4.3 Isosceles and Equilateral Triangles

Goal

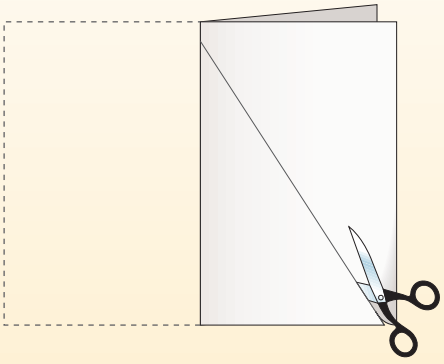
Use properties of isosceles and equilateral triangles.

Key Words

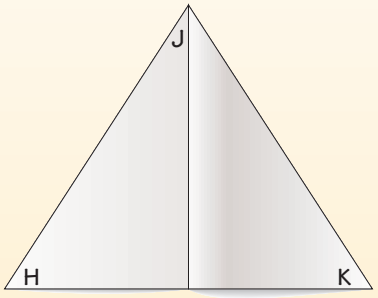
- legs of an isosceles triangle
- base of an isosceles triangle
- base angles

Geo-Activity Properties of Isosceles Triangles

1. Fold a sheet of paper in half. Use a straightedge to draw a line from the fold to the bottom edge. Cut along the line to form an isosceles triangle.



2. Unfold and label the angles as shown. Use a protractor to measure $\angle H$ and $\angle K$. What do you notice?



3. Repeat Steps 1 and 2 for different isosceles triangles. What can you say about $\angle H$ and $\angle K$ in the different triangles?

Student Help

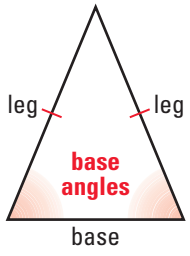
VOCABULARY TIP
Isos- means "equal," and *-sceles* means "leg." So, *isosceles* means equal legs.

The Geo-Activity shows that two angles of an isosceles triangle are always congruent. These angles are opposite the congruent sides.

The congruent sides of an isosceles triangle are called **legs**.

The other side is called the **base**.

The two angles at the base of the triangle are called the **base angles**.



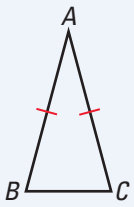
Isosceles Triangle

THEOREM 4.3

Base Angles Theorem

Words If two sides of a triangle are congruent, then the angles opposite them are congruent.

Symbols If $\overline{AB} \cong \overline{AC}$, then $\angle C \cong \angle B$.



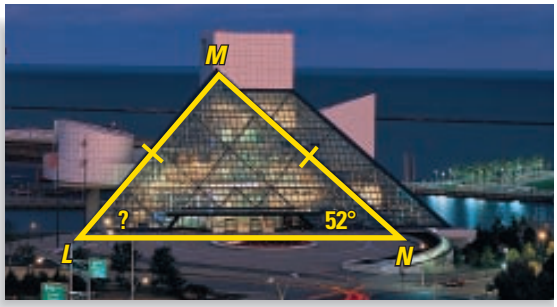
EXAMPLE 1 Use the Base Angles Theorem

Find the measure of $\angle L$.

Solution

Angle L is a base angle of an isosceles triangle. From the Base Angles Theorem, $\angle L$ and $\angle N$ have the same measure.

ANSWER ▶ The measure of $\angle L$ is 52°.



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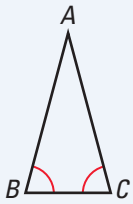
THEOREM 4.4

Converse of the Base Angles Theorem

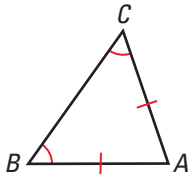
Words If two angles of a triangle are congruent, then the sides opposite them are congruent.

Symbols If $\angle B \cong \angle C$, then $\overline{AC} \cong \overline{AB}$.



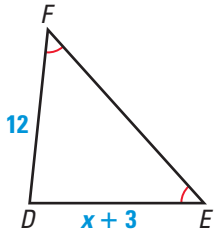
Visualize It!

Base angles don't have to be on the bottom of an isosceles triangle.



EXAMPLE 2 Use the Converse of the Base Angles Theorem

Find the value of x.



Solution

By the Converse of the Base Angles Theorem, the legs have the same length.

