

## 5.6

# Angle Bisectors and Perpendicular Bisectors

## Goal

Use angle bisectors and perpendicular bisectors.

## Key Words

- distance from a point to a line
- equidistant
- angle bisector p. 61
- perpendicular bisector

The **distance from a point to a line** is measured by the length of the perpendicular segment from the point to the line.

When a point is the same distance from one line as it is from another line, the point is **equidistant** from the two lines.



The fountain is equidistant from the jogging path and the bike path.

## THEOREM 5.3

### Angle Bisector Theorem

**Words** If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.



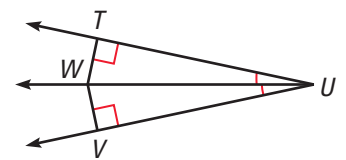
**Symbols** If  $m\angle 1 = m\angle 2$ , then  $\overline{BC} \cong \overline{BD}$ .

## EXAMPLE 1 Use the Angle Bisector Theorem

Prove that  $\triangle TWU \cong \triangle VWU$ .

**Given** ▶  $\overline{UW}$  bisects  $\angle TUV$ .  
 $\triangle UTW$  and  $\triangle UVW$  are right triangles.

**Prove** ▶  $\triangle TWU \cong \triangle VWU$ .



### Solution

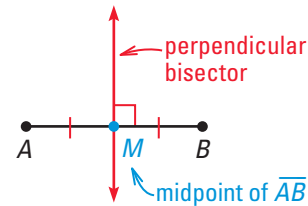
Statements	Reasons
1. $\overline{UW}$ bisects $\angle TUV$ .	1. Given
2. $\triangle UTW$ and $\triangle UVW$ are right triangles.	2. Given
H 3. $\overline{WU} \cong \overline{WU}$	3. Reflexive Prop. of Congruence
L 4. $\overline{WV} \cong \overline{WT}$	4. Angle Bisector Theorem
5. $\triangle TWU \cong \triangle VWU$	5. HL Congruence Theorem

## Student Help

### STUDY TIP

You can also show that the triangles in Example 1 are congruent by the AAS Congruence Theorem.

**Perpendicular Bisectors** A segment, ray, or line that is perpendicular to a segment at its midpoint is called a **perpendicular bisector**.

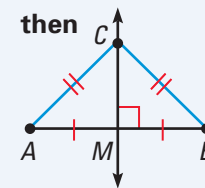
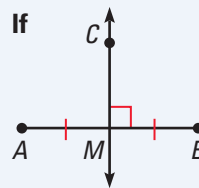


**THEOREM 5.4**

**Perpendicular Bisector Theorem**

**Words** If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

**Symbols** If  $C$  is on the perpendicular bisector of  $\overline{AB}$ , then  $\overline{CA} \cong \overline{CB}$ .



**xy** Using Algebra

**EXAMPLE 2 Use Perpendicular Bisectors**

Use the diagram to find  $AB$ .

**Solution**

In the diagram,  $\overleftrightarrow{AC}$  is the perpendicular bisector of  $\overline{DB}$ .

$$8x = 5x + 12$$

By the Perpendicular Bisector Theorem,  $AB = AD$ .

$$3x = 12$$

Subtract  $5x$  from each side.

$$\frac{3x}{3} = \frac{12}{3}$$

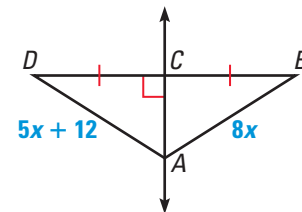
Divide each side by 3.

$$x = 4$$

Simplify.

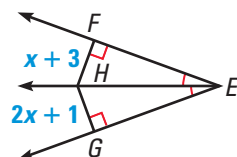
You are asked to find  $AB$ , not just the value of  $x$ .

**ANSWER**  $\blacktriangleright AB = 8x = 8 \cdot 4 = 32$

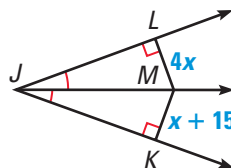


**Checkpoint** Use Angle Bisectors and Perpendicular Bisectors

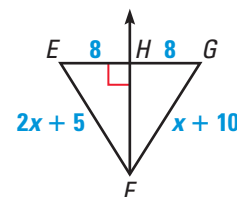
1. Find  $FH$ .



