**Goal**
Identify similar polygons.

**Key Words**
- similar polygons
- scale factor

In geometry, two figures that have the same shape are called *similar*. Two polygons are similar if corresponding angles are congruent and corresponding side lengths are proportional.

In the diagram, \(ABCD\) is similar to \(EFGH\). The symbol \(\sim\) indicates similarity. So, you can write \(ABCD \sim EFGH\). When you refer to similar polygons, list their corresponding vertices in the same order.

### Example 1: Use Similarity Statements

\( \triangle PRQ \sim \triangle STU \)

**a.** List all pairs of congruent angles.

\( \angle P \cong \angle S, \angle R \cong \angle T, \) and \( \angle Q \cong \angle U. \)

**b.** Write the ratios of the corresponding sides in a statement of proportionality.

\[
\frac{ST}{PR} = \frac{TU}{RQ} = \frac{US}{QP}
\]

**c.** Check that the ratios of corresponding sides are equal.

\[
\frac{ST}{PR} = \frac{8}{10} = \frac{4}{5}, \quad \frac{TU}{RQ} = \frac{16}{20} = \frac{4}{5}, \quad \text{and} \quad \frac{US}{QP} = \frac{12}{15} = \frac{4}{5}
\]

The ratios of corresponding sides are all equal to \(\frac{4}{5}\).
Scale Factor If two polygons are similar, then the ratio of the lengths of two corresponding sides is called the **scale factor**.

\[
\text{scale factor of } EFGH \text{ to } ABCD = \frac{EF}{AB} = \frac{FG}{BC} = \frac{GH}{CD} = \frac{HE}{DA}
\]

**EXAMPLE 2** Determine Whether Polygons are Similar

Determine whether the triangles are similar. If they are similar, write a similarity statement and find the scale factor of Figure B to Figure A.

### Solution

1. Check whether the corresponding angles are congruent.
   
   From the diagram, you can see that \( \angle G \cong \angle M, \angle H \cong \angle K, \) and \( \angle J \cong \angle L. \) Therefore, the corresponding angles are congruent.

2. Check whether the corresponding side lengths are proportional.

   \[
   \begin{align*}
   \frac{MK}{GH} &= \frac{12}{9} = \frac{4}{3} \\
   \frac{KL}{HJ} &= \frac{16}{12} = \frac{4}{3} \\
   \frac{LM}{JG} &= \frac{20}{15} = \frac{4}{3}
   \end{align*}
   \]

   All three ratios are equal, so the corresponding side lengths are proportional.

**ANSWER** By definition, the triangles are similar. \( \triangle GHJ \sim \triangle MKL. \)

The scale factor of Figure B to Figure A is \( \frac{4}{3}. \)

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**Checkpoint** Determine Whether Polygons are Similar

Determine whether the polygons are similar. If they are similar, write a similarity statement and find the scale factor of Figure B to Figure A.

1. \( \triangle XYZ \) and \( \triangle EDF \)

2. \( \quad \triangle ABC \) and \( \quad \square QRSU \)
### Example 3 Use Similar Polygons

\( \triangle RST \sim \triangle GHJ \).

Find the value of \( x \).

**Solution**

Because the triangles are similar, the corresponding side lengths are proportional. To find the value of \( x \), you can use the following proportion.

\[
\frac{GH}{RS} = \frac{JG}{TR}
\]

**Write proportion.**

\[
\frac{15}{10} = \frac{9}{x}
\]

Substitute 15 for \( GH \), 10 for \( RS \), 9 for \( JG \), and \( x \) for \( TR \).

\[
15 \cdot x = 10 \cdot 9
\]

Cross product property

\[
15x = 90
\]

Multiply.

\[
\frac{15x}{15} = \frac{90}{15}
\]

Divide each side by 15.

\[
x = 6
\]

Simplify.

### Example 4 Perimeters of Similar Polygons

The outlines of a pool and the patio around the pool are similar rectangles.

a. Find the ratio of the length of the patio to the length of the pool.

b. Find the ratio of the perimeter of the patio to the perimeter of the pool.

**Solution**

a. The ratio of the length of the patio to the length of the pool is

\[
\frac{\text{length of patio}}{\text{length of pool}} = \frac{48 \text{ feet}}{32 \text{ feet}} = \frac{48}{32} = \frac{3}{2}.
\]

b. The perimeter of the patio is \( 2(24) + 2(48) = 144 \) feet. The perimeter of the pool is \( 2(16) + 2(32) = 96 \) feet. The ratio of the perimeter of the patio to the perimeter of the pool is

\[
\frac{\text{perimeter of patio}}{\text{perimeter of pool}} = \frac{144 \text{ feet}}{96 \text{ feet}} = \frac{144}{96} = \frac{3}{2}.
\]
In Example 4 on the previous page, notice that the ratio of the perimeters of the similar figures is equal to the ratio of the side lengths. This observation is generalized in the following theorem.