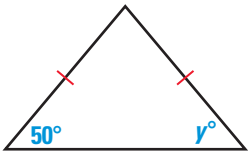
$DE = DF$	Converse of the Base Angles Theorem
$x + 3 = 12$	Substitute $x + 3$ for DE and 12 for DF.
$x = 9$	Subtract 3 from each side.

ANSWER ▶ The value of x is 9.

Checkpoint ✓ Use Isosceles Triangle Theorems

Find the value of y.

1.



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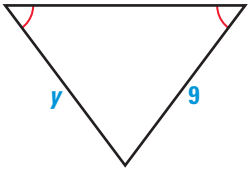

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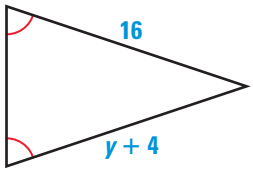


Student Help

LOOK BACK
For the definition of equilateral triangle, see p. 173.

THEOREMS 4.5 and 4.6

4.5 Equilateral Theorem

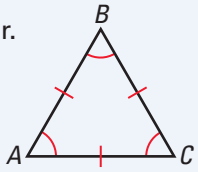
Words If a triangle is equilateral, then it is equiangular.

Symbols If $\overline{AB} \cong \overline{AC} \cong \overline{BC}$, then $\angle A \cong \angle B \cong \angle C$.

4.6 Equiangular Theorem

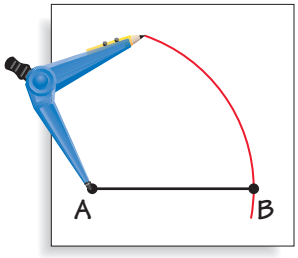
Words If a triangle is equiangular, then it is equilateral.

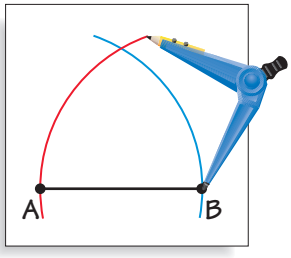
Symbols If $\angle B \cong \angle C \cong \angle A$, then $\overline{AB} \cong \overline{AC} \cong \overline{BC}$.

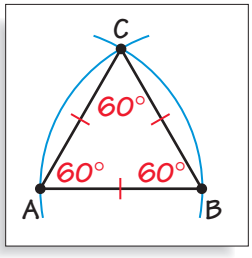


Constructing an Equilateral Triangle You can construct an equilateral triangle using a straightedge and compass.

1. Draw $\overline{AB}$. Draw an arc with center A that passes through B.
2. Draw an arc with center B that passes through A.
3. The intersection of the arcs is point C. $\triangle ABC$ is equilateral.





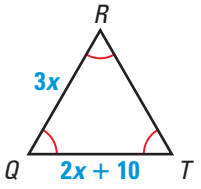


By the Triangle Sum Theorem, the measures of the three congruent angles in an equilateral triangle must add up to 180°. So, each angle in an equilateral triangle measures 60°.

Using Algebra

EXAMPLE 3 Find the Side Length of an Equiangular Triangle

Find the length of each side of the equiangular triangle.



Solution

The angle marks show that $\triangle QRT$ is equiangular. So, $\triangle QRT$ is also equilateral.

$3x = 2x + 10$ — Sides of an equilateral $\triangle$ are congruent.

$x = 10$ — Subtract $2x$ from each side.

$3(10) = 30$ — Substitute 10 for x.

ANSWER Each side of $\triangle QRT$ is 30.

4.3 Exercises

Guided Practice

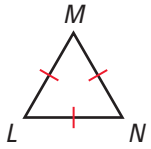
Vocabulary Check

1. What is the difference between *equilateral* and *equiangular*?

Skill Check

Tell which sides and angles of the triangle are congruent.

2.



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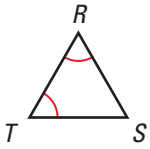

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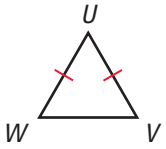
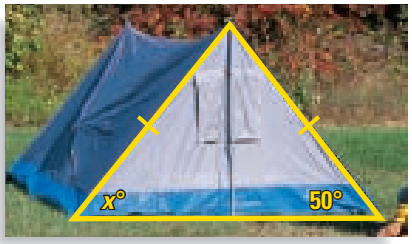


Find the value of *x*. Tell what theorem(s) you used.

