4.2 Angle Measures of Triangles

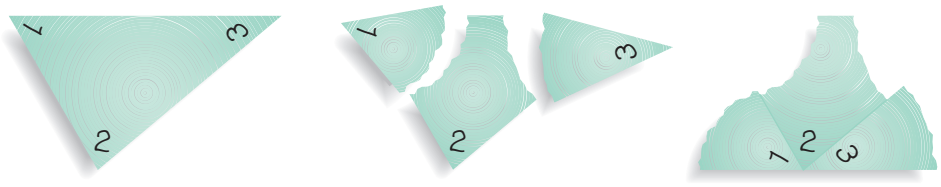
Goal

Find angle measures in triangles.

Key Words

- corollary
- interior angles
- exterior angles

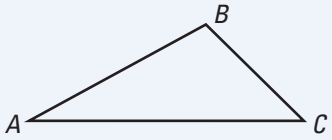
The diagram below shows that when you tear off the corners of any triangle, you can place the angles together to form a straight angle.



THEOREM 4.1

Triangle Sum Theorem

Words The sum of the measures of the angles of a triangle is 180°.



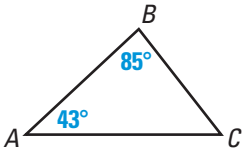
Symbols In $\triangle ABC$, $m\angle A + m\angle B + m\angle C = 180°$.

Student Help

READING TIP
Triangles are named by their vertices. $\triangle ABC$ is read "triangle *ABC*."

EXAMPLE 1 Find an Angle Measure

Given $m\angle A = 43°$ and $m\angle B = 85°$, find $m\angle C$.



Solution

$m\angle A + m\angle B + m\angle C = 180°$	Triangle Sum Theorem
$43° + 85° + m\angle C = 180°$	Substitute 43° for $m\angle A$ and 85° for $m\angle B$.
$128° + m\angle C = 180°$	Simplify.
$128° + m\angle C - 128° = 180° - 128°$	Subtract 128° from each side.
$m\angle C = 52°$	Simplify.

ANSWER ▶ $\angle C$ has a measure of 52°.

CHECK ✓ Check your solution by substituting 52° for $m\angle C$.
$43° + 85° + 52° = 180°$

A **corollary** to a theorem is a statement that can be proved easily using the theorem. The corollary below follows from the Triangle Sum Theorem.

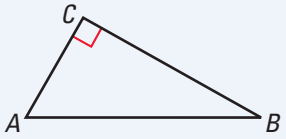
Student Help
LOOK BACK
For the definition of complementary angles, see p. 67.

COROLLARY

Corollary to the Triangle Sum Theorem

Words The acute angles of a right triangle are complementary.

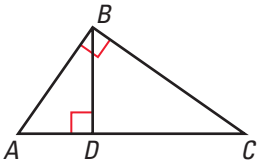
Symbols In $\triangle ABC$, if $m\angle C = 90°$, then $m\angle A + m\angle B = 90°$.



EXAMPLE 2 **Find Angle Measures**

$\triangle ABC$ and $\triangle ABD$ are right triangles. Suppose $m\angle ABD = 35°$.

a. Find $m\angle DAB$. **b.** Find $m\angle BCD$.



Student Help
CLASSZONE.COM
MORE EXAMPLES
More examples at classzone.com

Solution

a. $m\angle DAB + \mathbf{m\angle ABD} = 90°$ — Corollary to the Triangle Sum Theorem

$m\angle DAB + \mathbf{35°} = 90°$ — Substitute 35° for $m\angle ABD$.

$m\angle DAB + 35° \mathbf{- 35°} = 90° \mathbf{- 35°}$ — Subtract 35° from each side.

$m\angle DAB = 55°$ — Simplify.

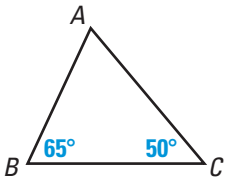
b. $\mathbf{m\angle DAB} + m\angle BCD = 90°$ — Corollary to the Triangle Sum Theorem

$\mathbf{55°} + m\angle BCD = 90°$ — Substitute 55° for $m\angle DAB$.

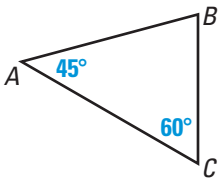
$m\angle BCD = 35°$ — Subtract 55° from each side.

Checkpoint **Find an Angle Measure**

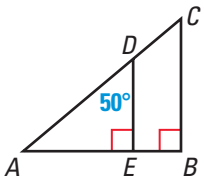
1. Find $m\angle A$.



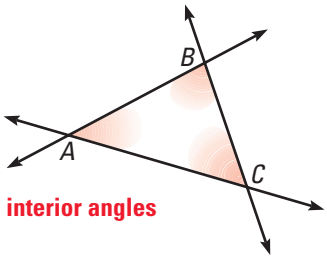
2. Find $m\angle B$.



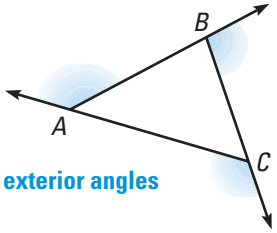
3. Find $m\angle C$.



