9.2 Surface Area of Prisms and Cylinders

Goal
Find the surface areas of prisms and cylinders.

Key Words
• prism
• surface area
• lateral face
• lateral area
• cylinder

A prism is a polyhedron with two congruent faces that lie in parallel planes.

To visualize the surface area of a prism, imagine unfolding it so that it lies flat. The flat representation of the faces is called a net.

The surface area of a polyhedron is the sum of the areas of its faces. The surface area of a prism is equal to the area of its net.

Example 1 Use the Net of a Prism

Find the surface area of the rectangular prism.

Solution
Add the areas of all the rectangles that form the faces of the prism.

<table>
<thead>
<tr>
<th>Congruent Faces</th>
<th>Dimensions</th>
<th>Area of Face</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left face and right face</td>
<td>8 in. by 5 in.</td>
<td>$8 \times 5 = 40 \text{ in.}^2$</td>
</tr>
<tr>
<td>Front face and back face</td>
<td>8 in. by 3 in.</td>
<td>$8 \times 3 = 24 \text{ in.}^2$</td>
</tr>
<tr>
<td>Top face and bottom face</td>
<td>3 in. by 5 in.</td>
<td>$3 \times 5 = 15 \text{ in.}^2$</td>
</tr>
</tbody>
</table>

Add the areas of all the faces to get the surface area.

\[ S = 40 + 40 + 24 + 24 + 15 + 15 \]
\[ S = 158 \]

ANSWER The surface area of the prism is 158 square inches.
**Lateral Faces and Area** The **lateral faces** of a prism are the faces of the prism that are not bases. **Lateral area** is the sum of the areas of the lateral faces.

**Surface Area of a Prism** One way to find the surface area of a prism is to use the method in Activity 9.2, summarized below.

\[
\text{Surface area} = 2(\text{area of base}) + \text{(perimeter of base)}(\text{height})
\]

**EXAMPLE 2** Find Surface Area of a Prism

Find the surface area of the prism.

**Solution**

1. Find the area of a triangular base.
   \[ B = \frac{1}{2} \cdot 4 \cdot 3 = 2 \cdot 3 = 6 \]

2. Find the perimeter of a base.
   \[ P = 3 + 4 + 5 = 12 \]

3. Find the height of the prism. In the diagram, \( h = 2 \).

4. Use the formula for surface area of a prism.
   \[ S = 2B + Ph \]
   \[ = 2 \cdot 6 + 12 \cdot 2 \]
   \[ = 12 + 24 \]
   \[ = 36 \]

**ANSWER** The surface area of the prism is 36 square meters.
Find the surface area of the prism.

1. 2 in.  \( \times \) 3 in.  \( \rightarrow \) 6 in.
2. 5 ft  \( \times \) 6 ft  \( \rightarrow \) 10 ft
3. 8 cm  \( \times \) 6 cm  \( \rightarrow \) 4 cm

Surface Area of a Cylinder

A **cylinder** is a solid with two congruent circular bases that lie in parallel planes. The **lateral area** of a cylinder is the area of the curved surface.

The **radius** of the cylinder is the radius of a base.

The **height** of the cylinder is the perpendicular distance between the bases.

The diagram below shows how to find the surface area of a cylinder.

**Surface area**

\[
S = 2B + Ch
\]

**Symbols**

\[
B = \pi r^2
\]

\[
C = 2\pi r
\]

When unwrapped, the label is a rectangle.

**SURFACE AREA OF A CYLINDER**

**Words**

Surface area = \( 2(\text{area of base}) + (\text{circumference of base})(\text{height}) \)

**Symbols**

\[
S = 2B + Ch
\]

\[
= 2\pi r^2 + 2\pi rh
\]
### Example 3 Find Surface Area of a Cylinder

Find the surface area of the cylinder. Round your answer to the nearest whole number.

**Solution**

The radius of the base is 3 feet and the height is 4 feet. Use these values in the formula for surface area of a cylinder.

\[
S = 2\pi r^2 + 2\pi rh
\]

Write the formula for surface area.

\[
= 2\pi(3^2) + 2\pi(3)(4)
\]

Substitute 3 for \(r\) and 4 for \(h\).

\[
= 18\pi + 24\pi
\]

Simplify.

\[
= 42\pi
\]

Add.

\[
= 132
\]

Multiply.

**Answer** The surface area is about 132 square feet.

### Example 4 Find Lateral Area

About how much plastic is used to make a straw that has a diameter of 5 millimeters and a height of 195 millimeters?

**Solution**

The straw is a cylinder with no bases. Use the formula for the surface area of a cylinder, but do not include the areas of the bases.

The diameter is 5 millimeters. So the radius is \(r = \frac{5}{2} = 2.5\).

\[
Lateral \ text{ area} = 2\pi rh
\]

Surface area formula without bases.

\[
= 2\pi(2.5)(195)
\]

Substitute 2.5 for \(r\) and 195 for \(h\).

\[
= 975\pi
\]

Simplify.

\[
= 3063
\]

Multiply.