5.



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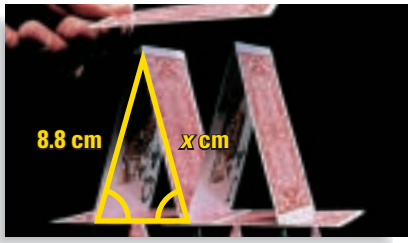


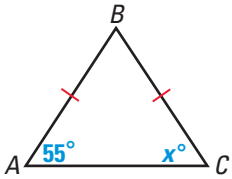
Practice and Applications

Extra Practice

See p. 681.

Finding Measures **Find the value of *x*. Tell what theorem(s) you used.**

7.



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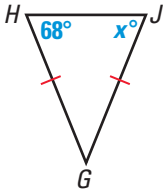
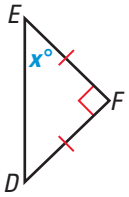
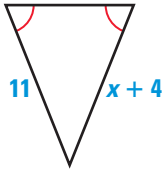


9.



Using Algebra **Find the value of *x*.**

10.



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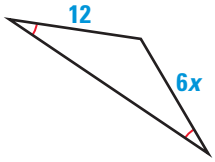

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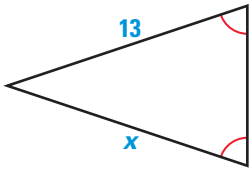

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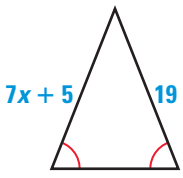

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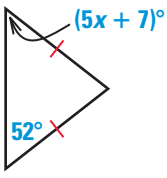
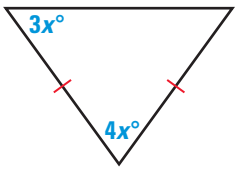


15.



Homework Help

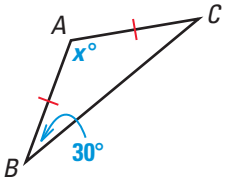
Example 1: Exs. 7–9, 14, 15, 17–19, 27, 28
Example 2: Exs. 10–13
Example 3: Exs. 20–25

16. **You be the Judge** Someone in your class tells you that all equilateral triangles are isosceles triangles. Do you agree? Use theorems or definitions to support your answer.

Student Help
CLASSZONE.COM
HOMEWORK HELP
Extra help with problem solving in Exs. 17–19 is at classzone.com

Using Algebra **Find the measure of $\angle A$.**

17.



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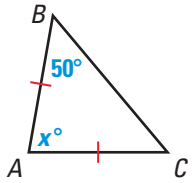

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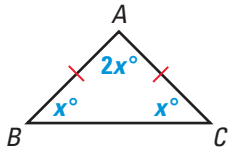
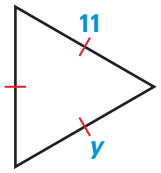


Using Algebra **Find the value of y.**

20.



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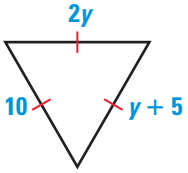

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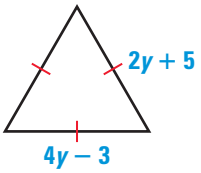

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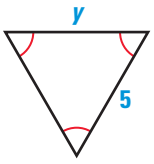

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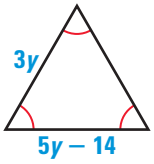

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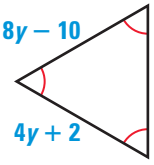
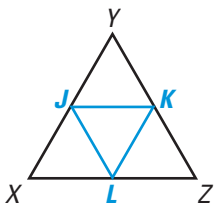


Link to Sports

ROCK CLIMBING The climber is using a method of rock climbing called *top roping*. If the climber slips, the anchors catch the fall.

Application Links
CLASSZONE.COM

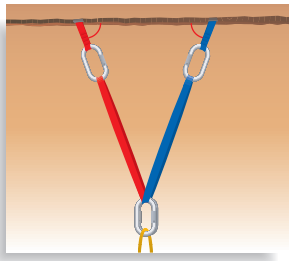
26. **Challenge** In the diagram at the right, $\triangle XYZ$ is equilateral and the following pairs of segments are parallel: $\overline{XY}$ and $\overline{LK}$; $\overline{ZY}$ and $\overline{LJ}$; $\overline{XZ}$ and $\overline{JK}$. Describe a plan for showing that $\triangle JKL$ must be equilateral.

