2.3 Complementary and Supplementary Angles

Goal
Find measures of complementary and supplementary angles.

Key Words
• complementary angles
• supplementary angles
• adjacent angles
• theorem

Two angles are **complementary angles** if the sum of their measures is 90°. Each angle is the **complement** of the other.

![Diagram](image1)

\[ \angle A \text{ and } \angle B \text{ are complementary angles.} \]

\[ m\angle A + m\angle B = 32° + 58° = 90° \]

Two angles are **supplementary angles** if the sum of their measures is 180°. Each angle is the **supplement** of the other.

![Diagram](image2)

\[ \angle C \text{ and } \angle D \text{ are supplementary angles.} \]

\[ m\angle C + m\angle D = 134° + 46° = 180° \]

**EXAMPLE 1** Identify Complements and Supplements

Determine whether the angles are complementary, supplementary, or neither.

a. \[ \angle 1 \text{ and } \angle 2 \text{ are complementary.} \]

b. \[ \angle 3 \text{ and } \angle 4 \text{ are supplementary.} \]

**Solution**

a. Because \(22° + 158° = 180°\), the angles are supplementary.

b. Because \(15° + 85° = 100°\), the angles are neither complementary nor supplementary.

c. Because \(55° + 35° = 90°\), the angles are complementary.

**Checkpoint** Identify Complements and Supplements

Determine whether the angles are complementary, supplementary, or neither.

1. \[ \angle 1 = 30° \]

2. \[ \angle 2 = 41° \]

3. \[ \angle 3 = 148° \]
Two angles are **adjacent angles** if they share a common vertex and side, but have no common interior points.

![Diagram of adjacent angles]

**EXAMPLE 2 Identify Adjacent Angles**

Tell whether the numbered angles are adjacent or nonadjacent.

a. Because the angles do not share a common vertex or side, \( \angle 1 \) and \( \angle 2 \) are nonadjacent.

b. Because the angles share a common vertex and side, and they do not have any common interior points, \( \angle 3 \) and \( \angle 4 \) are adjacent.

c. Although \( \angle 5 \) and \( \angle 6 \) share a common vertex, they do not share a common side. Therefore, \( \angle 5 \) and \( \angle 6 \) are nonadjacent.

**EXAMPLE 3 Measures of Complements and Supplements**

a. \( \angle A \) is a complement of \( \angle C \), and \( m\angle A = 47^\circ \). Find \( m\angle C \).

b. \( \angle P \) is a supplement of \( \angle R \), and \( m\angle R = 36^\circ \). Find \( m\angle P \).

**Solution**

a. \( \angle A \) and \( \angle C \) are complements, so their sum is 90°.

\[
\begin{align*}
    m\angle A + m\angle C &= 90^\circ \\
    47^\circ + m\angle C &= 90^\circ \\
    47^\circ + m\angle C - 47^\circ &= 90^\circ - 47^\circ \\
    m\angle C &= 43^\circ
\end{align*}
\]

b. \( \angle P \) and \( \angle R \) are supplements, so their sum is 180°.

\[
\begin{align*}
    m\angle P + m\angle R &= 180^\circ \\
    m\angle P + 36^\circ &= 180^\circ \\
    m\angle P + 36^\circ - 36^\circ &= 180^\circ - 36^\circ \\
    m\angle P &= 144^\circ
\end{align*}
\]

**Checkpoint Measures of Complements and Supplements**

4. \( \angle B \) is a complement of \( \angle D \), and \( m\angle D = 79^\circ \). Find \( m\angle B \).

5. \( \angle G \) is a supplement of \( \angle H \), and \( m\angle G = 115^\circ \). Find \( m\angle H \).
A theorem is a true statement that follows from other true statements. The two theorems that follow are about complementary and supplementary angles.

