Course Title: Advanced Algebra with Financial Applications

Department: Mathematics

Course #:   2215

Grade Level/s: 11-12

Length of Course: Year

Prerequisite/s: C or better in Geometry

UC/CSU (A-G) Req: C (Algebra II/Math III)

Brief Course Description: This course meets UC/CSU (c) Algebra II/Math III requirement. Advanced Algebra with Financial Applications (AAFA) provides students mathematical tools to become financially literate and responsible. Students will apply advanced mathematics to analyze and solve real-world problems in investments, credit, banking, auto insurance, mortgages, employment, income taxes, budgeting and planning for retirement. Field projects, computer spreadsheets, and graphing calculators are key components of this course. The interrelated instructional approach provides students with analytical understanding of fundamental business and finance issues while providing an engaging context to master the foundational Algebra II concepts. Students routinely use the standards for mathematical practice to make sense of problems, justify solutions and conclusions, model with mathematics, and strategically use technology to analyze and solve real-world problems.

I. GOALS

The students will:
A. Define basic business organization terminology in order to read, interpret and chart stock ownership and transaction data

B. Apply statistical analysis by using linear, quadratic and regression equations to obtain a complete picture of supply, demand, expense, revenue and profit as they relate to sale of a product

C. Explore function and computation of interest in short-term, long-term, single deposit and periodic deposit accounts

D. Use and manipulate credit formulas in order to make wise credit choices that fit their needs, current financial situation and future goals

Course Outline: Advanced Algebra with Financial Applications

    E. Analyze various functions and graphs that models purchasing, insuring, depreciating and driving a car

    F. Determine how salaries are computed, benefits bestowed and wage taxes calculated in order to make smart employment choices both before accepting a job and during the period of employment in that job

    G. Create functions and analyze graphic representations of these functions to gain insight into personal income reporting, credit and income tax obligations

    H. Compare calculations for moving, renting and mortgage

    I. Explore geometric demands of floor plans and design and investigate the relationship between area and probability

    J. Investigate retirement savings plans, both personal and federal, employee pension programs and life insurance

    K. Create, chart and use a responsible personal budget, living expenses and banking as well as level of education and related salaries

These goals are aligned with the California State Standards, including the Standards of Mathematical Practices.

II. OUTLINE OF CONTENT FOR MAJOR AREAS OF STUDY

Semester 1

A. Linear and Quadratic Equations Unit: *The Stock Market*
1. Compute financial responsibility of business ownership based on ratios and percents (A.CED.1, A.REI.3)
2. Use stock data to follow the daily progress of a corporate stocks (N.Q.1-3)
3. Write spreadsheet formulas (A.CED.1, A.REI.3)
4. Interpret and create a stock bar chart (N.Q.1, N.Q.2)
5. Interpret and create a stock candlestick chart (N.Q.1, N.Q.2)
6. Calculate simple moving averages using the arithmetic average formula or subtraction and addition method (N.Q.1, N.Q.2)
7. Graph simple moving averages using a spreadsheet (N.Q.1, N.Q.2)
8. Determine the total value of a trade and trade volumes from ticker information (N.Q.1, N.Q.2)
9. Define basic vocabulary of buying and selling stock (N.Q.1)
10. Compute gains and losses from stock trades (A.CED.1, A.CED.4, A.REI.3)
11. Compute the fees involved in buying and selling stocks (A.CED.2)
12. Calculate the post-split outstanding shares and share price for a traditional split and reverse split (A.CED.1, A.REI.3)
13. Calculate the fractional value amount that a shareholder receives after a split (A.CED.1, A.REI.3)
14. Compute dividend income, yield for a given stock, and interest earned on corporate bonds (A.SSE.1)

