### Goal
Find the measures of interior and exterior angles of polygons.

### Key Words
- interior angle  p. 181
- exterior angle  p. 181

The definitions for interior angles and exterior angles can be extended to include angles formed in any polygon. In the diagrams shown below, interior angles are red, and exterior angles are blue.

Activity 8.2 suggests the *Polygon Interior Angles Theorem* shown below.

#### THEOREM 8.1
**Polygon Interior Angles Theorem**

**Words** The sum of the measures of the interior angles of a convex polygon with \( n \) sides is \((n - 2) \cdot 180^\circ\).

**Symbols** \(m\angle 1 + m\angle 2 + \cdots + m\angle n = (n - 2) \cdot 180^\circ\)

#### EXAMPLE 1  Use the Polygon Interior Angles Theorem

Find the sum of the measures of the interior angles of a convex heptagon.

**Solution**
A heptagon has 7 sides. Use the Polygon Interior Angles Theorem and substitute 7 for \( n \).

\[
(n - 2) \cdot 180^\circ = (7 - 2) \cdot 180^\circ \\
= 5 \cdot 180^\circ \\
= 900^\circ 
\]

**ANSWER** The sum of the measures of the interior angles of a convex heptagon is 900°.
Example 2: Find the Measure of an Interior Angle

Find the measure of \( \angle A \) in the diagram.

Solution

The polygon has 6 sides, so the sum of the measures of the interior angles is:

\[
(n - 2) \cdot 180^\circ = (6 - 2) \cdot 180^\circ = 4 \cdot 180^\circ = 720^\circ.
\]

Add the measures of the interior angles and set the sum equal to 720°.

\[
136^\circ + 136^\circ + 88^\circ + 142^\circ + 105^\circ + m\angle A = 720^\circ
\]

\[
607^\circ + m\angle A = 720^\circ
\]

\[
m\angle A = 113^\circ
\]

The measure of \( \angle A \) is 113°.

Example 3: Interior Angles of a Regular Polygon

Find the measure of an interior angle of a regular octagon.

Solution

The sum of the measures of the interior angles of any octagon is:

\[
(n - 2) \cdot 180^\circ = (8 - 2) \cdot 180^\circ = 6 \cdot 180^\circ = 1080^\circ.
\]

Because the octagon is regular, each angle has the same measure. So, divide 1080° by 8 to find the measure of one interior angle.

\[
\frac{1080^\circ}{8} = 135^\circ
\]

The measure of an interior angle of a regular octagon is 135°.

Checkpoint: Find Measures of Interior Angles

In Exercises 1–3, find the measure of \( \angle G \).

1. \( \angle G \) in the diagram.
2. \( \angle G \) in the diagram.
3. \( \angle G \) in the diagram.
4. Find the measure of an interior angle of a regular polygon with twelve sides.
**Exterior Angles** The diagrams below show that the sum of the measures of the exterior angles of the convex polygon is 360°.

1. Shade one exterior angle at each vertex.
2. Cut out the exterior angles.
3. Arrange the exterior angles to form 360°.

The sum of the measures of the exterior angles of a convex polygon does not depend on the number of sides that the polygon has.

**THEOREM 8.2**

**Polygon Exterior Angles Theorem**

**Words** The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360°.

**Symbols** \( m\angle 1 + m\angle 2 + \cdots + m\angle n = 360° \)

**Example 4** Find the Measure of an Exterior Angle

Find the value of \( x \).

**Solution**

Using the Polygon Exterior Angles Theorem, set the sum of the measures of the exterior angles equal to 360°.

\[
95° + 85° + 2x° + x° = 360°
\]

Polygon Exterior Angles Theorem

\[
180 + 3x = 360
\]

Combine like terms.

\[
3x = 180
\]

Subtract 180 from each side.

\[
x = 60
\]

Divide each side by 3.

**Answer** The value of \( x \) is 60.
Chapter 8  Polygons and Area

8.2 Exercises

**Guided Practice**

**Vocabulary Check**

1. Name an interior angle and an exterior angle of polygon ABCDE shown at the right.

**Skill Check**

2. Write the formula that is used to find the sum of the measures of the interior angles of any convex polygon with \( n \) sides.