**THEOREMS 2.1 and 2.2**

**2.1 Congruent Complements Theorem**

**Words** If two angles are complementary to the same angle, then they are congruent.

**Symbols** If \( \angle 1 + \angle 2 = 90^\circ \) and \( \angle 2 + \angle 3 = 90^\circ \), then \( \angle 1 \cong \angle 3 \).

**2.2 Congruent Supplements Theorem**

**Words** If two angles are supplementary to the same angle, then they are congruent.

**Symbols** If \( \angle 4 + \angle 5 = 180^\circ \) and \( \angle 5 + \angle 6 = 180^\circ \), then \( \angle 4 \cong \angle 6 \).

You can use theorems in your reasoning about geometry, as shown in Example 4.

**EXAMPLE 4 Use a Theorem**

\( \angle 7 \) and \( \angle 8 \) are supplementary, and \( \angle 8 \) and \( \angle 9 \) are supplementary. Name a pair of congruent angles. Explain your reasoning.

**Solution**

\( \angle 7 \) and \( \angle 9 \) are both supplementary to \( \angle 8 \). So, by the Congruent Supplements Theorem, \( \angle 7 \cong \angle 9 \).

**Checkpoint Use a Theorem**

6. In the diagram, \( m\angle 10 + m\angle 11 = 90^\circ \), and \( m\angle 11 + m\angle 12 = 90^\circ \).

Name a pair of congruent angles.
Explain your reasoning.
1. Explain the difference between complementary angles and supplementary angles.

2. Complete the statement: Two angles are __ if they share a common vertex and a common side, but have no common interior points.

3–5. Determine whether the angles are complementary, supplementary, or neither. Also tell whether the angles are adjacent or nonadjacent.

6. \( \angle A \) is a complement of \( \angle B \), and \( m\angle A = 10^\circ \). Find \( m\angle B \).

7. \( \angle C \) is a supplement of \( \angle D \), and \( m\angle D = 109^\circ \). Find \( m\angle C \).

Extra Practice

See p. 677.

Identifying Angles Determine whether the angles are complementary, supplementary, or neither. Also tell whether the angles are adjacent or nonadjacent.

8. 

9. 

10. 

Identifying Angles Determine whether the two angles shown on the clock faces are complementary, supplementary, or neither.

11. 

12. 

13. 

14.
Finding Complements  Find the measure of a complement of the angle given.

15. \[41^\circ\]  \[61^\circ\]  \[76^\circ\]

16. \[86^\circ\]  \[8^\circ\]

17. \[24^\circ\]  \[66^\circ\]

18. \(\angle K\) is a complement of \(\angle L\), and \(m\angle K = 74^\circ\). Find \(m\angle L\).

19. \(\angle P\) is a complement of \(\angle Q\), and \(m\angle P = 9^\circ\). Find \(m\angle Q\).

Finding Supplements  Find the measure of a supplement of the angle given.

20. \[55^\circ\]  \[125^\circ\]

21. \[14^\circ\]  \[166^\circ\]

22. \[160^\circ\]  \[20^\circ\]

23. \(\angle A\) is a supplement of \(\angle B\), and \(m\angle A = 96^\circ\). Find \(m\angle B\).

24. \(\angle P\) is a supplement of \(\angle Q\), and \(m\angle P = 7^\circ\). Find \(m\angle Q\).

Finding Complements and Supplements  Find the measures of a complement and a supplement of the angle.

25. \(m\angle A = 39^\circ\)  \(m\angle B = 89^\circ\)  \(m\angle C = 54^\circ\)

28. Bridges  The Alamillo Bridge in Seville, Spain, was designed by Santiago Calatrava. In the bridge, \(m\angle 1 = 58^\circ\), and \(m\angle 2 = 24^\circ\). Find the measures of the supplements of both \(\angle 1\) and \(\angle 2\).

Naming Angles  In the diagram, \(\angle QPR\) is a right angle.

