4.5 The Converse of the Pythagorean Theorem

Goal

Use the Converse of Pythagorean Theorem.
Use side lengths to classify triangles.

Key Words

- converse p. 136

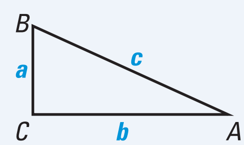
A gardener can use the Converse of the Pythagorean Theorem to make sure that the corners of a garden bed form right angles.

In the photograph, a triangle with side lengths 3 feet, 4 feet, and 5 feet ensures that the angle at one corner is a right angle.

THEOREM 4.8

The Converse of the Pythagorean Theorem

Words If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.



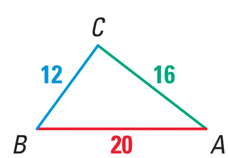
Symbols If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.

Student Help

LOOK BACK For the definition of converse, see p. 136.

EXAMPLE 1 Verify a Right Triangle

Is $\triangle ABC$ a right triangle?



Solution

Let c represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

$c^2 \stackrel{?}{=} a^2 + b^2$	Compare c^2 with $a^2 + b^2$.
$20^2 \stackrel{?}{=} 12^2 + 16^2$	Substitute 20 for c, 12 for a, and 16 for b.
$400 \stackrel{?}{=} 144 + 256$	Multiply.
$400 = 400$	Simplify.

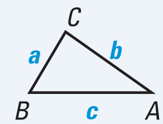
ANSWER ▸ It is true that $c^2 = a^2 + b^2$. So, $\triangle ABC$ is a right triangle.

Classifying Triangles You can determine whether a triangle is acute, right, or obtuse by its side lengths.

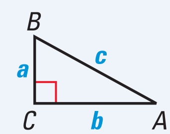
CLASSIFYING TRIANGLES

In $\triangle ABC$ with longest side c:

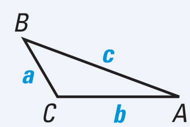
If $c^2 < a^2 + b^2$, then $\triangle ABC$ is *acute*.



If $c^2 = a^2 + b^2$, then $\triangle ABC$ is *right*.



If $c^2 > a^2 + b^2$, then $\triangle ABC$ is *obtuse*.

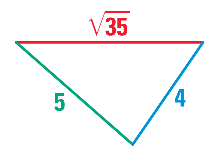


Student Help

STUDY TIP
This is the Converse of the Pythagorean Theorem.

EXAMPLE 2 Acute Triangles

Show that the triangle is an acute triangle.



Solution

Compare the side lengths.

$c^2 \stackrel{?}{=} a^2 + b^2$	Compare c^2 with $a^2 + b^2$.
$(\sqrt{35})^2 \stackrel{?}{=} 4^2 + 5^2$	Substitute $\sqrt{35}$ for c, 4 for a, and 5 for b.
$35 \stackrel{?}{=} 16 + 25$	Multiply.
$35 < 41$	Simplify.

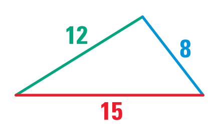
Student Help

STUDY TIP
$\sqrt{35} \approx 5.9$, so use $\sqrt{35}$ as the value of c, the longest side length of the triangle.

ANSWER ▶ Because $c^2 < a^2 + b^2$, the triangle is acute.

EXAMPLE 3 Obtuse Triangles

Show that the triangle is an obtuse triangle.



Solution

Compare the side lengths.

$c^2 \stackrel{?}{=} a^2 + b^2$	Compare c^2 with $a^2 + b^2$.
$(15)^2 \stackrel{?}{=} 8^2 + 12^2$	Substitute 15 for c, 8 for a, and 12 for b.
$225 \stackrel{?}{=} 64 + 144$	Multiply.
$225 > 208$	Simplify.

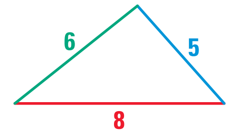
ANSWER ▶ Because $c^2 > a^2 + b^2$, the triangle is obtuse.

Student Help
CLASSZONE.COM

MORE EXAMPLES
More examples at classzone.com

EXAMPLE 4 Classify Triangles

Classify the triangle as *acute*, *right*, or *obtuse*.



Solution

Compare the square of the length of the longest side with the sum of the squares of the lengths of the two shorter sides.

$c^2 \stackrel{?}{=} a^2 + b^2$ Compare c^2 with $a^2 + b^2$.

$8^2 \stackrel{?}{=} 5^2 + 6^2$ Substitute 8 for *c*, 5 for *a*, and 6 for *b*.

