**3.1 Relationships Between Lines**

**Goal**
Identify relationships between lines.

**Key Words**
- parallel lines
- perpendicular lines
- skew lines
- parallel planes
- line perpendicular to a plane

Two lines are **parallel lines** if they lie in the same plane and do not intersect. On the building, lines \( r \) and \( s \) are parallel lines. You can write this as \( r \parallel s \). Triangles (\( \boxed{\triangle} \)) are used to indicate that the lines are parallel.

Two lines are **perpendicular lines** if they intersect to form a right angle. Lines \( s \) and \( t \) are perpendicular lines. You can write this as \( s \perp t \).

**Example 1** Identify Parallel and Perpendicular Lines

Determine whether the lines are parallel, perpendicular, or neither.

- **a.** \( n \) and \( m \)
- **b.** \( p \) and \( q \)
- **c.** \( n \) and \( p \)

**Solution**

- **a.** Lines \( n \) and \( m \) are parallel.
- **b.** Lines \( p \) and \( q \) are neither parallel nor perpendicular.
- **c.** Lines \( n \) and \( p \) are perpendicular.

**Example 2** Identify Skew Lines

Determine whether the lines are skew.

- **a.** \( f \) and \( h \)
- **b.** \( f \) and \( g \)

**Solution**

- **a.** Lines \( f \) and \( h \) are not skew lines because they intersect.
- **b.** Lines \( f \) and \( g \) are skew lines.
CheckPoint Identify Relationships Between Lines

Use the diagram.

1. Name a pair of parallel lines.
2. Name a pair of perpendicular lines.
3. Name a pair of skew lines.

Two planes are **parallel planes** if they do not intersect.
A **line perpendicular to a plane** is a line that intersects a plane in a point and that is perpendicular to every line in the plane that intersects it.

EXAMPLE 3 Identify Relationships in Space

a. Name a plane that appears parallel to plane B.
b. Name a line that appears perpendicular to plane B.

Solution
a. Plane C appears parallel to plane B.
b. Line ℓ appears perpendicular to plane B.

Think of each segment in the diagram as part of a line.
4. Name a line that is skew to \( \overrightarrow{VW} \).
5. Name a plane that appears parallel to plane VWX.
6. Name a line that is perpendicular to plane VWX.
1. How are skew lines and parallel lines alike? How are they different?

Fill in the blank with $\parallel$ or $\perp$ to make the statement true.

2. Line $k \ ? \ line m$.
3. Line $m \ ? \ line \ell$.
4. Line $\ell \ ? \ line j$.
5. Line $k \ ? \ line j$.

Match the photo with the corresponding description of the chopsticks.

6. skew
7. parallel
8. intersecting

Line Relationships Determine whether the lines are parallel, perpendicular, or neither.

9. $a$ and $c$
10. $q$ and $s$
11. $y$ and $z$

Skew Lines Determine whether the lines are skew. Explain.

12. $u$ and $w$
13. $m$ and $n$
14. $j$ and $k$
Identifying Relationships  In Exercises 15–19, think of each segment in the diagram as part of a line. Fill in the blank with parallel, perpendicular, or skew.

15. $\overrightarrow{DE}$, $\overrightarrow{AB}$, and $\overrightarrow{GC}$ appear to be __ ? __.

16. $\overrightarrow{DE}$ and $\overrightarrow{BE}$ are ? __.

17. $\overrightarrow{BE}$ and $\overrightarrow{GC}$ are ? __.

18. $\overrightarrow{BE}$ is __ ? __ to plane $DEF$.

19. Plane $GAD$ and plane $CBE$ appear to be __ ? __.

20. **Tightrope Walking** Philippe Petit sometimes uses a long pole to help him balance on the tightrope. Are the rope and the pole at the left intersecting, perpendicular, parallel, or skew?

Relationships in Space  Think of each segment in the diagram below as part of a line. There may be more than one correct answer.

21. Name a line that appears parallel to $\overrightarrow{QR}$.

22. Name a line perpendicular to $\overrightarrow{QR}$.

23. Name a line skew to $\overrightarrow{QR}$.

24. Name a plane that appears parallel to plane $QRS$.

Visualize It! Sketch a figure that fits the description.

25. Three lines that are parallel

26. A line that is perpendicular to two parallel lines

27. Two planes that intersect

28. A line that is perpendicular to two parallel planes

29. A line that is perpendicular to two skew lines (Hint: Start by sketching a figure like the one above in Exercises 21–24.)

Furniture Design  In Exercises 30–33, use the photo of the chair designed by Mario Botta shown at the right.

30. Name two pairs of parallel lines.

31. What kind of lines are $h$ and $m$?

32. Name two lines that are skew.

33. How many lines shown on the chair are perpendicular to $f$?
**Prisms** Sketch a prism with bases that have the given number of sides. Label the prism. Name two edges that appear parallel.

36. Three sides  
37. Four sides  
38. Five sides

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**Escalators** In Exercises 34 and 35, use the following information.

When a step on an up-escalator reaches the top, it flips over and goes back down to the bottom. On each step, let plane $A$ be the plane you stand on.

34. As each step moves around the escalator, is plane $A$ always parallel to the ground level? Explain.

35. When a person is standing on plane $A$, is it parallel to ground level? Explain.

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**Example** Sketch a Prism

A *prism* is a three-dimensional figure with two identical faces, called *bases*, that lie in parallel planes as shown below.

- **a.** Sketch a prism with bases that have six sides. Label the prism.
- **b.** Name two edges that appear parallel.

**Solution**

- **a.** To draw the prism, follow these steps.

  1. **Sketch** two identical bases in parallel planes. In this case, the bases have six sides.
  2. **Connect** the bases and make hidden edges dashed. Label the prism.

- **b.** Edges $AG$ and $DK$ appear parallel.
Challenge  Fill in the blank with always, sometimes, or never.
39. Two skew lines are ___ parallel.
40. Two perpendicular lines ___ intersect.
41. Two skew lines are ___ coplanar.

Standardized Test Practice

42. Multiple Choice  Two lines are ___ lines if they do not lie in the same plane and they do not intersect.
   A  perpendicular  B  parallel
   C  coplanar  D  skew

43. Multiple Choice  Use the diagram below to determine which of the following statements is false. Think of each segment in the diagram as part of a line.
   F  QP and MP are not parallel.
   G  MP and NR are skew.
   H  JM and KS are perpendicular.
   J  Plane KJM and plane QPM are not parallel.

Mixed Review

If-Then Statements  Identify the hypothesis and the conclusion of the if-then statement. (Lesson 2.5)
44. If the band plays, then each member gets $50.
45. If m∠5 = 120°, then ∠5 is obtuse.
46. If there is a sale, then the store will be crowded.
47. If we can get tickets, then we’ll go to the movies.

Properties of Congruence  Use the property to complete the statement. (Lesson 2.6)
48. Reflexive Property of Congruence: ___  ≅  ∠XYZ
49. Symmetric Property of Congruence: If ∠1  ≅  ∠2, then ___  ≅  ___.
50. Transitive Property of Congruence: If AB  ≅  EF and EF  ≅  ST, then ___  ≅  ___.

Algebra Skills

Reciprocals  Find the reciprocal. (Skills Review, p. 656)
51. 26  52. −7  53. 10  54. \(\frac{3}{8}\)

Integers  Evaluate. (Skills Review, p. 663)
55. 18 + (−3)  56. −4 ÷ 2  57. 17 + (−6)  58. 16 − (−5)
59. −5 + 31  60. 24 − 28  61. (−8)(−10)  62. −25 − 19

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