The Converse of the Pythagorean Theorem

A gardener can use the Converse of the Pythagorean Theorem to make sure that the corners of a garden bed form right angles.

In the photograph, a triangle with side lengths 3 feet, 4 feet, and 5 feet ensures that the angle at one corner is a right angle.

THEOREM 4.8

The Converse of the Pythagorean Theorem

Words If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

Symbols If \( c^2 = a^2 + b^2 \), then \( \triangle ABC \) is a right triangle.

EXAMPLE 1 Verify a Right Triangle

Is \( \triangle ABC \) a right triangle?

Solution

Let \( c \) represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation \( c^2 = a^2 + b^2 \).

\[
\begin{align*}
    c^2 & \geq a^2 + b^2 \\
    20^2 & \geq 12^2 + 16^2 \\
    400 & \geq 144 + 256 \\
    400 & = 400
\end{align*}
\]

\( \text{ANSWER} \) It is true that \( c^2 = a^2 + b^2 \). So, \( \triangle ABC \) is a right triangle.
4.5 The Converse of the Pythagorean Theorem

**Classifying Triangles** You can determine whether a triangle is acute, right, or obtuse by its side lengths.

**CLASSIFYING TRIANGLES**

In \( \triangle ABC \) with longest side \( c \):

- If \( c^2 < a^2 + b^2 \), then \( \triangle ABC \) is **acute**.

- If \( c^2 = a^2 + b^2 \), then \( \triangle ABC \) is **right**.

- If \( c^2 > a^2 + b^2 \), then \( \triangle ABC \) is **obtuse**.

**EXAMPLE 2 Acute Triangles**

Show that the triangle is an acute triangle.

**Solution**

Compare the side lengths.

\[
\sqrt{35} \leq 5.9, \text{ so use } \sqrt{35} \text{ as the value of } c, \text{ the longest side length of the triangle.}
\]

\[
\sqrt{35}^2 = a^2 + b^2 \\
35 \leq a^2 + b^2
\]

Substitute \( \sqrt{35} \) for \( c \), 4 for \( a \), and 5 for \( b \).

\[
35 \leq 16 + 25 \\
35 < 41 \quad \text{Multiply.}
\]

**Answer** Because \( c^2 < a^2 + b^2 \), the triangle is acute.

**EXAMPLE 3 Obtuse Triangles**

Show that the triangle is an obtuse triangle.

**Solution**

Compare the side lengths.

\[
c^2 \geq a^2 + b^2 \\
(15)^2 \geq 8^2 + 12^2
\]

Substitute 15 for \( c \), 8 for \( a \), and 12 for \( b \).

\[
225 \geq 64 + 144 \\
225 > 208 \quad \text{Simplify.}
\]

**Answer** Because \( c^2 > a^2 + b^2 \), the triangle is obtuse.
EXAMPLE 4 Classify Triangles

Classify the triangle as acute, right, or obtuse.

Solution

Compare the square of the length of the longest side with the sum of the squares of the lengths of the two shorter sides.

\[ c^2 \neq a^2 + b^2 \]  
Compare \( c^2 \) with \( a^2 + b^2 \).

\[ 8^2 \neq 5^2 + 6^2 \]  
Substitute 8 for \( c \), 5 for \( a \), and 6 for \( b \).

\[ 64 \neq 25 + 36 \]  
Multiply.

\[ 64 > 61 \]  
Simplify.

ANSWER Because \( c^2 > a^2 + b^2 \), the triangle is obtuse.

EXAMPLE 5 Classify Triangles

Classify the triangle with the given side lengths as acute, right, or obtuse.

a. \( 4, 6, 7 \)  

b. \( 12, 35, 37 \)

Solution

a. \[ c^2 \neq a^2 + b^2 \]

\[ 7^2 \neq 4^2 + 6^2 \]

\[ 49 \neq 16 + 36 \]

\[ 49 < 52 \]

The triangle is acute.

b. \[ c^2 \neq a^2 + b^2 \]

\[ 37^2 \neq 12^2 + 35^2 \]

\[ 1369 \neq 144 + 1225 \]

\[ 1369 = 1369 \]

The triangle is right.

Checkpoint

Classify the triangle as acute, right, or obtuse. Explain.

1. 

2. 

3. 

4. \( 7, 24, 24 \)  

5. \( 7, 24, 25 \)  

6. \( 7, 24, 26 \)
4.5 Exercises

Guided Practice

Vocabulary Check

1. Write the Converse of the Pythagorean Theorem in your own words.

Determine whether the triangle is acute, right, or obtuse.

