6.2 Properties of Parallelograms

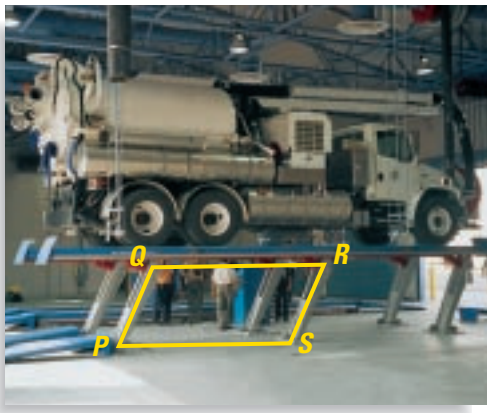
Goal

Use properties of parallelograms.

Key Words

- parallelogram

Parallelogram lifts, like the one shown in the photograph, are used to raise heavy-duty vehicles.



A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.

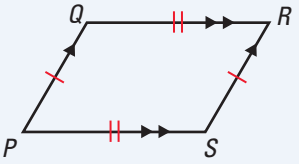
The symbol $\square PQRS$ is read "parallelogram $PQRS$."

In $\square PQRS$, $\overline{PQ} \parallel \overline{SR}$ and $\overline{QR} \parallel \overline{PS}$.

THEOREM 6.2

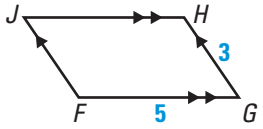
Words If a quadrilateral is a parallelogram, then its opposite sides are congruent.

Symbols In $\square PQRS$, $\overline{PQ} \cong \overline{SR}$ and $\overline{QR} \cong \overline{PS}$.



EXAMPLE 1 Find Side Lengths of Parallelograms

$FGHJ$ is a parallelogram. Find JH and FJ.



Solution

$JH = FG$ — Opposite sides of a $\square$ are congruent.

$= 5$ — Substitute 5 for FG.

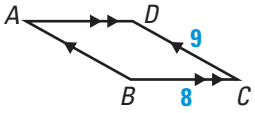
$FJ = GH$ — Opposite sides of a $\square$ are congruent.

$= 3$ — Substitute 3 for GH.

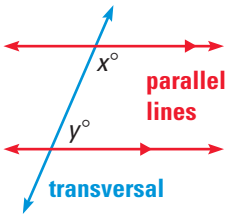
ANSWER In $\square FGHJ$, $JH = 5$ and $FJ = 3$.

Checkpoint Find Side Lengths of Parallelograms

1. $ABCD$ is a parallelogram. Find AB and AD.



Visualize It!



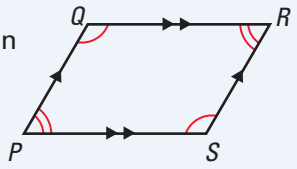
Consecutive angles of a parallelogram are like same-side interior angles. By Theorem 3.7, they are supplementary.

THEOREMS 6.3 and 6.4

Theorem 6.3

Words If a quadrilateral is a parallelogram, then its opposite angles are congruent.

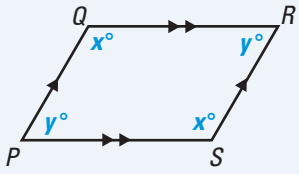
Symbols In $\square PQRS$, $\angle P \cong \angle R$ and $\angle Q \cong \angle S$.



Theorem 6.4

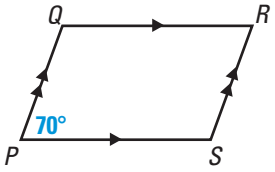
Words If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

Symbols In $\square PQRS$, $x° + y° = 180°$.



EXAMPLE 2 Find Angle Measures of Parallelograms

$PQRS$ is a parallelogram. Find the missing angle measures.