**THEOREM 7.1**

**Perimeters of Similar Polygons**

**Words** If two polygons are similar, then the ratio of their perimeters is equal to the ratio of their corresponding side lengths.

**Symbols** If \( \triangle ABC \sim \triangle DEF \), then \[
\frac{DE}{EF} = \frac{FD}{CA}.
\]

**Checkpoint Use Similar Polygons**

In the diagram, \( \triangle PQR \sim \triangle STU \).

3. Find the value of \( x \).

4. Find the ratio of the perimeter of \( \triangle STU \) to the perimeter of \( \triangle PQR \).

**7.2 Exercises**

**Guided Practice**

1. If two triangles are congruent, must they be similar? Explain.
2. If two triangles are similar, must they be congruent? Explain.

**Skill Check**

In Exercises 3–6, \( \triangle ABC \sim \triangle LMN \).

3. List all pairs of congruent angles.
4. Write the ratios of the corresponding sides in a statement of proportionality.
5. Find the scale factor of \( \triangle LMN \) to \( \triangle ABC \).
6. Find the value of \( x \).

7. Are the two rectangles shown at the right similar? Explain your reasoning.
Using Similarity Statements  List all pairs of congruent angles. Then write the ratios of the corresponding sides in a statement of proportionality.

8. \( \triangle PQR \sim \triangle DEF \)

9. \( ABCDE \sim QRSTU \)

10. Error Analysis  \( \triangle FGH \sim \triangle JKL \). A student was asked to list all pairs of congruent angles and write the ratios of the corresponding sides in a statement of proportionality. Copy the diagram and correct the student’s errors.

\[
\angle F \cong \angle J, \quad \angle G \cong \angle L, \quad \angle H \cong \angle K
\]

\[
\frac{FG}{JK} = \frac{KL}{GH} = \frac{FH}{JL} \quad \times
\]

Determining Similarity  Determine whether the polygons are similar. If they are similar, write a similarity statement and find the scale factor of Figure B to Figure A.

11.

12.

13.

14.

Homework Help

Example 1: Exs. 8–10, 22, 23
Example 2: Exs. 11–16, 29
Example 3: Exs. 17–21, 24, 25
Example 4: Exs. 26, 27

7.2 Similar Polygons
21. **Mural**  Alejandro Romero created the mural, *Chicago Federation of Labor*, by enlarging the 56 in. by 21 in. sketch shown below. Romero used a scale factor of about 3.5. What are the dimensions of the mural in inches? In feet?

22. Find $m\angle N$.
23. Find $m\angle F$.
24. Find the value of $x$.
25. Find the value of $y$.
26. Find the ratio of the perimeter of $KLMN$ to the perimeter of $FGHJ$.

27. **Perimeters of Similar Polygons**  $\triangle QRS$ is similar to $\triangle XYZ$. The ratio of one side of $\triangle XYZ$ to the corresponding side of $\triangle QRS$ is 6 : 7. What is the ratio of the perimeter of $\triangle XYZ$ to the perimeter of $\triangle QRS$?

28. **Challenge**  $\triangle JKL$ is similar to $\triangle STU$. The ratio of $ST$ to $JK$ is 5 to 2. The perimeter of $\triangle STU$ is 35 feet. Find the perimeter of $\triangle JKL$. 

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**Using Similar Polygons**  The two polygons are similar. Find the values of $x$ and $y$.

17. $JL = 18$, $JK = 21$, $x$

18. $EP = 20$, $EY = 12$, $x$ and $y$

19. $KM = 10$, $KL = 11$, $x$

20. $LM = 16$, $LN = 15$, $x$ and $y$
29. **Television Screens**
The aspect ratio of a television screen is the length-to-width ratio of the screen. A standard definition television has an aspect ratio of 4 : 3. A high definition projection television has an aspect ratio of 16 : 9. Are the television screens similar rectangles?

**Logical Reasoning**
Are the polygons always, sometimes, or never similar?

30. Two isosceles triangles  
31. Two rhombuses  
32. Two equilateral triangles  
33. A right and an isosceles triangle

**Standardized Test Practice**

34. **Multiple Choice** \( \triangle DEF \sim \triangle MNP \). Which statement may be false?

   A. \( \angle E \cong \angle N \)  
   B. \( \angle P \cong \angle D \)  
   C. \( \frac{MN}{DE} = \frac{NP}{EF} \)  
   D. \( \frac{MN}{DE} = \frac{PM}{FD} \)

**Mixed Review**

**Showing Triangles are Congruent** Does the diagram give enough information to show that the triangles are congruent? If so, state the postulate or theorem you would use. (Lessons 5.3, 5.4)

35.  
36.  
37.

**Algebra Skills**

**Writing Equivalent Fractions** Write two equivalent fractions.  
(Skills Review, p. 656)

38. \( \frac{1}{3} \)  
39. \( \frac{2}{5} \)  
40. \( \frac{4}{7} \)  
41. \( \frac{9}{4} \)