

2.5

If-Then Statements and Deductive Reasoning

Goal

Use if-then statements.
Apply laws of logic.

Key Words

- if-then statement
- hypothesis
- conclusion
- deductive reasoning

An **if-then statement** has two parts. The “if” part contains the **hypothesis**. The “then” part contains the **conclusion**.



Student Help

VOCABULARY TIP

If-then statements are also called *conditional statements*.

The following sentence is an example of an if-then statement:

If **the team wins the semi-final**,
then **the team will play in the championship**.

hypothesis

conclusion

EXAMPLE 1 Identify the Hypothesis and Conclusion

Identify the hypothesis and the conclusion of the if-then statement.

If I pass the driving test, then I will get my driver's license.

Solution

I pass the driving test is the hypothesis.

I will get my driver's license is the conclusion.

EXAMPLE 2 Write If-Then Statements

Rewrite the statement as an if-then statement.

- Every game on my computer is fun to play.
- I will buy the CD if it costs less than \$15.

Solution

- If a game is on my computer, then it is fun to play.
- If the CD costs less than \$15, then I will buy it.

Student Help

STUDY TIP

Example 2, part (b), shows that the conclusion of a statement is not always the last part.

Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to make a logical argument. This form of reasoning differs from *inductive reasoning*, in which previous examples and patterns are used to form a conjecture.

LAWS OF LOGIC

Law of Detachment

If the hypothesis of a true if-then statement is true, then the conclusion is also true.

Law of Syllogism

If **statement p** , then **statement q** .
 If **statement q** , then **statement r** .
 If **statement p** , then **statement r** .

← If these statements are true,
 ← then this statement is true.

EXAMPLE 3 Use the Law of Detachment

What can you conclude from the following true statements?

If you wash the cotton T-shirt in hot water, then it will shrink.

You wash the cotton T-shirt in hot water.

Solution

The hypothesis (you wash the cotton T-shirt in hot water) of a true if-then statement is true. By the Law of Detachment, the conclusion must be true.

ANSWER ▶ You can conclude that the cotton T-shirt shrinks.

Checkpoint If-Then Statements and the Law of Detachment

Rewrite the statement as an if-then statement. Then underline the hypothesis and circle the conclusion.

- All teachers at East High School have taught for at least 5 years.
- An angle is obtuse if its measure is 170° .

What can you conclude from the given true statements?

- If x has a value of 7, then $2x - 3$ has a value of 11. The value of x is 7.
- If you study at least two hours for the test, then you will pass the test. You study three hours for the test.



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MORE EXAMPLES

More examples at
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EXAMPLE 4 Use the Law of Detachment

Which argument is correct? Explain your reasoning.

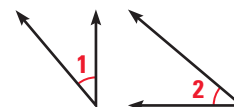
Argument 1: If two angles are vertical angles, then they are congruent. $\angle 1$ and $\angle 2$ are congruent. So, $\angle 1$ and $\angle 2$ are vertical angles.

Argument 2: If two angles are vertical angles, then they are congruent. $\angle 1$ and $\angle 2$ are vertical angles. So, $\angle 1$ and $\angle 2$ are congruent.

Solution

Argument 2 is correct. The hypothesis (two angles are vertical angles) is true, which implies that the conclusion (they are congruent) is true.

You can use the following counterexample to show that Argument 1 is false. In the diagram at the right, $\angle 1 \cong \angle 2$, but they are not vertical angles.



Checkpoint Use the Law of Detachment

5. Which argument is correct? Explain your reasoning.

Argument 1: If two angles form a linear pair, then they are supplementary. $\angle 3$ and $\angle 4$ form a linear pair. So, $\angle 3$ and $\angle 4$ are supplementary.

Argument 2: If two angles form a linear pair, then they are supplementary. $\angle 3$ and $\angle 4$ are supplementary. So, $\angle 3$ and $\angle 4$ form a linear pair.

EXAMPLE 5 Use the Law of Syllogism

Write the statement that follows from the pair of true statements.

If the daily high temperature is 32°F or less, then the water in the pipe is frozen.

If the water in the pipe is frozen, then the pipe will break.

Solution

Use the Law of Syllogism.

If the daily high temperature is 32°F or less, then the pipe will break.

Checkpoint Use the Law of Syllogism

6. Write the statement that follows from the pair of true statements.

If the ball is thrown at the window, it will hit the window.

If the ball hits the window, then the window will break.

2.5 Exercises

Guided Practice

Vocabulary Check

Complete the statement.

- The “if” part of an if-then statement contains the ?.
- The “then” part of an if-then statement contains the ?.

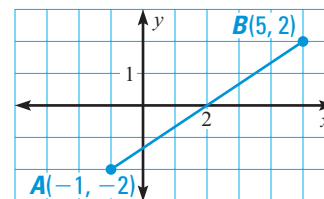
Skill Check

In Exercises 3 and 4, rewrite the statement as an if-then statement.

- Adjacent angles share a common side.
- You will be late to school if you miss the bus.
- What can you conclude from the following true statements?

If the endpoints of a segment have coordinates $(-1, -2)$ and $(5, 2)$, then the midpoint of the segment is at $(2, 0)$.

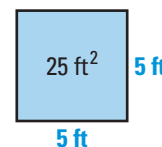
The endpoints of \overline{AB} are $A(-1, -2)$ and $B(5, 2)$.