Course Outline: Advanced Algebra with Financial Applications

B. Linear and Quadratic Functions Unit: *Modeling a Business*
   1. Graph bivariate data and draw lines and curves of best fit (N.Q.2, N.Q.3)
   2. Interpret trends based on scatterplots (F.IF.7a, F.IF.8, S.ID.6, S.ID.9)
   3. Find and interpret correlation coefficient (N.Q.1-3, A.CED.2)
   4. Make predictions based on lines of best fit (F.IF.1, F.IF.7a, F.IF.8, S.ID.6c, S.ID.8)
   5. Understand slopes of supply and demand curves and determine points of equilibrium (A.CED.2, F.IF.1, F.IF.4-5, F.IF.7a, F.IF.8, S.ID.6c, S.ID.8)
   6. Compare/Contrast fixed and variable expenses (A.REI.2, A.REI.6, A.REI.12)
   7. Create an expense equation based on fixed and variable expenses (A.CED.2-3, F.IF.8, S.ID.8)
   8. Write, graph and interpret expense function and revenue function (A.SSE.1a, A.CED.2, A.CED.3, F.IF.7a, F.IF.8)
   9. Identify breakeven points and explain them in the context of the problem (N.Q.1, N.Q.2, N.Q.3, A.REI.4b, A.REI.10)
   10. Determine a profit equation given the expense and revenue equations, as well as the maximum profit, and the prices at which that maximum is attained (A.CED.2-3, A.REI.7, A.REI.10, A.REI.11 F.IF.7a, F.IF.8)
   11. Recognize the transitive property of dependence as it is used in a business model (N.Q.1, N.Q.2)
   12. Use multiple pieces of information, equations, and methodologies to model a new business (A.CED.2-3, A.REI.7, A.REI.10, A.REI.11 F.IF.7a, F.IF.8, CA Modeling Standard)

C. Exponential and Logarithmic Functions Unit: *Banking Services*
   1. Explain how checking and savings accounts work (A.SSE.1, A.SSE.3, F.BF.1a, CA Modeling Standard)
   2. Verify a checking account with a bank statement by hand and by using a spreadsheet (A.CED.4, CA Modeling Standard)
   3. Compute simple interest using the simple interest formula (A.CED.4)
   4. Compute compound interest using a table— understand the concept of getting interest on your interest (A.SSE.1a, A.SSE.1b)
   5. Analyze the derivation of the compound interest formula (A.SSE.3c, F.IF.8b)
   6. Compute interest on account that is continuously compounded (N.RN.1, N.RN.2, A.SSE.1b, A.SSE.3)
   7. Calculate the future value of a periodic deposit investment (F.IF.8b)
   8. Graph and interpret the graph of the future value function (F.IF.8b)
   9. Calculate and compare the present value of a single deposit investment and a periodic deposit investment (F.IF.4, A.SSE.3, A.CED.4, F.IF.8b)

D. Exponential and Logarithmic Functions Unit: *Consumer Credit*
   1. Define basic vocabulary of credit terms and be familiar with types of lending institutions (N.Q.1, CA Modeling Standard)
   2. Compute finance charges for installment purchases (A.SSE.3, A.CED.3, F.BF.1a)
   3. Analyze monthly payments from a table (A.SSE.1b, F.LE.5)
   4. Compute monthly payments using a formula and finance charges on loans (A.SSE.2, A.SSE.3c, F.IF.8b)
   5. Calculate and model loan computations (S.ID.6a)
   6. Computer an average daily balance (N.Q.1, N.Q.2)
   7. Identify and use the various entries in a credit card statement (N.Q.1, A.SSE.1)

E. Functions and Their Graphs & Statistics Unit: *Automobile Ownership*
   1. Compute the cost of classified ads for used cars and sales tax on automobiles (A.CED.2, F.IF.1, F.IF.2, F.IF.7b)
   2. Compare and contrast mean, median, mode, range, quartiles, and interquartile range in buying or selling a car (S.ID.2, S.ID.3, S.ID.4)
   3. Create a frequency distribution from a set of data (S.ID.1, S.ID.2)
   4. Use box-and-whisker plots and stem-and-leaf plots to display information (S.ID.1, S.ID.2, S.ID.3)
   5. Use linear regression to negotiate the purchase or sale of a used car (S.ID.2, S.ID.3, S.ID.4)
   6. Discuss different types of auto insurance coverage (CA Modeling Standard)
   7. Compute insurance costs and payments on insurance claims (F.IF.7a, S.ID.7)
   8. Write, interpret, and graph a straight line depreciation and exponential depreciation equations (A.CED.2, A.CED.3, F.IF.6, F.IF.7a, F.IF.9, F.LE.1b, F.LE.5)
   9. Manipulate the exponential depreciation equation in order to determine time, original price and depreciated value (A.CED.2, A.CED.3, F.IF.7e, F.IF.8b, F.LE.1c, F.LE.5, S.ID.6)
   10. Write, interpret, and use the distance formula to determine the relationship between distance, fuel economy, and gas usage (A.CED.4)
   11. Calculate reaction time and distance in the English Standard System (A.SSE.1b, A.SSE.3)
   12. Calculate and use the braking distance and total stopping distance in both the English Standard and Metric Systems (A.SSE.1b, A.SSE.3)
   13. Determine and compare/contrast the minimum skid speed using the skid mark formula and yaw mark formula (A.REI.2, G.C.5, F.IF.4)