$64 \stackrel{?}{=} 25 + 36$ Multiply.

$64 > 61$ Simplify.

ANSWER ▶ Because $c^2 > a^2 + b^2$, the triangle is obtuse.

EXAMPLE 5 Classify Triangles

Classify the triangle with the given side lengths as *acute*, *right*, or *obtuse*.

a. 4, 6, 7

b. 12, 35, 37

Solution

a. $c^2 \stackrel{?}{=} a^2 + b^2$

$7^2 \stackrel{?}{=} 4^2 + 6^2$

$49 \stackrel{?}{=} 16 + 36$

$49 < 52$

The triangle is acute.

b. $c^2 \stackrel{?}{=} a^2 + b^2$

$37^2 \stackrel{?}{=} 12^2 + 35^2$

$1369 \stackrel{?}{=} 144 + 1225$

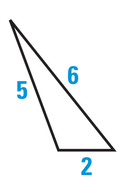
$1369 = 1369$

The triangle is right.

Checkpoint ✓ Classify Triangles

Classify the triangle as *acute, right,* or *obtuse*. Explain.

1.



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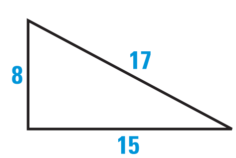

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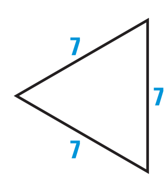


Use the side lengths to classify the triangle as *acute, right,* or *obtuse*.

4. 7, 24, 24

5. 7, 24, 25

6. 7, 24, 26

4.5 Exercises

Guided Practice

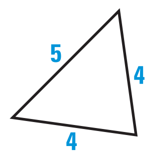
Vocabulary Check

1. Write the Converse of the Pythagorean Theorem in your own words.

Skill Check

Determine whether the triangle is *acute, right,* or *obtuse*.

2.



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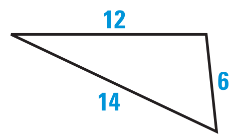

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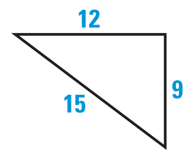


Match the side lengths of a triangle with the best description.

5. 2, 10, 11	**A.** right
6. 8, 5, 7	**B.** acute
7. 5, 5, 5	**C.** obtuse
8. 6, 8, 10	**D.** equiangular

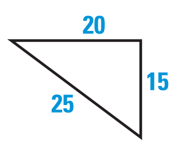
Practice and Applications

Extra Practice

See p. 682.

Verifying Right Triangles **Show that the triangle is a right triangle.**

9.



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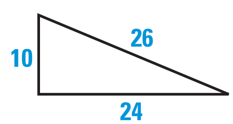

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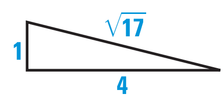
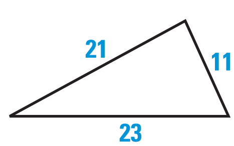


Verifying Acute Triangles **Show that the triangle is an acute triangle.**

12.



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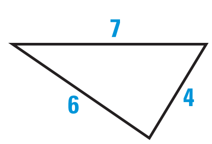

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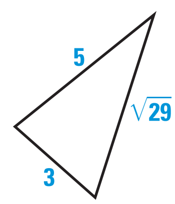
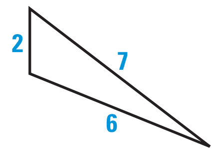


Homework Help

Example 1: Exs. 9–11, 24
Example 2: Exs. 12–14
Example 3: Exs. 15–17
Example 4: Exs. 18–23, 37–38
Example 5: Exs. 25–36

Verifying Obtuse Triangles **Show that the triangle is an obtuse triangle.**

15.



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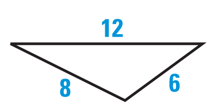

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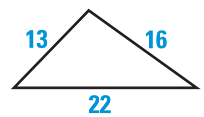
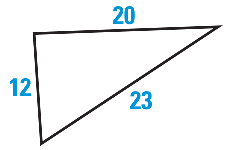


Classifying Triangles **Classify the triangle as *acute, right,* or *obtuse*.**

18.



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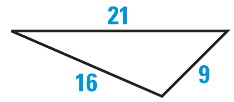

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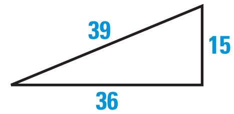

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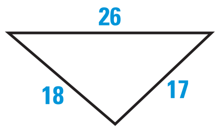

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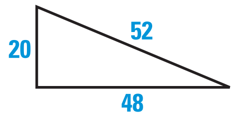

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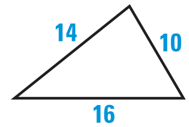


Link to History

EARLY MATHEMATICS
This photograph shows part of a Babylonian clay tablet made around 350 B.C. The tablet contains a table of numbers.

