor arc.

The \textit{measure of a minor arc} is the measure of its central angle.

The \textit{measure of a major arc} is the difference of $360^\circ$ and the measure of the related minor arc.

A \textit{semicircle} is an arc whose central angle measures $180^\circ$. A semicircle is named by three points. Its measure is $180^\circ$.

**EXAMPLE 1** Name and Find Measures of Arcs

Name the red arc and identify the type of arc. Then find its measure.

a. $\overline{DG}$ is a minor arc. Its measure is $40^\circ$.

b. $\overline{LMN}$ is a major arc. Its measure is $360^\circ - 110^\circ = 250^\circ$. 
Two circles are congruent if they have the same radius.

Two arcs of the same circle or of congruent circles are congruent if they have the same measure.

1. \( BC \) and \( EF \)
2. \( BC \) and \( CD \)
3. \( CD \) and \( DE \)
4. \( BFE \) and \( CBF \)

**Arc Addition Postulate**

**Words** The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

**Symbols** \( m\overarc{ACB} = m\overarc{AC} + m\overarc{CB} \)

**Example 2** Find Measures of Arcs

Find the measure of \( \overarc{GEF} \).

**Solution**

\[
m\overarc{GEF} = m\overarc{GH} + m\overarc{HE} + m\overarc{EF} = 40° + 80° + 110° = 230°
\]

Two circles are congruent circles if they have the same radius.

Two arcs of the same circle or of congruent circles are congruent arcs if they have the same measure.

**Example 3** Identify Congruent Arcs

Find the measures of the blue arcs. Are the arcs congruent?

**Solution**

a. Notice that \( \overarc{AB} \) and \( \overarc{DC} \) are in the same circle. Because \( m\overarc{AB} = m\overarc{DC} = 45° \), \( \overarc{AB} \equiv \overarc{DC} \).

b. Notice that \( \overarc{XY} \) and \( \overarc{ZW} \) are not in the same circle or in congruent circles. Therefore, although \( m\overarc{XY} = m\overarc{ZW} = 65° \), \( \overarc{XY} \not\equiv \overarc{ZW} \).

**Checkpoint** Identify Congruent Arcs

Find the measures of the arcs. Are the arcs congruent?

1. \( \overarc{BC} \) and \( \overarc{EF} \)
2. \( \overarc{BC} \) and \( \overarc{CD} \)
3. \( \overarc{CD} \) and \( \overarc{DE} \)
4. \( \overarc{BFE} \) and \( \overarc{CBF} \)
**Arc Length**

An **arc length** is a portion of the circumference of a circle. You can write a proportion to find arc length.

\[
\frac{\text{arc length}}{2\pi r} = \frac{m_{\text{arc AB}}}{360^\circ}
\]

**Words**

In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.

**Symbols**

Arc length of \( \overline{AB} = \frac{m_{\overline{AB}}}{360^\circ} \cdot 2\pi r \)

---

**Example 4 Find Arc Lengths**

Find the length of the red arc.

a. \( \overline{AB} \) with a central angle of 50°

Solution

a. Arc length of \( \overline{AB} = \frac{50^\circ}{360^\circ} \cdot 2\pi(5) = 4.36 \) centimeters

b. Arc length of \( \overline{CD} = \frac{50^\circ}{360^\circ} \cdot 2\pi(7) = 6.11 \) centimeters

c. Arc length of \( \overline{EF} = \frac{98^\circ}{360^\circ} \cdot 2\pi(7) = 11.97 \) centimeters

---

**Checkpoint Find Arc Lengths**

Find the length of the red arc. Round your answer to the nearest hundredth.

5. \( \overline{AB} \) with a central angle of 120°

6. \( \overline{DE} \) with a central angle of 180°

7. \( \overline{MN} \) with a central angle of 90°
11.3 Exercises

Guided Practice

Vocabulary Check
1. In the diagram at the right, identify a major arc, a minor arc, and a semicircle.