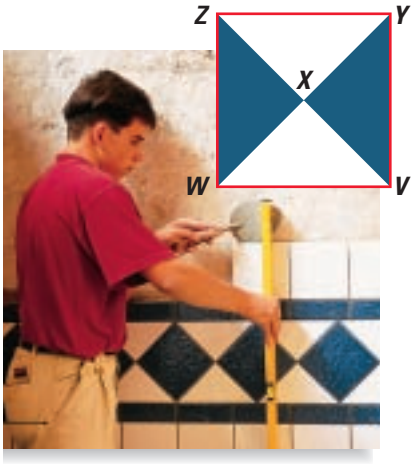


Rock Climbing **In one type of rock climbing, climbers tie themselves to a rope that is supported by anchors. The diagram shows a red and a blue anchor in a horizontal slit in a rock face.**

27. If the red anchor is longer than the blue anchor, are the base angles congruent?

28. If a climber adjusts the anchors so they are the same length, do you think that the base angles will be congruent? Why or why not?





Tiles **In Exercises 29–31, use the diagram at the left. In the diagram, $\overline{VX} \cong \overline{WX} \cong \overline{YX} \cong \overline{ZX}$.**

29. Copy the diagram. Use what you know about side lengths to mark your diagram.

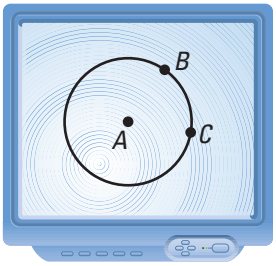
30. Explain why $\angle XWV \cong \angle XVW$.

31. Name four isosceles triangles.

32. Technology Use geometry software to complete the steps.

1. Construct circle A.
2. Draw points B and C on the circle.
3. Connect the points to form $\triangle ABC$.

Is $\triangle ABC$ isosceles? Measure the sides of the triangle to check your answer.



Standardized Test Practice

Multiple Choice **In Exercises 33 and 34, use the diagram below.**

33. What is the measure of $\angle EFD$?

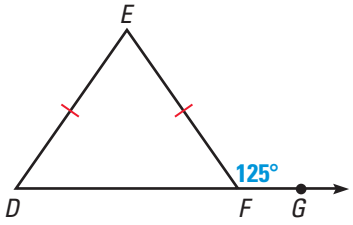
Ⓐ 55° Ⓑ 65°

Ⓒ 125° Ⓓ 180°

34. What is the measure of $\angle DEF$?

Ⓕ 50° Ⓖ 70°

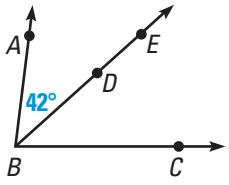
Ⓗ 125° Ⓙ 180°



Mixed Review

Angle Bisectors **$\overrightarrow{BE}$ is the angle bisector. Find $m\angle DBC$ and $m\angle ABC$.** *(Lesson 2.2)*

35.



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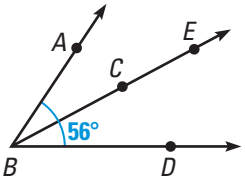

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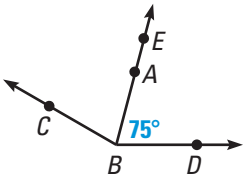
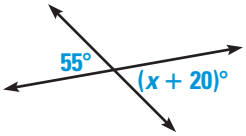
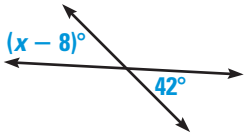


Vertical Angles **Find the value of the variable.** *(Lesson 2.4)*

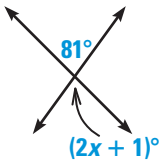
38.



39.



40.



Algebra Skills

Evaluating Square Roots **Evaluate.** *(Skills Review, p. 668)*

41. $\sqrt{49}$ **42.** $\sqrt{121}$ **43.** $\sqrt{1}$ **44.** $\sqrt{400}$