29. Name a straight angle.

30. Name two congruent supplementary angles.

31. Name two supplementary angles that are not congruent.

32. Name two complementary angles.
Beach Chairs  Adjustable beach chairs form angles that are supplementary. Find the value of $x$.

33. 

34. 

Using Algebra  $\angle ABD$ and $\angle DBC$ are complementary angles. Find the value of the variable.

35. 

36. 

37. 

38. Complementary Angles  $\angle ABD$ and $\angle DBE$ are complements, and $\angle CBE$ and $\angle DBE$ are complements. Can you show that $\angle ABD \cong \angle CBE$? Explain.

39. Technology  Use geometry software to draw two intersecting lines. Measure three of the four angles formed. Drag the points and observe the angle measures. What theorem does this illustrate?

Complements and Supplements  Find the angle measure described.

40. $\angle 1$ and $\angle 2$ are both supplementary to $\angle 3$, and $m\angle 1 = 43^\circ$. Find the measure of $\angle 2$.

41. $\angle 4$ and $\angle 6$ are both complementary to $\angle 5$, and $m\angle 5 = 85^\circ$. Find the measure of $\angle 4$.

42. $\angle P$ is supplementary to $\angle Q$, $\angle R$ is supplementary to $\angle P$, and $m\angle Q = 60^\circ$. Find the measure of $\angle R$.

43. Challenge  $\angle C$ and $\angle D$ are supplementary angles. The measure of $\angle D$ is eight times the measure of $\angle C$. Find $m\angle C$ and $m\angle D$. 

Chapter 2  Segments and Angles
44. **Multiple Choice** What is the measure of a complement of a 27° angle?

- **A** 53°  
- **B** 63°  
- **C** 117°  
- **D** 163°

45. **Multiple Choice** \(\angle 1\) and \(\angle 2\) are supplementary. Suppose that \(m\angle 1 = 60°\) and \(m\angle 2 = (2x + 20)°\). What is the value of \(x\)?

- **F** 5  
- **G** 10  
- **H** 50  
- **J** 100

## Segment Addition Postulate
Find the length. *(Lesson 1.5)*

46. Find \(FH\).

47. Find \(KL\).

## Midpoint Formula
Find the coordinates of the midpoint of \(\overline{AB}\). *(Lesson 2.1)*

48. \(A(0, 0), B(8, 2)\)

49. \(A(-6, 0), B(2, 4)\)

50. \(A(4, 1), B(10, 3)\)

51. \(A(-2, 5), B(-2, 7)\)

52. \(A(3, -8), B(-1, 0)\)

53. \(A(-5, -9), B(11, 5)\)

## Algebra Skills
**Evaluating Decimals** Evaluate. *(Skills Review, p. 655)*

54. \(2.58 + 8.04\)

55. \(5.17 - 1.96\)

56. \(1.4 \times 3.1\)

57. \(0.61 \times 0.38\)

58. \(11.2 \div 1.4\)

59. \(2 \times 5.4 \times 3.9\)

## Quiz 1

1. In the diagram, \(K\) is the midpoint of \(\overline{JL}\). Find \(KL\) and \(JL\). *(Lesson 2.1)*

   ![Diagram](image)

   Find the coordinates of the midpoint of \(\overline{AB}\). *(Lesson 2.1)*

2. \(A(1, 3), B(7, -1)\)

3. \(A(-4, -2), B(6, 4)\)

4. \(A(-5, 3), B(3, -3)\)

In Exercises 5–7, \(\overline{KM}\) bisects \(\angle JKL\). Find the angle measure. *(Lesson 2.2)*

5. Find \(m\angle JKM\).

6. Find \(m\angle JKL\).

7. Find \(m\angle JKL\).

8. \(\angle F\) is a supplement of \(\angle G\), and \(m\angle F = 101°\). Find \(m\angle G\). *(Lesson 2.3)*

9. The measure of \(\angle D\) is 83°. Find the measure of a complement and a supplement of \(\angle D\). *(Lesson 2.3)*