3. Use your answer from Exercise 2 to find the sum of the measures of the interior angles of a convex polygon with 9 sides.

**Write an equation to find the measure of \( \angle 1 \). Do not solve the equation.**

4. 5. 6. 7.
**Practice and Applications**

**Extra Practice**
See p. 689.

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**Sum of Interior Angle Measures** Find the sum of the measures of the interior angles of the convex polygon.

8.  

9.  

10.  

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**Polygons with n Sides** Find the sum of the measures of the interior angles of the convex polygon with \( n \) sides.

11. \( n = 10 \)  
12. \( n = 15 \)  
13. \( n = 20 \)  
14. \( n = 30 \)  
15. \( n = 52 \)  
16. \( n = 100 \)

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**Interior Angle Measures** Find the measure of \( \angle A \).

17.  

18.  

19.  

20.  

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**Interior Angles of Regular Polygons** Find the measure of an interior angle of the regular polygon.

21.  

22.  

23.  

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**Using Algebra** Find the value of \( x \).

24.  

25.  

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**Student Help**

**HOMEWORK HELP** Extra help with problem solving in Exs. 21–23 is at classzone.com

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**Homework Help**

Example 1: Exs. 8–16  
Example 2: Exs. 17–20  
Example 3: Exs. 21–23  
Example 4: Exs. 24, 25

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8.2 Angles in Polygons
Chapter 8  Polygons and Area

**Outdoor Furniture**  You are constructing a regular hexagonal wooden bench like the one shown. On the bench, \( \angle 1 \equiv \angle 2 \). Find the measures of \( \angle 1 \) and \( \angle 2 \) so that you know what angle to use to cut the pieces of wood.

**Challenge**  Find the number of sides of the regular polygon with the given exterior angle measure.

<table>
<thead>
<tr>
<th>Exterior Angle Measure</th>
<th>Number of Sides</th>
</tr>
</thead>
<tbody>
<tr>
<td>35. 60°</td>
<td>36</td>
</tr>
<tr>
<td>36. 20°</td>
<td>37</td>
</tr>
<tr>
<td>37. 72°</td>
<td>38</td>
</tr>
<tr>
<td>38. 10°</td>
<td></td>
</tr>
</tbody>
</table>

**Logical Reasoning**  Select the word that makes the statement true.

31. The sum of the measures of the exterior angles of a convex polygon, one at each vertex, is (always, sometimes, never) 360°.

32. The sum of the measures of the interior angles of a convex octagon is (always, sometimes, never) 1440°.

33. The measures of the exterior angles of a convex polygon are (always, sometimes, never) equal.

**Visualize It!**  Cut a strip of lined paper and tie it into an overhand knot as shown. Gently flatten the knot to form a pentagon. The pentagon should be regular.

29. Find the measure of an interior angle of a regular pentagon.

30. Measure the interior angles of your knot with a protractor. Use your answer to Exercise 29 to determine whether your pentagon is regular.

**Baseball**  A home plate for a baseball field is a pentagon as shown.

26. Is the polygon regular? Explain why or why not.

27. What is the sum of the interior angles?

28. Find the measures of \( \angle C \) and \( \angle E \).
**Standardized Test Practice**

**Multiple Choice** In Exercises 39 and 40, use the diagram below.

39. What is the value of $x$?
   - A 18
   - B 78
   - C 117
   - D 198

40. What is the measure of $\angle P$?
   - F 39°
   - G 78°
   - H 156°
   - J 234°

**Mixed Review**

**Using a Centroid** $E$ is the centroid of $\triangle ABC$. Find $BE$ and $ED$.

(Lesson 4.6)

41. $BD = 6$
42. $BD = 33$
43. $BD = 52$

**Algebra Skills**

**Absolute Value** Evaluate. (Skills Review, p. 662)

44. $|-5|$
45. $|11|$
46. $|-48|$
47. $|0|$
48. $|13.2|$
49. $|-2|$
50. $|-0.001|$
51. $|-1.11|$

**Quiz 1**

Decide whether the polygon is regular. Explain your answer.
(Lesson 8.1)

1. 
2. 
3. 

Find the measure of $\angle 1$. (Lesson 8.2)

4. 
5. 
6. 
7. 
8.