2.2 Angle Bisectors

Goal
Bisect an angle.

Key Words
• angle bisector

An angle bisector is a ray that divides an angle into two angles that are congruent. In the photograph of the hang glider, \( \overline{BD} \) bisects \( \angle ABC \) because it divides the angle into two congruent angles.

\[ \angle ABD \cong \angle DBC \]

If \( \overline{BD} \) bisects \( \angle ABC \), then the measures of \( \angle ABD \) and \( \angle DBC \) are half the measure of \( \angle ABC \). Also, the measure of \( \angle ABC \) is twice the measure of \( \angle ABD \) or \( \angle DBC \).

EXAMPLE 1 Find Angle Measures

\( \overline{BD} \) bisects \( \angle ABC \), and \( m\angle ABC = 110^\circ \).
Find \( m\angle ABD \) and \( m\angle DBC \).

Solution
\[
m\angle ABD = \frac{1}{2}(m\angle ABC) \quad \overline{BD} \text{ bisects } \angle ABC.
\]
\[
= \frac{1}{2}(110^\circ) \quad \text{Substitute } 110^\circ \text{ for } m\angle ABC.
\]
\[
= 55^\circ \quad \text{Simplify.}
\]
\( \angle ABD \) and \( \angle DBC \) are congruent, so \( m\angle DBC = m\angle ABD \).

ANSWER
So, \( m\angle ABD = 55^\circ \), and \( m\angle DBC = 55^\circ \).

Checkpoint Find Angle Measures

\( \overline{HK} \) bisects \( \angle GHJ \). Find \( m\angle GHK \) and \( m\angle KHJ \).

1. \[ \begin{array}{c}
\angle JKH = 52^\circ \\
\end{array} \]
2. \[ \begin{array}{c}
\angle JKH = \text{missing angle}
\end{array} \]
3. \[ \begin{array}{c}
\angle JKH = 161^\circ
\end{array} \]
EXAMPLE 2 Find Angle Measures and Classify an Angle

\(\overrightarrow{MP}\) bisects \(\angle LMN\), and \(m\angle LMP = 46^\circ\).

a. Find \(m\angle PMN\) and \(m\angle LMN\).

b. Determine whether \(\angle LMN\) is acute, right, obtuse, or straight. Explain.

Solution

a. \(\overrightarrow{MP}\) bisects \(\angle LMN\), so \(m\angle LMP = m\angle PMN\).

You know that \(m\angle LMP = 46^\circ\). Therefore, \(m\angle PMN = 46^\circ\).

The measure of \(\angle LMN\) is twice the measure of \(\angle LMP\).

\[
m\angle LMN = 2(m\angle LMP) = 2(46^\circ) = 92^\circ
\]

So, \(m\angle PMN = 46^\circ\), and \(m\angle LMN = 92^\circ\).

b. \(\angle LMN\) is obtuse because its measure is between 90° and 180°.

Checkpoint

Find Angle Measures and Classify an Angle

\(\overrightarrow{QS}\) bisects \(\angle PQR\). Find \(m\angle SQP\) and \(m\angle PQR\). Then determine whether \(\angle PQR\) is acute, right, obtuse, or straight.

EXAMPLE 3 Use Angle Bisectors

In the kite, \(\angle DAB\) is bisected by \(\overrightarrow{AC}\), and \(\angle BCD\) is bisected by \(\overrightarrow{CA}\). Find \(m\angle DAB\) and \(m\angle BCD\).

Solution

\[
m\angle DAB = 2(m\angle BAC)\quad \overrightarrow{AC}\text{ bisects }\angle DAB.
\]
\[
= 2(45^\circ)\quad \text{Substitute } 45^\circ \text{ for } m\angle BAC.
\]
\[
= 90^\circ\quad \text{Simplify.}
\]

\[
m\angle BCD = 2(m\angle ACB)\quad \overrightarrow{CA}\text{ bisects }\angle BCD.
\]
\[
= 2(27^\circ)\quad \text{Substitute } 27^\circ \text{ for } m\angle ACB.
\]
\[
= 54^\circ\quad \text{Simplify.}
\]

ANSWER The measure of \(\angle DAB\) is 90°, and the measure of \(\angle BCD\) is 54°.
7. \( \overline{KM} \) bisects \( \angle JKL \).
   Find \( m\angle JKM \) and \( m\angle MKL \).

8. \( \overline{UV} \) bisects \( \angle WUT \).
   Find \( m\angle WUV \) and \( m\angle WUT \).

**EXAMPLE 4**

Use Algebra with Angle Measures

\( \overline{RQ} \) bisects \( \angle PRS \). Find the value of \( x \).

**Solution**

\[
m\angle PRQ = m\angle QRS \\
(6x + 1)° = 85°
\]
Substitute given measures.

\[
6x + 1 - 1 = 85 - 1 \\
6x = 84
\]
Subtract 1 from each side.

\[
6x = 84 \\
\frac{6x}{6} = \frac{84}{6}
\]
Simplify.

\[
x = 14
\]
Divide each side by 6.