2. Find  $MK$ .

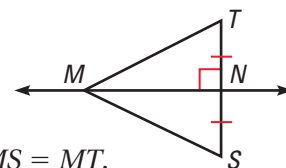


3. Find  $EF$ .



**EXAMPLE 3** Use the Perpendicular Bisector Theorem

In the diagram,  $\overleftrightarrow{MN}$  is the perpendicular bisector of  $\overline{ST}$ . Prove that  $\triangle MST$  is isosceles.

**Solution**

To prove that  $\triangle MST$  is isosceles, show that  $MS = MT$ .

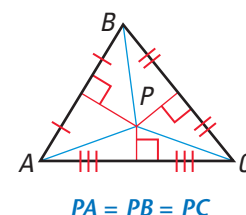
**Statements**

- $\overleftrightarrow{MN}$  is the  $\perp$  bisector of  $\overline{ST}$ .
- $MS = MT$
- $\triangle MST$  is isosceles.

**Reasons**

- Given
- Perpendicular Bisector Theorem
- Def. of isosceles triangle

**Intersecting Bisectors** One consequence of the Perpendicular Bisector Theorem is that the perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.

**Link to Careers****FACILITIES PLANNER**

By finding a location for a warehouse that is easily accessible to all its stores, a facilities planner helps a company save money and run more efficiently.

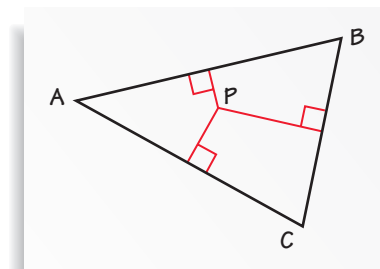
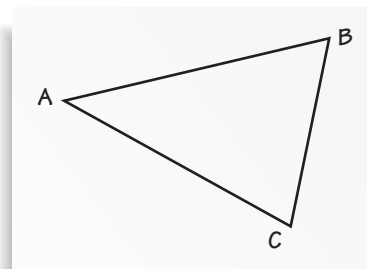
**EXAMPLE 4** Use Intersecting Bisectors of a Triangle

A company plans to build a warehouse that is equidistant from each of its three stores, A, B, and C. Where should the warehouse be built?

**Solution**

Think of the stores as the vertices of a triangle. The point where the perpendicular bisectors intersect will be equidistant from each store.

- Trace the location of the stores on a piece of paper. Connect the points of the locations to form  $\triangle ABC$ .
- Draw the perpendicular bisectors of  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{CA}$ . Label the intersection of the bisectors  $P$ .



**ANSWER** ▶ Because  $P$  is equidistant from each vertex of  $\triangle ABC$ , the warehouse should be built near location  $P$ .

# 5.6 Exercises

## Guided Practice

### Vocabulary Check

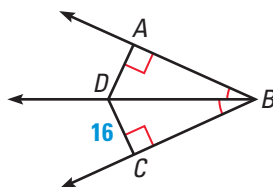
Complete the statement.

- If a point is on the bisector of an angle, then it is   ? from the two sides of the angle.
- If  $D$  is on the   ? of  $\overline{AB}$ , then  $D$  is equidistant from  $A$  and  $B$ .

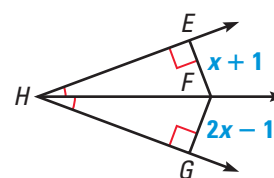
### Skill Check

Use the information in the diagram to find the measure.

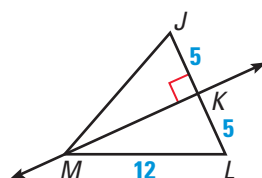
3. Find  $AD$ .



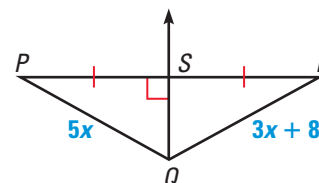
4. Find  $EF$ .



5. Find  $JM$ .



6. Find  $QR$ .



## Practice and Applications

### Extra Practice

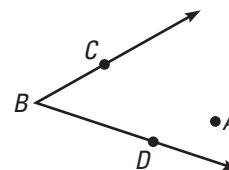
See p. 684.

**Visualize It!** Copy each diagram on a piece of paper. Then draw a segment that represents the distance from  $A$  to  $\overline{BC}$ .

7.  $A \bullet$



8.



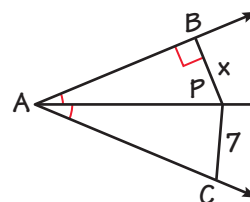
9. **Error Analysis** Explain why Paige cannot make this conclusion, given the diagram shown.