Solution

1. By Theorem 6.3, the opposite angles of a parallelogram are congruent, so $m\angle R = m\angle P = 70°$.

2. By Theorem 6.4, the consecutive angles of a parallelogram are supplementary.

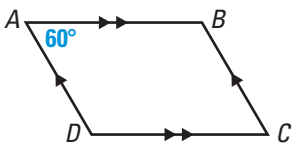
$$m\angle Q + m\angle P = 180° \quad \text{Consecutive angles of a } \square \text{ are supplementary.}$$
$$m\angle Q + 70° = 180° \quad \text{Substitute } 70° \text{ for } m\angle P.$$
$$m\angle Q = 110° \quad \text{Subtract } 70° \text{ from each side.}$$

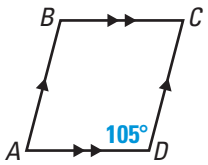
3. By Theorem 6.3, the opposite angles of a parallelogram are congruent, so $m\angle S = m\angle Q = 110°$.

ANSWER The measure of $\angle R$ is 70°, the measure of $\angle Q$ is 110°, and the measure of $\angle S$ is 110°.

Checkpoint Find Angle Measures of Parallelograms

***ABCD* is a parallelogram. Find the missing angle measures.**

2. 

3. 

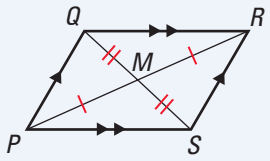
Student Help

LOOK BACK
To review the definition of bisect, see p. 53.

THEOREM 6.5

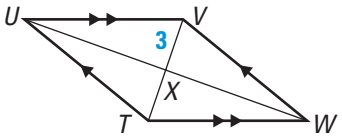
Words If a quadrilateral is a parallelogram, then its diagonals bisect each other.

Symbols In $\square PQRS$, $\overline{QM} \cong \overline{MS}$ and $\overline{PM} \cong \overline{MR}$.



EXAMPLE 3 Find Segment Lengths

$TUVW$ is a parallelogram. Find TX.



Solution

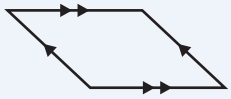
$TX = XV$ Diagonals of a $\square$ bisect each other.

$= 3$ Substitute 3 for XV.

SUMMARY PROPERTIES OF PARALLELOGRAMS

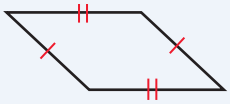
Definition of parallelogram, p. 310

If a quadrilateral is a parallelogram, then both pairs of opposite sides are parallel.



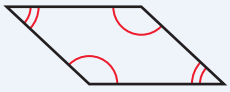
Theorem 6.2, p. 310

If a quadrilateral is a parallelogram, then its opposite sides are congruent.



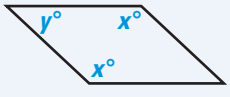
Theorem 6.3, p. 311

If a quadrilateral is a parallelogram, then its opposite angles are congruent.



Theorem 6.4, p. 311

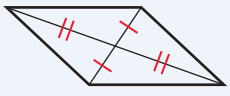
If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.



$x° + y° = 180°$

Theorem 6.5, p. 312

If a quadrilateral is a parallelogram, then its diagonals bisect each other.



6.2 Exercises

Guided Practice

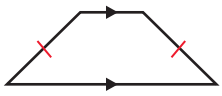
Vocabulary Check

1. Complete the statement: A(n) __?__ is a quadrilateral with both pairs of opposite sides parallel.

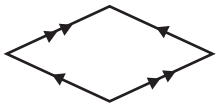
Skill Check

Decide whether the figure is a parallelogram. If it is not, explain why.

2.



3.



Complete the statement. Give a reason for your answer.