**CHECK**

You can check your answer by substituting 14 for \( x \).

\[
m\angle PRQ = (6x + 1)° = (6 \cdot 14 + 1)° = (84 + 1)° = 85°
\]
2.2 Exercises

Guided Practice

Vocabulary Check

What kind of geometric figure is an angle bisector?

Skill Check

1. BD bisects \( \angle ABC \). Find the angle measure.

   - Find \( m \angle ABD \).
   - Find \( m \angle DBC \).
   - Find \( m \angle DBC \).

   - Find \( m \angle ABC \).
   - Find \( m \angle CBA \).
   - Find \( m \angle ABC \).

Practice and Applications

Extra Practice

See p. 677.

Finding Angle Measures

\( QS \) bisects \( \angle PQR \). Find \( m \angle PQS \) and \( m \angle SQR \).

- \( Q \) \( S \) \( P \) \( R \)
  - \( m \angle PQS = 50° \)
  - \( m \angle SQR = 108° \)

- \( Q \) \( S \) \( P \) \( R \)
  - \( m \angle PQS = 124° \)
  - \( m \angle SQR = 67° \)

- \( Q \) \( S \) \( P \) \( R \)
  - \( m \angle PQS = 91° \)
  - \( m \angle SQR = 75° \)

Fans

\( QS \) bisects \( \angle PQR \). Find \( m \angle PQS \) and \( m \angle SQR \).

- \( Q \) \( S \) \( P \) \( R \)
  - \( m \angle PQS = 87° \)
  - \( m \angle SQR = 180° \)

- \( Q \) \( S \) \( P \) \( R \)
  - \( m \angle PQS = 87° \)
  - \( m \angle SQR = 120° \)
Finding Angle Measures  \( \overrightarrow{BD} \) bisects \( \angle ABC \). Find \( m\angle ABD \) and \( m\angle ABC \). Then determine whether \( \angle ABC \) is acute, right, obtuse, or straight.

17.  

18.  

19.  

20.  

21.  

22.  

23. Paper Airplanes  The diagram at the right represents an unfolded piece of paper used to make a paper airplane like the one shown below. Using the diagram, name the angles that are bisected by \( \overrightarrow{AK} \).

Lasers In Exercises 24–27, use the diagram below. When light is reflected by a smooth surface, the angle of incidence is equal to the angle of reflection.

24. Name an angle bisector in the diagram.

25. If the angle of reflection is 67°, what is the angle of incidence?

26. If \( m\angle KJL = 109^\circ \), what is the angle of reflection?

27. **You be the Judge** Can you determine whether \( \angle JKP \) is congruent to \( \angle LKQ \) in the diagram above? Explain your reasoning.
Using Algebra  \( \overrightarrow{KM} \) bisects \( \angle JKL \). Find the value of the variable.

28. \( y - 29 \)°

29. 25°

30. \( 8x - 13 \)°

31. **Technology** Use geometry software to draw \( \triangle ABC \). Construct the angle bisector of \( \angle BAC \). Then find the midpoint of \( BC \). Drag any of the points. Does the angle bisector *always* pass through the midpoint of the opposite side? Does it *ever* pass through the midpoint?

32. **Challenge** \( \overrightarrow{BD} \) bisects \( \angle ABE \) and \( \overrightarrow{BF} \) bisects \( \angle EBC \). Use the diagram shown to find \( m\angle DBF \).

33. **Multiple Choice** In the diagram below, \( \overrightarrow{BD} \) bisects \( \angle ABC \) and \( m\angle ABC = 23° \). What is \( m\angle ABD \)?
   - A 11.5°  
   - B 12.5°  
   - C 23°  
   - D 46°

Mixed Review

**Describing Number Patterns** Describe a pattern in the numbers. Write the next number you expect in the pattern. *(Lesson 1.1)*

34. 1, 9, 17, 25, . . .

35. \(-13, -8, -3, 2, \ldots\)

36. 5.6, 5.16, 5.116, 5.1116, . . .

37. 60, 30, 15, 7.5, . . .

**Classifying Angles** Use the diagram below to classify the angle as *acute*, *right*, *obtuse*, or *straight*. *(Lesson 1.6)*

38. \( \angle EBC \)

39. \( \angle ABE \)

40. \( \angle DBC \)

41. \( \angle ABC \)

**Algebra Skills** Solve the equation. *(Skills Review, p. 673)*

42. \( 2x - 15 = 9 \)

43. \( 3a + 12 = 48 \)

44. \( 10 - 3y = 52 \)

45. \( 5m - 11 = -46 \)

46. \( -2z + 4 = 8 \)

47. \( 3 = -n + 23 \)