### Homework Help

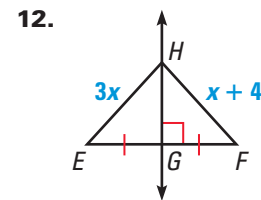
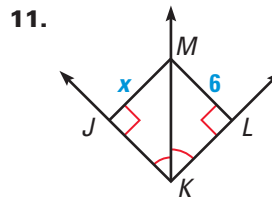
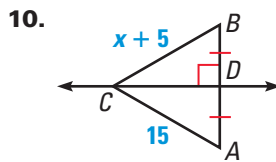
- Example 1: Exs. 32, 33
- Example 2: Exs. 10–12, 14–19
- Example 3: Exs. 32, 33
- Example 4: Exs. 23–29

Paige

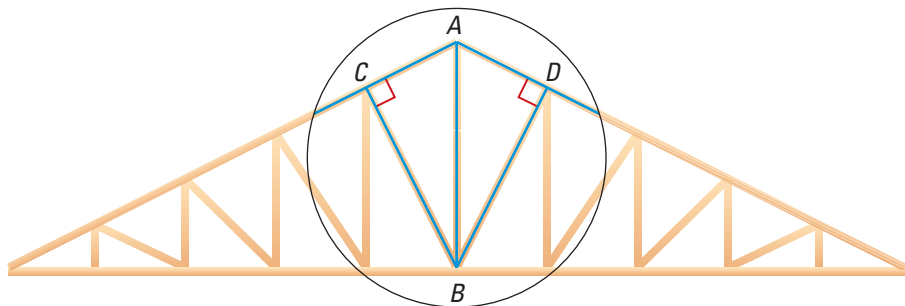
~~By the Angle Bisector Theorem,  $x = 7$ .~~



**Using Algebra** Find the value of  $x$ .

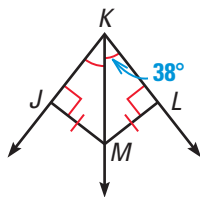


13. **Roof Trusses** In the diagram of the roof truss shown below, you are given that  $\overline{AB}$  bisects  $\angle CAD$  and that  $\angle ACB$  and  $\angle ADB$  are right angles. What can you say about  $\overline{BC}$  and  $\overline{BD}$ ? Why?

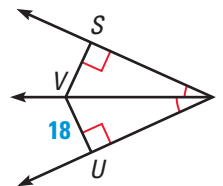


**Using Bisectors** Use the diagram to find the indicated measure(s).

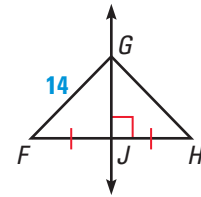
14. Find  $m\angle JKM$ .



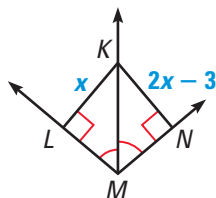
15. Find  $SV$ .



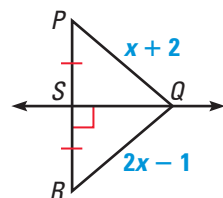
16. Find  $HG$ .



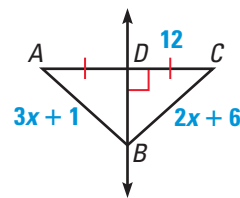
17. Find  $LK$ .



18. Find  $PQ$ .



19. Find  $AD$  and  $BC$ .



**Link to Careers**

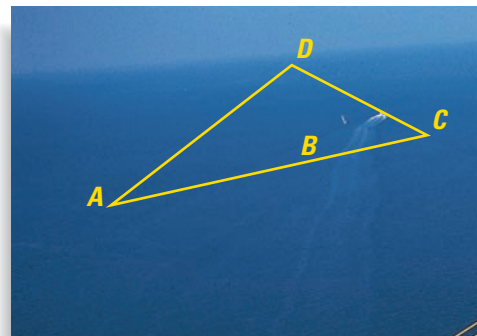


**CIVIL ENGINEERS** plan and build large construction projects, such as bridges, canals, and tunnels.



20. **Bridges** In the photo, the road is perpendicular to the support beam and  $\overline{AB} \cong \overline{CB}$ .

What theorem allows you to conclude that  $\overline{AD} \cong \overline{CD}$ ? Explain.

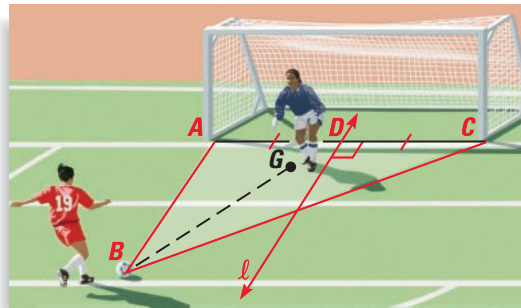


**Student Help**

**LOOK BACK**

For more about soccer, see p. 230.