2. 

3. 

4. 

Skill Check

Match the side lengths of a triangle with the best description.

5. 2, 10, 11

6. 8, 5, 7

7. 5, 5, 5

8. 6, 8, 10

A. right

B. acute

C. obtuse

D. equiangular

Practice and Applications

Extra Practice

See p. 682.

Verifying Right Triangles Show that the triangle is a right triangle.

9. 

10. 

11. 

Verifying Acute Triangles Show that the triangle is an acute triangle.

12. 

13. 

14. 

Verifying Obtuse Triangles Show that the triangle is an obtuse triangle.

15. 

16. 

17. 

Homework Help

Example 1: Exs. 9–11, 24
Example 2: Exs. 12–14
Example 3: Exs. 15–17
Example 4: Exs. 18–23, 37–38
Example 5: Exs. 25–36

4.5 The Converse of the Pythagorean Theorem
Classifying Triangles  Classify the triangle as acute, right, or obtuse.

18. 19. 20.

21. 22. 23.

24. Early Mathematics  The Babylonian tablet shown at the left contains several sets of triangle side lengths, suggesting that the Babylonians may have been aware of the relationships among the side lengths of right triangles. The side lengths in the table below show several sets of numbers from the tablet. Use a calculator to verify that each set of side lengths satisfies the Pythagorean Theorem.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>120</td>
<td>119</td>
<td>169</td>
</tr>
<tr>
<td>4,800</td>
<td>4,601</td>
<td>6,649</td>
</tr>
<tr>
<td>13,500</td>
<td>12,709</td>
<td>18,541</td>
</tr>
</tbody>
</table>

Classifying Triangles  Classify the triangle with the given side lengths as acute, right, or obtuse.

25. 20, 99, 101  26. 21, 28, 35  27. 26, 10, 17
28. 7, 10, 11  29. 4, $\sqrt{67}$, 9  30. $\sqrt{13}$, 6, 7
31. 468, 595, 757  32. 10, 11, 14  33. 4, 5, 5
34. 17, 144, 145  35. 10, 49, 50  36. $\sqrt{5}$, 5, 5.5

Air Travel  In Exercises 37 and 38, use the map below.

37. Use the distances given on the map to tell whether the triangle formed by the three cities is a right triangle.

38. Cincinnati is directly west of Washington, D.C. Is Tallahassee directly south of Cincinnati? Explain your answer.
39. **You be the Judge** A classmate tells you if you find three side lengths that form a right triangle and double each of them, the sides will form an obtuse triangle. Is your classmate correct? Explain.

**Challenge** Graph points \( P, Q, \) and \( R \). Connect the points to form \( \triangle PQR \). Decide whether \( \triangle PQR \) is **acute**, **right**, or **obtuse**.

40. \( P(-3, 4), Q(5, 0), R(-6, -2) \)

41. \( P(-1, 2), Q(4, 1), R(0, -1) \)

42. **Multi-Step Problem** A double play occurs in baseball when two outs are made on a single play. In the diagram shown, the ball is hit to the player at point \( A \). A double play is made when the player at point \( A \) throws the ball to the player at point \( B \) who in turn throws it to the player at point \( C \).

- **a.** Use the diagram to determine what kind of triangle is formed by points \( A, B, \) and \( C \).
- **b.** What kind of triangle is formed by points \( A, B, \) and \( C \) if the distance between points \( A \) and \( C \) is 99 feet?
- **c.** **Critical Thinking** Find values for \( AB \) and \( AC \) that would make \( \triangle ABC \) in the diagram a right triangle if \( BC = 90 \) feet.

### Mixed Review

**Finding Measures** Find the value of \( x \). *(Lesson 4.3)*

43. \[ \angle x = 67^\circ \]

44. \[ 3x - 4 = 8 \]

45. \[ 5x^\circ \]

### Algebra Skills

**Multiplying Fractions** Multiply. Write the answer as a fraction or a mixed number in simplest form. *(Skills Review, p. 659)*

46. \[ \frac{1}{2} \times \frac{4}{5} \]

47. \[ \frac{3}{8} \times \frac{3}{4} \]

48. \[ \frac{3}{11} \times \frac{11}{12} \]

49. \[ \frac{3}{5} \times \frac{5}{9} \]

50. \[ \frac{3}{4} \times 6 \]

51. \[ 8 \times 1\frac{3}{4} \]

52. \[ \frac{1}{3} \times \frac{4}{9} \]

53. \[ \frac{5}{4} \times \frac{2}{3} \]

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