Semester 2

A. Systems of Equations Unit: *Employment Basics and Income Taxes*
   1. Compute periodic salary based on annual contract salary (A.CED.2)
   2. Express classified ad prices as piecewise functions (A.CED.2, F.IF.2)
   3. Calculate weekly, semimonthly, and biweekly earnings given annual salary (A.CED.4)
   4. Determine hourly pay and overtime pay given hourly rate (A.CED.4)
   5. Compute and compare pay-based on percent commission with piecework pay (A.CED.1, F.IF.2)
   6. Examine the advantages and disadvantages of pay based on production (A.CED.1, F.IF.2)
   7. Analyze and calculate the value of certain employee benefits (A.CED.1, A.REI.3, F.BF.1, F.LE.1)
   8. Calculate paycheck deductions for Social Security and Medicare (F.IF.4, F.IF.7b)
   9. Express tax schedules algebraically (A.CED.3)
   10. Compute federal income taxes using a tax table and tax schedules (A.CED.3)
   11. Construct income tax graphs using compound equations (F.IF.1, F.IF.2, F.IF.7b, F.IF.8, F.BF.1a)
   12. Interpret and use the information on a pay stub, W-2 form, and 1099 form (A.SSE.1, F.BF.1)
   13. Complete tax forms 1040EZ, 1040A, and 1040 with itemized deductions (A.SSE.1, A.CED.3)

14. Differentiate a tax credit from a tax deduction (A.SSE.1, A.CED.3)

B. Radical and Rational Functions Unit: *Independent Living*
    1. Calculate lease signing costs, moving expenses, and the affordability of monthly rent (A.CED.2, A.CED.3, A.REI.6)
    2. Determine the relationship between square footage and monthly rent (S.ID.6a, S.ID.6c, S.ID.8)
    3. Find the perimeter and area of a polygon and irregular polygons (G.GMD.1)
    4. Determine volumes of rectangular solids (G.GMD.3)
    5. Compute the monthly cost of paying for a house and understand the necessary research before purchasing a home (A.SSE.1, A.CED.3, A.APR.6, F.BF.1)
    6. Create amortization tables for a fixed rate mortgage and a fixed rate mortgage with extra payments (A.SSE.1, F.BF.1)
    7. Investigate the amortization table for an adjustable rate mortgage (A.SSE.1, F.BF.1)
    8. Estimate closing costs (A.SSE.1)
    9. Calculate costs of purchasing a cooperative or a condominium (A.SSE.1b, F.BF.1, F.LE.1)
    10. Examine advantages and disadvantages of different forms of homes (CA Modeling Standard)

C. Mathematical Modeling: *Planning for Retirement Unit*
    1. Calculate future values of retirement investments that are both single deposit and periodic (F.IF.8b)
    2. Compare the tax savings by making contributions to pre-tax retirement savings accounts (F.IF.8b)
    3. Determine an employer's matching contribution to a retirement account (F.IF.8b)
    4. Examine how benefits are computed and the benefits paid by Social Security (A.SSE.1, A.CED.3)
    5. Compute federal income tax on benefits that are paid under Social Security (A.SSE.1, A.CED.3)
    6. Analyze calculated pension benefits using various formulas (F.BF.1)
    7. Calculate pension benefits during and after vesting periods (F.BF.1)
    8. Compute the cost of different types of life insurance (S.MD.1, S.MD.2, S.MD.4, S.MD.5)
    9. Discuss the advantages and disadvantage of different types of life insurance (CA Modeling Standard)

D. Mathematical Modeling: *Preparing a Budget Unit*
    1. Compute the cost of electric, gas, oil, and water for the home (N.Q.1, N.Q.2)
    2. Compute the cost of using specific appliances for specific lengths of time (N.Q.1, N.Q.2)
    3. Calculate the time for an energy-saving appliance to pay for itself (A.SSE.1a, A.SSE.1b)
    4. Compare different plans for various electronic utilities such as cell phone, internet service, and cable television (A.SSE.1a, F.IF.7a, F.IF.7b)
    5. Create and use a budget check-off matrix (N.VM.6, A.REI.10)
    6. Visualize and interpret a budget using a pie chart, a bar graph, a line graph, and budget line graph (A.REI.10, F.IF.4, F.IF.5, F.IF.7a)

