10.5 Sine and Cosine Ratios

Goal

Find the sine and cosine of an acute angle.

Key Words

- sine
- cosine

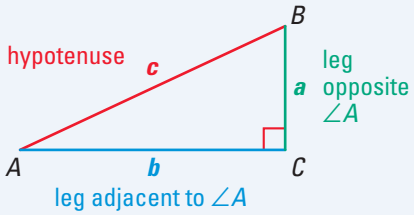
Two other trigonometric ratios are **sine** and **cosine**. These are abbreviated as *sin* and *cos*. Unlike the tangent ratio, these ratios involve the hypotenuse of a right triangle.

SINE AND COSINE RATIOS

For any acute angle A of a right triangle:

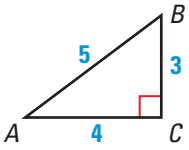
$$\sin A = \frac{\text{leg opposite } \angle A}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos A = \frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}} = \frac{b}{c}$$



EXAMPLE 1 Find Sine and Cosine Ratios

Find sin A and cos A.



Solution

$$\sin A = \frac{\text{leg opposite } \angle A}{\text{hypotenuse}}$$ Write ratio for sine.

$$= \frac{3}{5}$$ Substitute.

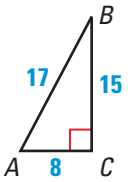
$$\cos A = \frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}}$$ Write ratio for cosine.

$$= \frac{4}{5}$$ Substitute.

Checkpoint Find Sine and Cosine Ratios

Find sin *A* and cos *A*.

1.



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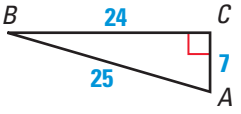

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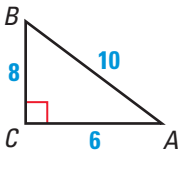
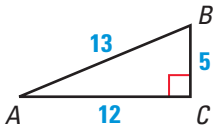


EXAMPLE 2 Find Sine and Cosine Ratios

Find sin A and cos A. Write your answers as fractions and as decimals rounded to four decimal places.

Solution

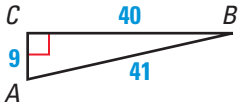


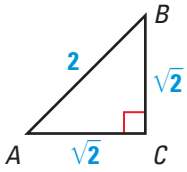
$$\sin A = \frac{\text{leg opposite } \angle A}{\text{hypotenuse}} = \frac{5}{13} \approx 0.3846$$

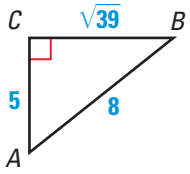
$$\cos A = \frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}} = \frac{12}{13} \approx 0.9231$$

Checkpoint ✓ Find Sine and Cosine Ratios

Find sin *A* and cos *A*. Write your answers as fractions and as decimals rounded to four decimal places.

4. 

5. 

6. 

Sine and Cosine Functions You can use the SIN and COS functions on a calculator to approximate the sine and cosine of an angle. You can also use the table of trigonometric ratios on page 705.

EXAMPLE 3 Use a Calculator for Sine and Cosine

Use a calculator to approximate sin 74° and cos 74°. Round your answers to four decimal places.

Solution

Calculator keystrokes	Display	Rounded value
74 SIN *or* SIN 74 ENTER	0.961261696	0.9613
74 COS *or* COS 74 ENTER	0.275637356	0.2756

Checkpoint ✓ Use a Calculator for Sine and Cosine

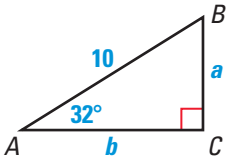
Use a calculator to approximate the value to four decimal places.

7. sin 43° **8.** cos 43° **9.** sin 15° **10.** cos 15°

11. cos 72° **12.** sin 72° **13.** cos 90° **14.** sin 90°

EXAMPLE 4 Find Leg Lengths

Find the lengths of the legs of the triangle. Round your answers to the nearest tenth.



Solution

$$\sin A = \frac{\text{leg opposite } \angle A}{\text{hypotenuse}} \qquad \cos A = \frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}}$$

$$\sin 32° = \frac{a}{10} \qquad \cos 32° = \frac{b}{10}$$

$$10(\sin 32°) = a \qquad 10(\cos 32°) = b$$

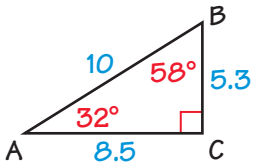
$$10(0.5299) \approx a \qquad 10(0.8480) \approx b$$

$$5.3 \approx a \qquad 8.5 \approx b$$

ANSWER In the triangle, BC is about 5.3 and AC is about 8.5.

Student Help

VISUAL STRATEGY

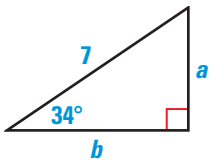


You can label side lengths and angle measures in different colors. See p. 536.