**Answer** The straw is made with about 3063 square millimeters of plastic.

### Checkpoint

**Find Surface Area of Cylinders**

Find the area described. Round your answer to the nearest whole number.

4. surface area

5. surface area

6. lateral area
# 9.2 Exercises

## Guided Practice

### Vocabulary Check
Tell whether the statement is true or false.

1. The solid is a triangular prism.
2. The blue face is a lateral face of the solid.
3. The red face is a lateral face of the solid.
4. The blue face is a base of the solid.

### Skill Check
Find the surface area of the solid. If necessary, round your answer to the nearest whole number.

5. 6. 7.

## Practice and Applications

### Extra Practice
See p. 691.

### Identifying Parts of a Prism
In Exercises 8–10, use the diagram at the right.

8. What is the height of the solid?
9. What is the area of a base?
10. What is the perimeter of a base?

### Identifying Parts of a Cylinder
In Exercises 11–13, use the diagram at the right.

11. What is the height of the solid?
12. What is the area of a base?
13. What is the circumference of a base?

### Identifying Parts of a Prism
In Exercises 14–16, use the diagram at the right.

14. What is the height of the solid?
15. What is the area of a base?
16. What is the perimeter of a base?
Surface Area of a Prism  Find the surface area of the prism.

17. 18. 19.

20. You be the Judge
The height of Prism A is twice the height of Prism B. Is the surface area of Prism A twice the surface area of Prism B? Explain.

Analyzing Nets  Name the solid that can be folded from the net.

21. 22. 23.

Finding Surface Area of a Cylinder  Find the surface area of the cylinder. Round your answer to the nearest whole number.


Finding Surface Area  Find the surface area of the solid. Round your answer to the nearest whole number.

27. 28. Cube with 16 mm sides

29. 30. 31.
32. **Error Analysis** Juanita is trying to find the surface area of the cylinder shown below. What did she do wrong?

\[
S = 2\pi(12^2) + 2\pi(12)(10) \\
= 2\pi(24) + 2\pi(120) \\
= 288\pi \\
\approx 905 \text{ in.}^2
\]

**Visualize It!** Sketch the solid that results after the net has been folded. Use the shaded faces as bases.

**33.** [Sketch]

**34.** [Sketch]

**35.** [Sketch]

**Visualize It!** Sketch the solid described and find its surface area.

36. A rectangular prism with a height of 10 feet, a length of 3 feet, and a width of 6 feet.

37. A cylinder with a radius of 3 meters and a height of 7 meters.

38. A triangular prism with a height of 5 inches and a base that is a right triangle with legs of 8 inches and 6 inches.

**Finding Lateral Area** Find the lateral area of the solid. If necessary, round your answer to the nearest whole number.

39. [Sketch]

40. [Sketch]

41. [Sketch]

42. **Clothes Rod** Find the lateral area of a clothes rod that has a radius of 2 centimeters and a height of 90 centimeters.

43. **Wax Cylinder Records** In the late 1800’s, a standard-sized cylinder record was about 2 inches in diameter and 4 inches long. Find the lateral area of the cylinder described.

44. **Compact Discs** A standard compact disc has an outer radius of 60 millimeters and a height of 1.2 millimeters. Find the lateral area of the disc.

**WAX CYLINDER RECORDS** The first type of phonograph records were hollow wax cylinders. The cylinder is rotated on a phonograph to produce the music.
Architecture  In Exercises 45 and 46 use the following information. Suppose a skyscraper is a prism that is 415 meters tall and each base is a square that measures 64 meters on a side.

45. What is the lateral area of this skyscraper?

46. Challenge What is the surface area of this skyscraper? (Hint: The ground is not part of the surface area of the skyscraper.)

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47. **Multiple Choice** What is the approximate surface area of the cylinder shown?

- **A** 502 cm²
- **B** 628 cm²
- **C** 785 cm²
- **D** 1570 cm²

48. **Multiple Choice** What is the height of the prism shown if the surface area is 104 square feet?

- **F** 4 feet
- **G** 5 feet
- **H** 6 feet
- **J** 7 feet

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**Finding Area** Find the area of the polygon made up of rectangles and triangles. *(Lessons 8.3, 8.4)*

49. 50. 51.

52. A parallelogram has a height of 5 feet and an area of 70 square feet. Find the base. *(Lesson 8.5)*

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**Algebra Skills** Solve the proportion. *(Skills Review, p. 660)*

53. \( \frac{6}{x} = \frac{2}{5} \)

54. \( \frac{1}{10} = \frac{4}{x} \)

55. \( \frac{3}{5} = \frac{x}{35} \)

56. \( \frac{4}{7} = \frac{16}{x} \)

57. \( \frac{33}{x} = \frac{11}{13} \)

58. \( \frac{x}{32} = \frac{5}{8} \)