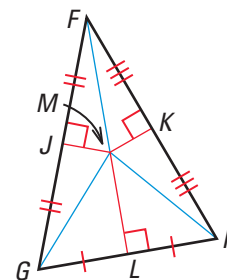
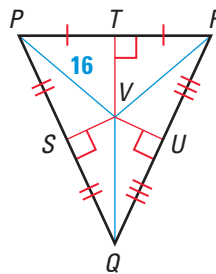
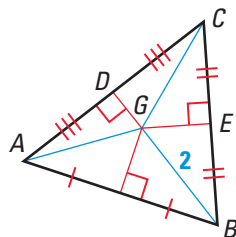
**Soccer** One way a goalie can determine a good defensive position is to imagine a triangle formed by the goal posts and the ball.



21. When the ball is far from the goal, the goalie most likely stands on line  $l$ . How is  $l$  related to the goal line  $(\overline{AC})$ ?
22. As the ball moves closer, the goalie moves from line  $l$  to other places in front of the goal. How should  $\overrightarrow{BG}$  relate to  $\angle ABC$ ? Explain.

**Using Perpendicular Bisectors** Use the information in the diagram.

23. Find  $CG$  and  $AG$ .
24. Find  $VR$  and  $VQ$ .
25. Name all congruent segments.



**Student Help**  
CLASSZONE.COM

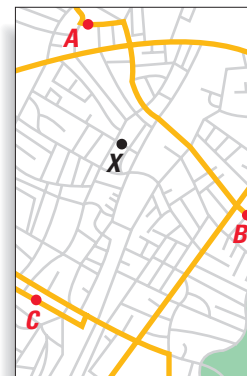
**HOMEWORK HELP**

Extra help with problem solving in Exs. 26–29 is at classzone.com

**Analyzing a Map** In Exercises 26–29, use the map shown and the following information.

A city planner is trying to decide whether a new household at point  $X$  should be covered by fire station  $A$ ,  $B$ , or  $C$ .

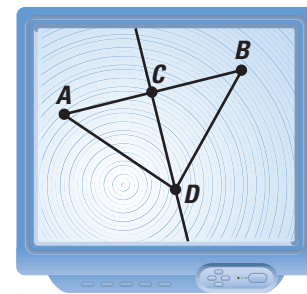
26. Trace the points  $A$ ,  $B$ ,  $C$ , and  $X$  on a piece of paper and draw the segments  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{CA}$ .
27. Draw the perpendicular bisectors of  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{CA}$ . Check that they meet at a point.
28. The perpendicular bisectors divide the town into three regions. Shade the region closest to fire station  $A$  red. Shade the region closest to fire station  $B$  blue. Shade the region closest to fire station  $C$  gray.



29. **Writing** In an emergency at household  $X$ , which fire station should respond? Explain your choice.

**Technology** In Exercises 30 and 31, use geometry drawing software to complete the steps below.

- 1 Draw  $\overline{AB}$ . Find the midpoint of  $\overline{AB}$  and label it  $C$ .
- 2 Construct the perpendicular bisector of  $\overline{AB}$  through  $C$ .
- 3 Construct point  $D$  along the perpendicular bisector. Construct  $\overline{DA}$  and  $\overline{DB}$ .



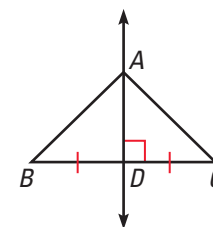
30. What is the relationship between  $\overline{DA}$  and  $\overline{DB}$ ? Measure  $\overline{DA}$  and  $\overline{DB}$  to verify your answer.

31. Move  $D$  to another point along the perpendicular bisector. Will the relationship between  $\overline{DA}$  and  $\overline{DB}$  stay the same? Why?

32. **Proving the Perpendicular Bisector Theorem** Fill in the missing statements and reasons.

**Given** ▶  $\overleftrightarrow{AD}$  is the perpendicular bisector of  $\overline{BC}$ .

**Prove** ▶  $AB = AC$



**Statements**

1.  $\overleftrightarrow{AD}$  is the perpendicular bisector of  $\overline{BC}$ .
2.  $\overline{DB} \cong \overline{DC}$
3. \_\_\_\_\_ ?
4. \_\_\_\_\_ ?
5. \_\_\_\_\_ ?
6.  $\triangle ADB \cong \triangle ADC$
7.  $\overline{AB} \cong \overline{AC}$
8. \_\_\_\_\_ ?