2. Draw a circle with a pair of congruent arcs.

3. What is the difference between arc measure and arc length?

Skill Check
Find the measure in $\odot T$.
4. $m\overline{RS}$
5. $m\overline{RPS}$
6. $m\overline{PQR}$
7. $m\overline{QS}$
8. $m\overline{QSP}$
9. $m\angle QTR$

Find the blue arc length. Round your answer to the nearest hundredth.
10. Length of $\overline{AB}$
11. Length of $\overline{DE}$
12. Length of $\overline{FGH}$

Practice and Applications

Extra Practice
See p. 695.

Naming Arcs Name the blue minor arc and find its measure.
13.
14.
15.

Naming Arcs Name the blue major arc and find its measure.
16.
17.
18.
Types of Arcs Determine whether the arc is a **minor arc**, a **major arc**, or a **semicircle** of $\odot R$. $PT$ and $QU$ are diameters.

19. $PQ$ 20. $SU$
21. $PQT$ 22. $QT$
23. $TUQ$ 24. $TUP$
25. $QUT$ 26. $PUQ$

Finding the Central Angle Find the measure of $\angle ACB$.

27. 28. 29.

Finding the Central Angle Find the measure of $\angle ACB$.

30. $mKL$ 31. $mMN$
32. $mLKN$ 33. $mMKN$
34. $mNJK$ 35. $m\angle MQL$
36. $mML$ 37. $m\angle JQN$
38. $m\angle M$ 39. $m\angle LN$

Measuring Arcs and Central Angles $KN$ and $JL$ are diameters. Find the measure.

Time Zone Wheel In Exercises 40–42, use the following information.
To find the time in Tokyo when it is 4 P.M. in San Francisco, rotate the small wheel until 4 P.M. and San Francisco line up as shown. Then look at Tokyo to see that it is 9 A.M. there.

When it is 9 A.M. in Tokyo . . .

. . . it is 4 P.M. in San Francisco

40. What is the arc measure for each time zone on the wheel?

41. What is the measure of the minor arc from the Tokyo zone to the Anchorage zone?

42. If two cities differ by 180° on the wheel, then it is 3:00 P.M. in one city when it is ___ in the other city.
Naming Congruent Arcs  Are the blue arcs congruent? Explain.

43. 

44. 

45. 

46. 

Finding Arc Length  Find the length of $\overline{AB}$. Round your answer to the nearest hundredth.

47. 

48. 

49. 

50. 

51. 

52. 

53. You be the Judge  A friend tells you two arcs from different circles have the same arc length if their central angles are equal. Is your friend correct? Explain your reasoning.

54. Challenge  Engineers reduced the lean of the Leaning Tower of Pisa. If they moved it back 0.46°, what was the arc length of the move? Round your answer to the nearest whole number.
55. **Multiple Choice** What is the length of $\overline{AC}$ in $\odot P$ shown below?

- **A** 5.6 ft
- **B** 16.8 ft
- **C** 19.5 ft
- **D** 25.1 ft

Mixed Review

**Finding Leg Lengths** Find the lengths of the legs of the triangle. Round your answers to the nearest tenth. (*Lesson 10.5*)

56. 

57. 

58.

Algebra Skills

**Simplifying Ratios** Simplify the ratio. (*Skills Review, p. 660*)

59. $\frac{2 \text{ km}}{400 \text{ km}}$

60. $\frac{5 \text{ ft}}{72 \text{ in.}}$

61. $\frac{3 \text{ yards}}{27 \text{ ft}}$

62. $\frac{4 \text{ ounces}}{8 \text{ pounds}}$

Quiz 1

Tell whether the given line, segment, or point is best described as a chord, a secant, a tangent, a diameter, a radius, or a point of tangency. (*Lesson 11.1*)

1. $\overline{AB}$
2. $\overrightarrow{JH}$
3. $\overline{GE}$
4. $\overrightarrow{JH}$
5. $\overline{CE}$
6. $D$

$\overline{PQ}$ and $\overline{PR}$ are tangent to $\odot C$. Find the value of $x$. (*Lesson 11.2*)

7. $x$

8. $x - 1$

9. $3x - 8$

Find the length of $\overline{AB}$. Round your answer to the nearest hundredth. (*Lesson 11.3*)

10. 

11. 

12.