When the sides of a triangle are extended, other angles are formed. The three original angles are the **interior angles**.



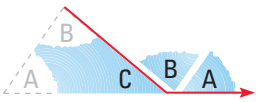
interior angles



exterior angles

The angles that are adjacent to the interior angles are the **exterior angles**. It is common to show only *one* exterior angle at each vertex.

Visualize It!



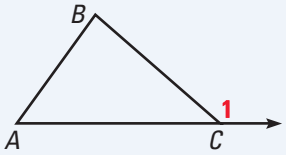
You can tear off two angles and place them together to form an exterior angle.

THEOREM 4.2

Exterior Angle Theorem

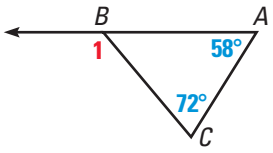
Words The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

Symbols $m\angle 1 = m\angle A + m\angle B$



EXAMPLE 3 Find an Angle Measure

Given $m\angle A = 58°$ and $m\angle C = 72°$, find $m\angle 1$.



Solution

$m\angle 1 = m\angle A + m\angle C$ — Exterior Angle Theorem

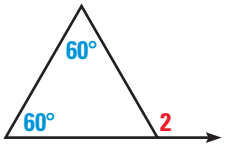
$= 58° + 72°$ — Substitute 58° for $m\angle A$ and 72° for $m\angle C$.

$= 130°$ — Simplify.

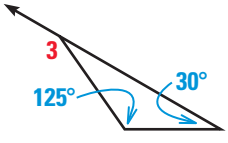
ANSWER ▶ $\angle 1$ has a measure of 130°.

Checkpoint ✓ **Find an Angle Measure**

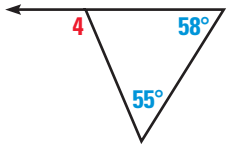
4. Find $m\angle 2$.



5. Find $m\angle 3$.



6. Find $m\angle 4$.

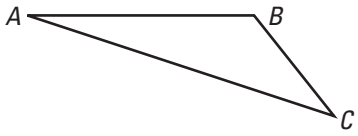


4.2 Exercises

Guided Practice

Vocabulary Check

1. Copy $\triangle ABC$ and label its *interior* angles 1, 2, and 3. Then draw three of its *exterior* angles and label the angles 4, 5, and 6.

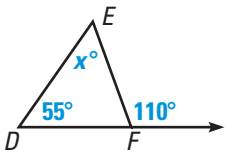


Skill Check

2. Use the diagram to determine which equation can be used to find $m\angle DEF$.

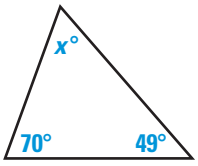
A. $55° + x° = 110°$ **B.** $55° + 110° = x°$

C. $55° - x° = 110°$ **D.** $55° - 110° = x°$



Find the value of x.

3.



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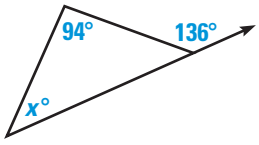

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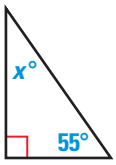


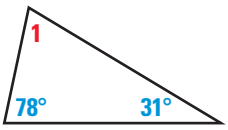
Practice and Applications

Extra Practice

See p. 681.

Finding Angle Measures Find the measure of $\angle 1$.

6.



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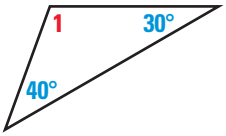

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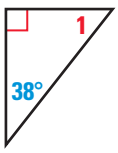

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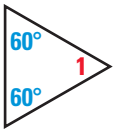

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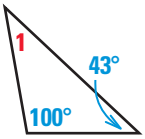

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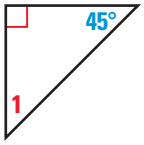
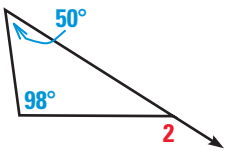


Homework Help

Example 1: Exs. 6–11, 15–21, 23, 24
Example 2: Exs. 6–11, 15–21
Example 3: Exs. 12–14, 18–22

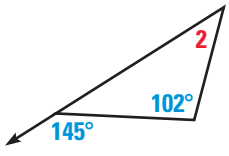
Exterior Angles Find the measure of $\angle 2$.

12.

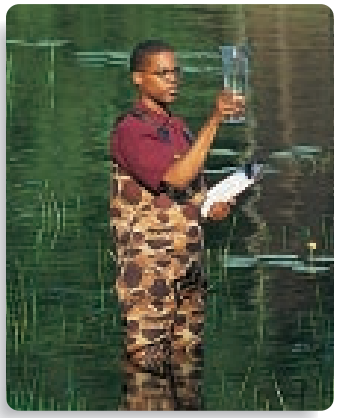


13.

14.