**Reasons**

1. \_\_\_\_\_ ?
2. \_\_\_\_\_ ?
3.  $\perp$  lines form right angles.
4. Right angles are congruent.
5. Reflexive Prop. of Congruence
6. \_\_\_\_\_ ?
7. \_\_\_\_\_ ?
8. Def. of congruent segments

**Student Help**

**LOOK BACK**

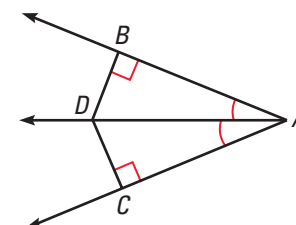
For help with writing proofs, see p. 243.

33. **Challenge** Use the diagram and the information below to prove the Angle Bisector Theorem.

**Given** ▶  $D$  is on the bisector of  $\angle BAC$ .  
 $\overline{DB} \perp \overline{AB}$ ,  $\overline{DC} \perp \overline{AC}$

**Prove** ▶  $\overline{DB} \cong \overline{DC}$

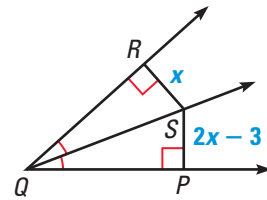
*Hint:* First prove that  $\triangle ADB \cong \triangle ADC$ .



### Standardized Test Practice

34. **Multiple Choice** In the figure at the right, what is  $SR$ ?

- (A) 2
- (B) 3
- (C) 4
- (D) 5

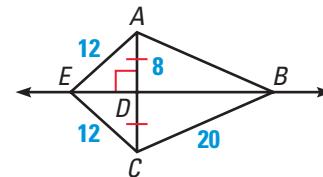


35. **Multiple Choice** In the figure above, what is  $SP$ ?

- (F) 2
- (G) 3
- (H) 4
- (J) 5

36. **Multiple Choice** What can you say about the figure below, in which  $\overleftrightarrow{BE}$  is the perpendicular bisector of  $\overline{AC}$ ?

- (A)  $AB = 20$
- (B)  $AC = 16$
- (C)  $DC = 8$
- (D) All of these

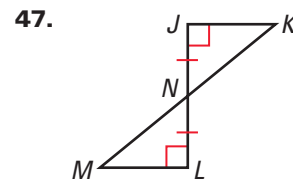
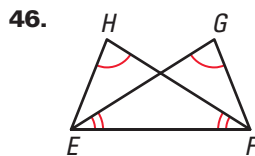
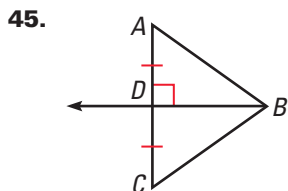


### Mixed Review

**Translations in a Coordinate Plane** Find the image of the given point using the translation  $(x, y) \rightarrow (x + 3, y - 6)$ . (Lesson 3.7)

- 37. (5, 1)
- 38. (-2, 3)
- 39. (-4, -4)
- 40. (0, -6)
- 41. (6, 2)
- 42. (2, -5)
- 43. (10, 12)
- 44. (-1, -1)

**Determining Congruent Triangles** What theorem or postulate, if any, can you use to show that the triangles are congruent? Explain your reasoning. (Lesson 5.5)



### Algebra Skills

**Ordering Numbers** Write the numbers in order from least to greatest. (Skills Review, p. 662)

- 48. 3, -3, 0.3, -0.3, 0.6, 0
- 49. -0.25, 1, -0.75, 4, -1.25, 0.25
- 50. -0.4, 0.1, 0, 4.0, -0.1, -4
- 51. -3.3, 3.1, 3.8, -3.9, -3, 3.5
- 52. 0.55, -1, 1.1, 1, 0.5, -0.1, 0
- 53. 3.2, 1, 2.1, 3.25, -2.5, 5

**Solving Equations** Solve the equation. (Skills Review, p. 673)

- 54.  $4x + 3 = 11$
- 55.  $2y - 9 = -11$
- 56.  $5d - 35 = 90$
- 57.  $4a + 9a = 39$
- 58.  $x + 2 = 3x - 4$
- 59.  $4r - 2 = 5r + 6$
- 60.  $q = 2q - 9$
- 61.  $2z + 5 = 4z - 1$
- 62.  $10t + 10 = 12t$