Checkpoint Find Leg Lengths

Find the lengths of the legs of the triangle. Round your answers to the nearest tenth.

15.



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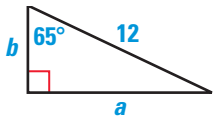

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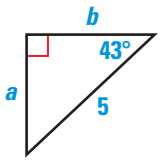
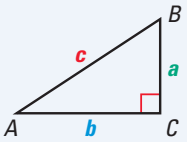


SUMMARY TRIGONOMETRIC RATIOS

For any acute angle A of a right triangle:

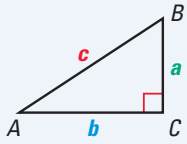
Tangent of ∠A

$$\tan A = \frac{\text{leg opposite } \angle A}{\text{leg adjacent to } \angle A} = \frac{a}{b}$$



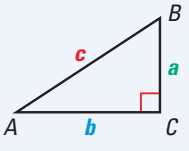
Sine of ∠A

$$\sin A = \frac{\text{leg opposite } \angle A}{\text{hypotenuse}} = \frac{a}{c}$$



Cosine of ∠A

$$\cos A = \frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}} = \frac{b}{c}$$



10.5 Exercises

Guided Practice

Vocabulary Check

Use the diagram shown at the right to match the trigonometric ratios.

1. $\cos D$

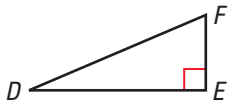
2. $\sin D$

3. $\tan D$

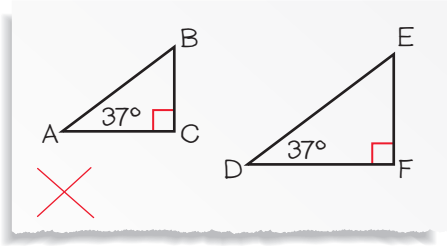
A. $\frac{EF}{DE}$

B. $\frac{DE}{DF}$

C. $\frac{EF}{DF}$



4. Error Analysis A student says that $\sin D > \sin A$ because the side lengths of $\triangle DEF$ are greater than the side lengths of $\triangle ABC$. Explain why the student is incorrect.



Skill Check

In Exercises 5–10, use the diagram shown below to find the trigonometric ratio.

5. $\sin A$

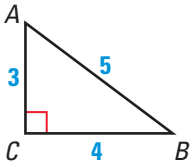
6. $\cos A$

7. $\tan A$

8. $\sin B$

9. $\cos B$

10. $\tan B$



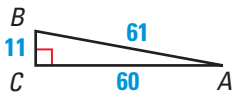
Practice and Applications

Extra Practice

See p. 694.

Finding Sine and Cosine Ratios **Find sin *A* and cos *A*. Write your answers as fractions in simplest form.**

11.



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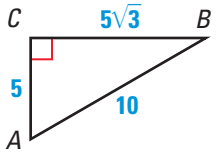

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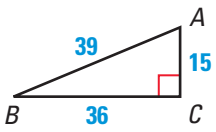
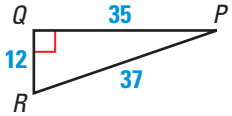


Finding Sine and Cosine Ratios **Find sin *P* and cos *P*. Write your answers as fractions in simplest form and as decimals rounded to four decimal places.**

14.



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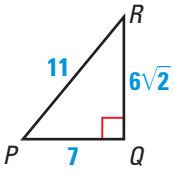

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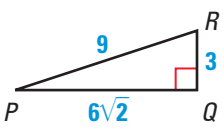


Homework Help

Example 1: Exs. 11–13
Example 2: Exs. 14–16
Example 3: Exs. 17–24
Example 4: Exs. 25–30

Calculator **Use a calculator to approximate the value to four decimal places.**

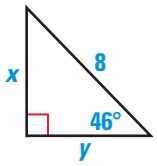
17. sin 40° **18.** cos 23° **19.** sin 80° **20.** cos 5°

21. sin 59° **22.** cos 61° **23.** sin 90° **24.** cos 77°

Student Help
CLASSZONE.COM
HOMEWORK HELP
Extra help with problem solving in Exs. 25–30 is at classzone.com

Finding Leg Lengths **Find the lengths of the legs of the triangle. Round your answers to the nearest tenth.**

25.