4. $\overline{JK} \cong$ __?__

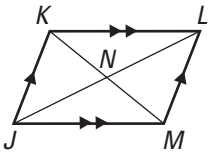
5. $\angle MLK \cong$ __?__

6. $\angle JKL \cong$ __?__

7. $\overline{JN} \cong$ __?__

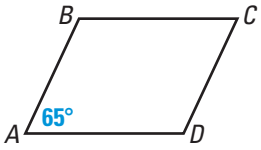
8. $\angle MNL \cong$ __?__

9. $\overline{NM} \cong$ __?__

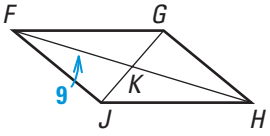


Find the measure in the parallelogram.

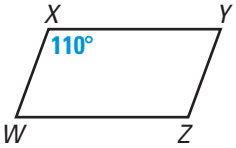
10. Find $m\angle C$.



11. Find HK.



12. Find $m\angle Y$.



Practice and Applications

Extra Practice

See p. 685.

Congruent Segments **Match the segment in $\square PQRS$ with a congruent one. Give a reason for your answer.**

13. $\overline{PT}$

14. $\overline{QR}$

15. $\overline{QT}$

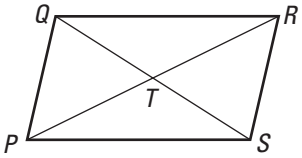
16. $\overline{PQ}$

A. $\overline{RS}$

B. $\overline{RT}$

C. $\overline{PS}$

D. $\overline{ST}$



Homework Help

Example 1: Exs. 13–16, 22–24
Example 2: Exs. 17–20, 25–27
Example 3: Exs. 13–16, 28–30

Congruent Angles **Match the angle in $\square VWXY$ with a congruent one. Give a reason for your answer.**

17. $\angle VZY$

18. $\angle WVY$

19. $\angle WXZ$

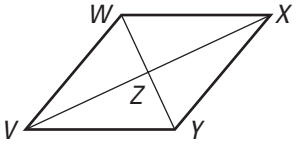
20. $\angle VYX$

E. $\angle WZX$

F. $\angle VWX$

G. $\angle YVZ$

H. $\angle YXW$



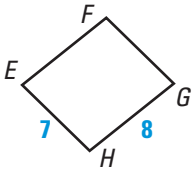
Student Help

VISUAL STRATEGY
In Ex. 21, use lined paper to help you sketch a parallelogram, as shown on on p. 302.

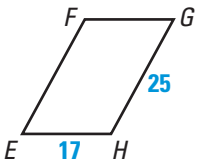
21. You be the Judge $EFGH$ is a parallelogram. Is $\overline{EF}$ parallel to $\overline{HG}$ or $\overline{GF}$? Explain your answer.

Finding Side Lengths ***EFGH*** **is a parallelogram. Find *EF* and *FG*.**

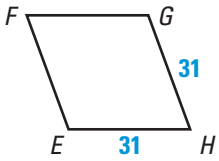
22.



23.

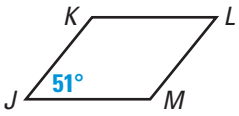


24.



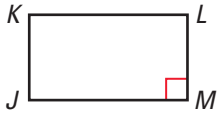
Finding Angle Measures ***JKLM*** **is a parallelogram. Find the missing angle measures.**

25.



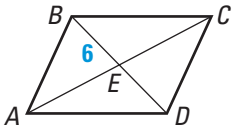
26.

27.

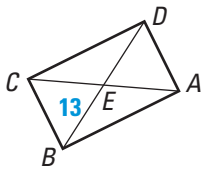


Finding Segment Lengths ***ABCD*** **is a parallelogram. Find *DE*.**

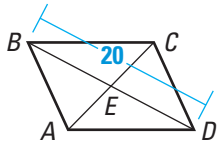
28.



29.



30.

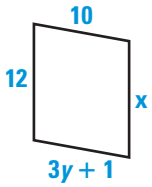


Link to Photography

SCISSORS LIFT
Photographers can use scissors lifts for overhead shots. The crossing beams of the lift form parallelograms that raise and lower the platform. For more about scissors lifts, see p. 300.

