An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle.

The arc that lies in the interior of an inscribed angle and has endpoints on the angle is called the **intercepted arc** of the angle.

Activity 11.5 shows the relationship between an inscribed angle and its intercepted arc.

### Theorem 11.7

**Measure of an Inscribed Angle**

**Words** If an angle is inscribed in a circle, then its measure is half the measure of its intercepted arc.

**Symbols** \( m\angle ADB = \frac{1}{2}m\overline{AB} \)

### Example 1 Find Measures of Inscribed Angles and Arcs

Find the measure of the inscribed angle or the intercepted arc.

**a.**

Find \( m\angle NMP \) or \( m\overline{NP} \).

**Solution**

\[ m\angle NMP = \frac{1}{2}m\overline{NP} \]

\[ = \frac{1}{2}(100^\circ) \]

\[ = 50^\circ \]

Substitute 100° for \( m\overline{NP} \).

Simplify.

**b.**

Find \( m\angle ZYX \) or \( m\overline{ZWX} \).

**Solution**

\[ m\angle ZYX = \frac{1}{2}m\overline{ZWX} \]

\[ 105^\circ = \frac{1}{2}m\overline{ZWX} \]

Substitute 105° for \( m\angle ZYX \).

\[ 210^\circ = m\overline{ZWX} \]

Multiply each side by 2.
Find the measure of the inscribed angle or the intercepted arc.

1. \( \angle BAC = 90° \)
2. \( \angle DEF = 160° \)
3. \( \angle KNP = 120° \)

**Inscribed and Circumscribed** If all the vertices of a polygon lie on a circle, the polygon is **inscribed** in the circle and the circle is **circumscribed** about the polygon. The polygon is an **inscribed polygon** and the circle is a **circumscribed circle**.

**THEOREM 11.8**

**Words** If a triangle inscribed in a circle is a right triangle, then the hypotenuse is a diameter of the circle.

If a side of a triangle inscribed in a circle is a diameter of the circle, then the triangle is a right triangle.

**EXAMPLE 2 Find Angle Measures**

Find the values of \( x \) and \( y \).

**Solution**

Because \( \triangle ABC \) is inscribed in a circle and \( AB \) is a diameter, it follows from Theorem 11.8 that \( \triangle ABC \) is a right triangle with hypotenuse \( AB \).

Therefore, \( x = 90° \). Because \( \angle A \) and \( \angle B \) are acute angles of a right triangle, \( y = 90° - 50° = 40° \).
Find the values of \( x \) and \( y \) in \( \odot C \).

\( \begin{align*}
4. & \quad \triangle C \quad 35^\circ \quad \gamma \quad C \\
5. & \quad \triangle C \quad x^\circ \quad y^\circ \\
6. & \quad \triangle C \quad 60^\circ \quad x^\circ 
\end{align*} \)

**EXAMPLE 3** Find Angle Measures

Find the values of \( y \) and \( z \).

**Solution**

Because \( RSTU \) is inscribed in a circle, by Theorem 11.9 opposite angles must be supplementary.

\[ \begin{align*}
\angle S \text{ and } \angle U & \text{ are opposite angles.} \\
\angle R \text{ and } \angle T & \text{ are opposite angles.}
\end{align*} \]

\[ \begin{align*}
m\angle S + m\angle U & = 180^\circ \\
120^\circ + y^\circ & = 180^\circ \\
y & = 60 \\
m\angle R + m\angle T & = 180^\circ \\
z^\circ + 80^\circ & = 180^\circ \\
z & = 100
\end{align*} \]

**Checkpoint** Find Angle Measures

Find the values of \( x \) and \( y \) in \( \odot C \).

\( \begin{align*}
7. & \quad \triangle C \quad 100^\circ \quad y^\circ \\
8. & \quad \square C \quad x^\circ \quad y^\circ \\
9. & \quad \triangle C \quad 50^\circ \quad x^\circ \quad y^\circ 
\end{align*} \)
In Exercises 1 and 2, use the diagram at the right.

1. Name the inscribed angles.

2. Identify the two pairs of opposite angles in the inscribed quadrilateral.

Find the measure of the blue intercepted arc.

3. 4. 5.

Find the value of each variable.

6. 7. 8.

Practice and Applications

Extra Practice
See p. 696.

Angle Measures Find the measure of the inscribed angle.

9. 10. 11.

**Arc Measures** Find the measure of the blue intercepted arc.

15. \[ \text{arc } AB \]  
16. \[ \text{arc } BC \]  
17. \[ \text{arc } CA \]  
18. \[ \text{arc } ST \]  
19. \[ \text{arc } PQ \]  
20. \[ \text{arc } XZ \]  

**Arc and Angle Measures** In Exercises 21–26, use the diagram below to find the intercepted arc or inscribed angle.

21. \[ m\overarc{BE} \]  
22. \[ m\angle BDE \]  
23. \[ m\angle AED \]  
24. \[ m\overarc{AD} \]  
25. \[ m\angle ABD \]  
26. \[ m\overarc{DE} \]  

27. Are \( \triangle ABC \) and \( \triangle DEC \) similar? Explain your reasoning.

**Inscribed Right Triangles** Find the value of each variable. Explain your reasoning.

28. \( \triangle ABC \)  
29. \( \triangle LKM \)  
30. \( \triangle PQR \)  

31. **Carpenter's Square** A carpenter's square is an L-shaped tool used to draw right angles. Suppose you are making a toy truck. To make the wheels you trace a circle on a piece of wood. How could you use a carpenter's square to find the center of the circle?
Inscribed Quadrilaterals

Find the values of $x$ and $y$.

32. 
33. 
34.

You be the Judge

Can the quadrilateral always be inscribed in a circle? Explain your answer.

35. square
36. isosceles trapezoid
37. rhombus
38. rectangle

Standardized Test Practice

39. Multiple Choice

In the diagram at the right, if $\angle ACB$ is a central angle and $m\angle ACB = 80^\circ$, what is $m\angle ADB$?

A) 20°  
B) 40°  
C) 80°  
D) 160°

40. Multiple Choice

In the diagram at the right, what are the values of $x$ and $y$?

F) $x = 80, y = 95$  
G) $x = 85, y = 100$  
H) $x = 95, y = 80$  
J) $x = 95, y = 85$

Mixed Review

Multiplying Radicals

Multiply the radicals. Then simplify if possible. (Lesson 10.1)

41. $\sqrt{5} \cdot \sqrt{7}$
42. $\sqrt{2} \cdot \sqrt{2}$
43. $\sqrt{6} \cdot \sqrt{14}$
44. $(8\sqrt{2})^2$
45. $(3\sqrt{3})^2$
46. $2\sqrt{5} \cdot \sqrt{10}$

Solving Right Triangles

Solve the right triangle. Round decimals to the nearest tenth. (Lesson 10.6)

47.
48.
49.

Algebra Skills

Evaluating Expressions

Evaluate the expression when $x = 2$.

(Skills Review, p. 670)

50. $3x + 5$
51. $8x - 7$
52. $x^2 + 9$
53. $(x + 4)(x - 4)$
54. $x^2 + 3x - 2$
55. $x^3 + x^2$