24. Early Mathematics The Babylonian tablet shown at the left contains several sets of triangle side lengths, suggesting that the Babylonians may have been aware of the relationships among the side lengths of right triangles. The side lengths in the table below show several sets of numbers from the tablet. Use a calculator to verify that each set of side lengths satisfies the Pythagorean Theorem.

a	*b*	*c*
120	119	169
4,800	4,601	6,649
13,500	12,709	18,541

Classifying Triangles **Classify the triangle with the given side lengths as *acute, right,* or *obtuse*.**

25. 20, 99, 101

26. 21, 28, 35

27. 26, 10, 17

28. 7, 10, 11

29. 4, $\sqrt{67}$, 9

30. $\sqrt{13}$, 6, 7

31. 468, 595, 757

32. 10, 11, 14

33. 4, 5, 5

34. 17, 144, 145

35. 10, 49, 50

36. $\sqrt{5}$, 5, 5.5

Air Travel **In Exercises 37 and 38, use the map below.**

37. Use the distances given on the map to tell whether the triangle formed by the three cities is a right triangle.

38. Cincinnati is directly west of Washington, D.C. Is Tallahassee directly south of Cincinnati? Explain your answer.

39. ***You be the Judge*** A classmate tells you if you find three side lengths that form a right triangle and double each of them, the sides will form an obtuse triangle. Is your classmate correct? Explain.

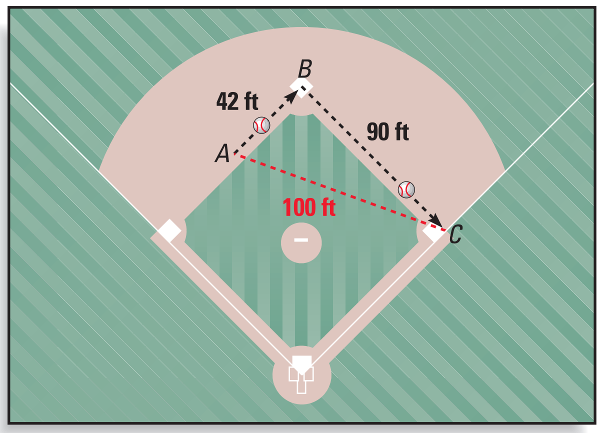
Challenge Graph points *P, Q,* and *R*. Connect the points to form $\triangle PQR$. Decide whether $\triangle PQR$ is *acute, right,* or *obtuse*.

40. $P(-3, 4)$, $Q(5, 0)$, $R(-6, -2)$

41. $P(-1, 2)$, $Q(4, 1)$, $R(0, -1)$

Standardized Test Practice

42. **Multi-Step Problem** A double play occurs in baseball when two outs are made on a single play. In the diagram shown, the ball is hit to the player at point *A*. A double play is made when the player at point *A* throws the ball to the player at point *B* who in turn throws it to the player at point *C*.



a. Use the diagram to determine what kind of triangle is formed by points *A*, *B*, and *C*.

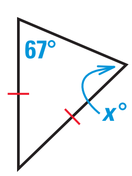
b. What kind of triangle is formed by points *A*, *B*, and *C* if the distance between points *A* and *C* is 99 feet?

c. **Critical Thinking** Find values for *AB* and *AC* that would make $\triangle ABC$ in the diagram a right triangle if $BC = 90$ feet.

Mixed Review

Finding Measures Find the value of *x*. *(Lesson 4.3)*

43.



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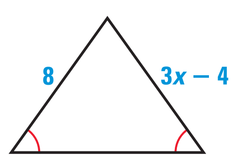

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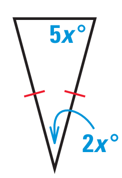

Algebra Skills

Multiplying Fractions Multiply. Write the answer as a fraction or a mixed number in simplest form. *(Skills Review, p. 659)*

46. $\frac{1}{2} \times \frac{4}{5}$

47. $\frac{3}{8} \times \frac{3}{4}$

48. $\frac{3}{11} \times \frac{11}{12}$

49. $\frac{3}{5} \times \frac{5}{9}$

50. $\frac{3}{4} \times 6$

51. $8 \times 1\frac{3}{4}$

52. $1\frac{1}{3} \times \frac{4}{9}$

53. $5\frac{1}{4} \times \frac{2}{3}$