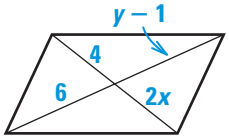
Using Algebra **Find the values of *x* and *y* in the parallelogram.**

31.



32.

33.



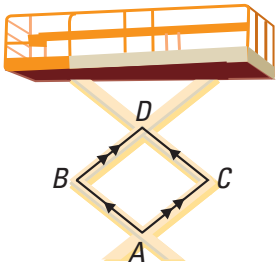
Scissors Lift **Use the diagram of the scissors lift below.**

34. What is $m\angle B$ when $m\angle A$ is 120°?

35. Suppose you decrease $m\angle A$. What happens to $m\angle B$?

36. Suppose you decrease $m\angle A$. What happens to AD?

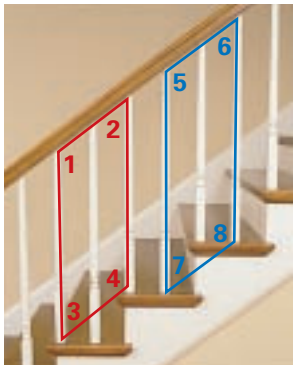
37. Suppose you decrease $m\angle A$. What happens to the overall height of the scissors lift?



Student Help
CLASSZONE.COM

HOMEWORK HELP
Extra help with problem solving in Exs. 38–41 is at classzone.com

Staircases **In the diagram below, the red quadrilateral and the blue quadrilateral are parallelograms.**



38. Which angle in the red parallelogram is congruent to $\angle 1$?

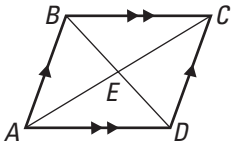
39. Which angles in the blue parallelogram are supplementary to $\angle 6$?

40. Which postulate can be used to prove that $\angle 1 \cong \angle 5$?

41. Challenge Is the red parallelogram congruent to the blue parallelogram? Explain your reasoning.

Standardized Test Practice

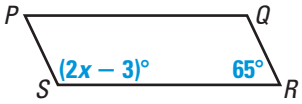
42. Multiple Choice Which of the following statements is *not* necessarily true about $\square ABCD$?



Ⓐ $AE = CE$ Ⓑ $AD = BC$

Ⓒ $BE = DE$ Ⓓ $AC = BD$

43. Multiple Choice $PQRS$ is a parallelogram. What is the value of x?



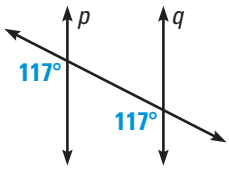
Ⓕ 28 Ⓖ 34

Ⓗ 59 Ⓙ 121

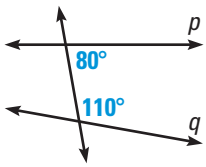
Mixed Review

Parallel Lines **Are lines p and q parallel? Explain.** *(Lesson 3.5)*

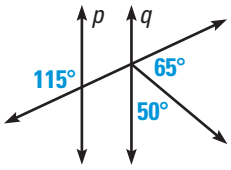
44.



45.

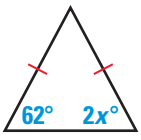


46.

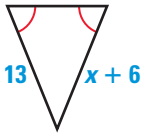


Isosceles and Equilateral Triangles **Find the value of x.** *(Lesson 4.3)*

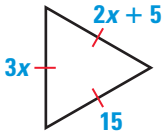
47.



48.



49.



Algebra Skills

Finding Slope **Find the slope of the line that passes through the points.** *(Skills Review, p. 665)*

50. (1, 3) and (6, 5)

51. (3, −8) and (7, 4)

52. (2, 1) and (−1, 0)

53. (−4, 2) and (5, −1)

54. (6, −2) and (12, 14)

55. (0, −3) and (−5, −6)