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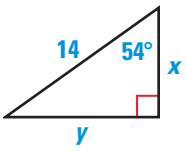

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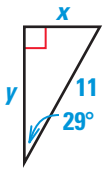

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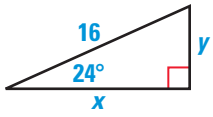

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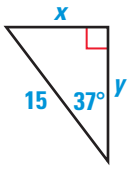

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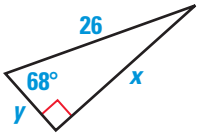
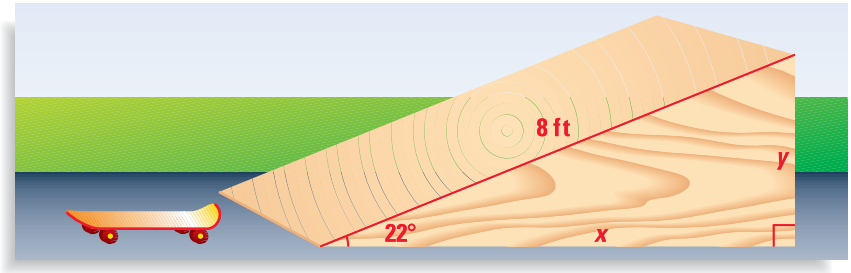


31. **Visualize It!** A ladder that is 15 feet long is leaning against a wall. The ladder makes an angle of 70° with the ground. Make a sketch. Then determine how high up the wall the ladder reaches. Round your answer to the nearest foot.

32. **Skateboard Ramp** You are constructing a skateboarding ramp like the one shown below. Your ramp will be 8 feet long and the ramp angle will be about 22°. Find the lengths of the legs of the triangles that support the ramp. Round your answers to the nearest inch.

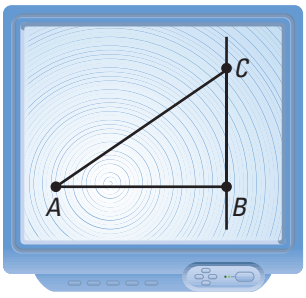


Link to Skateboarding

SKATEBOARD RAMPS built with curves, called half pipes, allow freestyle skateboard riders to perform acrobatic maneuvers.

Technology **In Exercises 33–35, use geometry software.**

1. Draw $\overline{AB}$.
2. Construct a perpendicular to $\overline{AB}$ through B.
3. Add point C on the perpendicular.
4. Draw $\overline{AC}$.



33. Find $m\angle A$, $\sin A$, and $\cos A$.

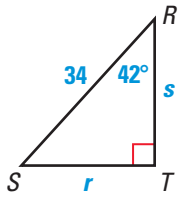
34. Calculate $(\sin A)^2 + (\cos A)^2$.

35. Drag point C. What do you notice?

36. Challenge Let A be any acute angle of a right triangle. Use the definitions of sin A, cos A, and tan A to prove the following result.

$$\frac{\sin A}{\cos A} = \tan A$$

37. You be the Judge One student uses the ratio $\sin 42° = \frac{r}{34}$ to find the length of $\overline{ST}$. Another student uses the ratio $\cos 48° = \frac{r}{34}$. Assuming the students make no errors in calculation, who will get the correct answer?

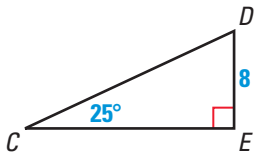


Standardized Test Practice

38. Multiple Choice Use the diagram below. Which expression could be used to find CD?

Ⓐ $8(\cos 25°)$ Ⓑ $8(\sin 25°)$

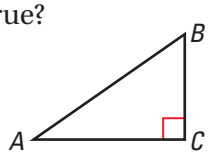
Ⓒ $\frac{8}{\sin 25°}$ Ⓓ $\frac{8}{\cos 25°}$



39. Multiple Choice Which statement *cannot* be true?

Ⓕ $\sin A = 0.55$ Ⓖ $\sin A = 0.61$

Ⓗ $\sin A = 1.2$ Ⓙ $\sin A = 0.4869$

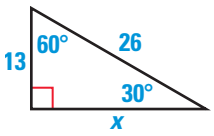


Not drawn to scale

Mixed Review

Finding Leg Lengths **Find the value of each variable. Write your answer in radical form.** *(Lesson 10.3)*

40.



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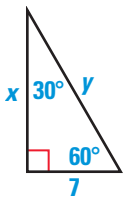

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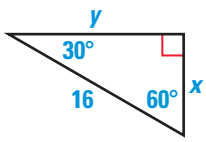

Using a Calculator **Use a calculator to approximate the value to four decimal places.** *(Lesson 10.4)*

43. tan 32° **44.** tan 88° **45.** tan 56°

46. tan 24° **47.** tan 17° **48.** tan 49°

Algebra Skills

Ordering Numbers **Write the numbers in order from least to greatest.** *(Skills Review, p. 662)*

49. $-0.8, 1.8, -8, 0.08, -18, 0, -1.8$

50. $2641, 2146, 2614, 2416, 2164, 2461$

51. $-0.56, -0.47, -0.61, -0.5, -0.6$