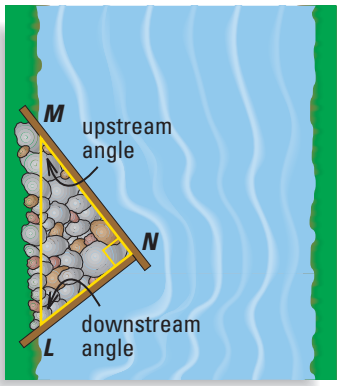
Link to Careers



WATER RESOURCES
A hydrologist studies how water circulates. A hydrologist might use a structure as shown on the right to minimize erosion.

Career Links CLASSZONE.COM

Water Resources **In Exercises 15–17, use the diagram.**
A structure built with rocks is used to redirect the flow of water in a stream and increase the rate of the water's flow. Its shape is a right triangle.



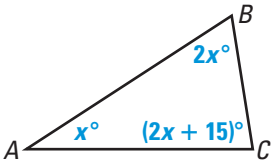
15. Identify the side opposite $\angle MNL$.

16. If the measure of the upstream angle is 37°, what is the measure of the downstream angle?

17. It is generally recommended that the upstream angle should be between 30° and 45°. Give a range of angle measures for the downstream angle.

Using Algebra **Find the value of each variable.**

18.



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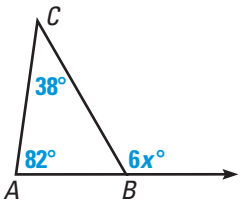

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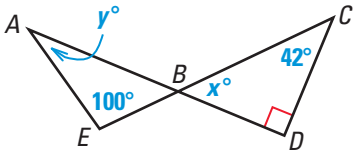
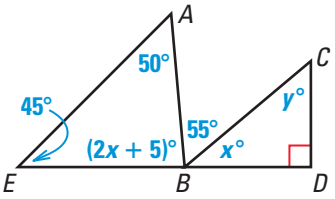
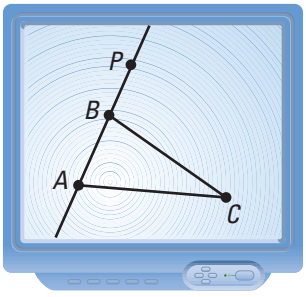


21.



22. Technology Use geometry software to complete the steps below.



1. Draw A, B, C and $\triangle ABC$.
2. Draw $\overleftrightarrow{AB}$ and a point P on it as shown.
3. Find $m\angle PBC$.
4. Find $m\angle BAC + m\angle BCA$.
5. Move point C.

What do you notice? What theorem does this demonstrate?

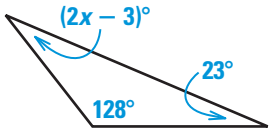
Student Help
VISUAL STRATEGY
In Ex. 23, draw a sketch with angle measures that are roughly correct, as shown on p. 172.

23. Angle Measures in a Triangle The measure of one interior angle of a triangle is 26°. The other interior angles are congruent. Find their measures.

24. **Using Algebra** In $\triangle PQR$, the measure of $\angle P$ is 36°. The measure of $\angle Q$ is five times the measure of $\angle R$. Find $m\angle Q$ and $m\angle R$.

Standardized Test Practice

25. **Multiple Choice** Find the value of x.



Ⓐ 8 Ⓑ 13

Ⓒ 16 Ⓓ 29

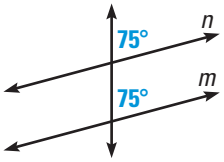
26. **Multiple Choice** Suppose a triangle has interior angle measures of 50°, 60°, and 70°. Which of the following is *not* an exterior angle measure?

Ⓕ 100° Ⓖ 110° Ⓗ 120° Ⓙ 130°

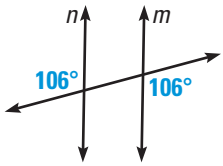
Mixed Review

Showing Lines are Parallel **Explain how you would show that $m \parallel n$. State any theorems or postulates that you would use.** *(Lesson 3.5)*

27.



28.



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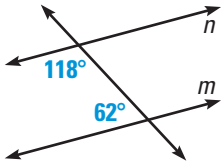


Algebra Skills

Comparing Numbers **Compare the two numbers. Write the answer using <, >, or =.** *(Skills Review, p. 662)*

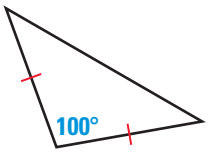
30. 1015 and 1051 **31.** 3.5 and 3.06 **32.** 8.09 and 8.1

33. 1.75 and 1.57 **34.** 0 and 0.5 **35.** 2.055 and 2.1

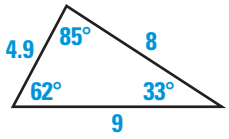
Quiz 1

Classify the triangle by its angles and by its sides. *(Lesson 4.1)*

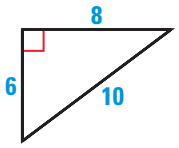
1.



2.

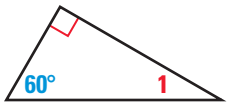


3.

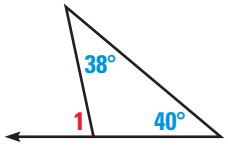


Find the measure of $\angle 1$. *(Lesson 4.2)*

4.



5.



6.