- Use the Law of Syllogism to write the if-then statement that follows from the pair of true statements.

If the perimeter of a square is 20 feet, then the length of a side of the square is 5 feet.

If the length of a side of a square is 5 feet, then the area is 25 square feet.



Practice and Applications

Extra Practice

See p. 678.

Parts of an If-Then Statement Identify the hypothesis and the conclusion of the if-then statement.

- If the car is running, then the key is in the ignition.
- If the measure of an angle is 60° , then the angle is acute.

Writing If-Then Statements Rewrite the statement as an if-then statement. Then underline the hypothesis and circle the conclusion.

- A number divisible by 6 is also divisible by 3 and 2.
- Eagles are fish-eating birds.
- A shape has four sides if it is a square.
- Two angles are supplementary if they form a linear pair.

Homework Help

- Example 1:** Exs. 7–12,
20, 21, 24–27
- Example 2:** Exs. 9–12,
20, 21, 24–27
- Example 3:** Exs. 13–15
- Example 4:** Ex. 16
- Example 5:** Exs. 17–19

Using the Law of Detachment In Exercises 13–15, what can you conclude from the given true statements?

13. If two planes intersect, then their intersection is a line. Two planes are intersecting.
14. If x has a value of 4, then $2x$ has a value of 8. The value of x is 4.
15. If Central High School wins the championship, then the school will celebrate. Central High School wins the championship.

16.  **You be the Judge** Which argument is correct? Explain your reasoning.

Argument 1: If two angles measure 40° and 50° , then the angles are complementary. The measure of $\angle 1$ is 40° and the measure of $\angle 2$ is 50° . So, $\angle 1$ and $\angle 2$ are complementary.

Argument 2: If two angles measure 40° and 50° , then the angles are complementary. $\angle 1$ and $\angle 2$ are complementary. So, $m\angle 1 = 40^\circ$ and $m\angle 2 = 50^\circ$.

Using the Law of Syllogism Write the if-then statement that follows from the pair of true statements.

17. If the sun is shining, then it is a beautiful day.
If it is a beautiful day, then we will have a picnic.
18. If the stereo is on, then the volume is loud.
If the volume is loud, then the neighbors will complain.
19. If Todd goes to the movies, then Gabriela will go to the movies.
If Chris goes to the movies, then Todd will go to the movies.

Advertising Use the following advertising slogan: “Want a great selection of used cars? Come and see Bargain Bob’s Used Cars!”



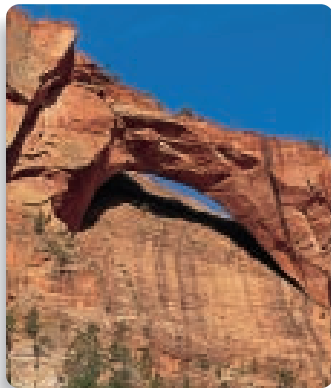
20. Rewrite the slogan as an if-then statement. Then underline the hypothesis and circle the conclusion.
21. Find an advertising slogan that can be written as an if-then statement. Then repeat Exercise 20 using the slogan.

Link to Careers

ADVERTISING COPYWRITERS work with graphic designers to develop the text for advertisements.



Link to Geology



THE KOLOB ARCH is the world's widest natural arch. The arch, located in Springdale, Utah, spans 310 feet.

Logical Reasoning Select the word or phrase that makes the concluding statement true.

22. The Oak Terrace apartment building does not allow dogs. Serena lives at Oak Terrace. So, Serena (*must, may, must not*) keep a dog.
23. The Kolob Arch is the world's widest natural arch. The world's widest natural arch is in Zion National Park. So, the Kolob Arch (*is, may be, is not*) in Zion National Park.

Quotes of Wisdom Rewrite the statement as an if-then statement. Then underline the hypothesis and circle the conclusion.

24.

If you tell the truth,
you don't have to
remember anything.
Mark Twain

25.

You have to expect
things of yourself
before you can
do them.
Michael Jordan

26.

If one is lucky, a solitary
fantasy can totally
transform one million
realities.
Maya Angelou

27.

Whoever is happy
will make others
happy too.
Anne Frank

Standardized Test Practice

28. **Multiple Choice** What is the *conclusion* of the following if-then statement?

If the storm passes, then our plane will take off.

- (A) The storm passes. (B) Our plane will take off.
(C) Then our plane will take off. (D) None of these

Mixed Review

Classifying Angles Classify the angle as *acute, right, obtuse, or straight*. (Lesson 1.6)

29. $m\angle A = 20^\circ$

30. $m\angle B = 154^\circ$

31. $m\angle C = 90^\circ$

32. $m\angle D = 7^\circ$

33. $m\angle E = 180^\circ$

34. $m\angle F = 89^\circ$

Midpoint Formula Find the coordinates of the midpoint of \overline{AB} . (Lesson 2.1)

35. $A(0, 0), B(-2, 6)$

36. $A(2, 3), B(8, 5)$

37. $A(0, -8), B(6, 6)$

Algebra Skills

Solving Equations Solve the equation. (Skills Review, p. 673)

38. $4y - 2 = 4$

39. $6 = -2t + 1$

40. $4x = -5 + x$

41. $w + 4 = 3w$

42. $12x + 33 = 9x$

43. $14b - 17 = -3b$