7. Develop and interpret a cash flow chart, a frequency budget plan, and a year-long expense budget plan (A.SSE.1, F.BF.1, CA Modeling Standard)

III. ACCOUNTABILITY DETERMINANTS

A. Key Assignments
1. Students select a corporation, traded on the New York Stock Exchange, and produce a background paper, PowerPoint presentation or poster board display on that corporation. Students chart the open, close, high, low and volume data for 15 consecutive trading days. Students graph the data using two different formats and then discuss trends that the data shows. Students also calculate three different cluster-lengths of moving averages and, using those clusters, will create superimposed line graphs. Students discuss trading implications based upon stated domains of graph pairs before and after any intersection points. Finally, students determine the closing price curve of best fit using regression analysis. Students must state the regression equation and support why their stated curve best fits the data of closing prices. Students then use the curve of best fit to predict a closing price on the 16th trading day. Students compare predicted price with the actual closing price on the 16th day and find a percent error.
2. Students pick a college and find out the cost of tuition, room and board (if necessary) and fees over the past ten years. Students determine a regression line or curve of best fit to predict the cost of a college education in 18 years. Students use the prevailing interest rate and future value formula to determine the monthly periodic deposit necessary to have the full college cost saved after 18 years.
3. Students are given a market research scenario for a new product, attained from a focus group question. The research contains a list of ordered pairs in the form (p, q) where p is a potential price and q is the quantity of the product that the focus group member would purchase if it were set at that price. Using these ordered pairs, students construct a scatterplot, determine the correlation coefficient, and identify a linear regression equation in which q is the independent variable and p is the dependent variable. With given information on expenses, students determine a linear expense function in terms of the quantity demanded. The quadratic revenue and profit equations are determined and graphed on the same axes with the expense function. Students identify and interpret the breakeven points, the coordinates of the maximum point on the revenue graph, the coordinates of the maximum point on the profit graph, and the price at which the product should be sold in order to maximize profit. Finally, students are told the initial price per share for the company's stock and asked to determine the number of shares that must be sold in order to have enough money to start this business.
4. The tax tables give taxpayers a function in which the independent variable is the taxable income and the dependent variable is the tax. It is convoluted and has confused taxpayers for years. Within the last decade, the IRS created a worksheet that uses the slope-intercept form of the equations of a line to simplify calculations for the taxpayer. Students interpret the IRS Schedule, express the domains using compound inequality notation, and create the piecewise function that models the IRS intentions. Students convert this function, which is a translated version of point-slope form, into the slope-intercept form to create the tax worksheet.

5. Using the monthly payment rational function, students graph the cost C of purchasing a new car, using the down payment as the *y*-intercept, and the monthly payment as the slope. Students investigate three types of depreciation: straight-line, exponential, and historical bathtub graphs. Students graph the cost and depreciation functions on the same set of axes to find the month at which the total cost C of owning the car surpasses its value V as it depreciates. They identify and interpret the domains on which C > V and C < V.

6. Students use formulas to determine reaction distance, braking distance, and figure out the speed a car was going based on its skid marks. The braking-distance formula is a quadratic function, with speed as the independent variable. The skid speed formula is an irrational function that has three independent variables. Students also use geometry of the circle to compute the radius of a given yaw mark, which is a curved skid mark, and use the radius and friction factor to find the speed the car was going when it began to skid. Students prepare a PowerPoint or poster presentation (for the driver's education class at school).

7. Students are given a scenario in which a family must make a decision about the affordability of a loan based on the principal, the loan-length, the APR and the maximum affordable monthly payment the family is able to make towards loan debt reduction. Students determine the affordability of the loan in three different ways: using the monthly payment function, interpreting the graphs of the system of equations defined by the exponential monthly payment function and the linear maximum affordable monthly payment, and using the logarithmic loan length function. Students construct two spreadsheets: a monthly payment spreadsheet that charts the monthly payment as loan length time varies from 1 to 20 years, and a loan length spreadsheet that charts time as monthly payments vary from $100 to $1000. Finally, students write a summary analysis for this situation explaining how the algebraic modeling by the spreadsheet formulas supports their prior work.

8. Students use the monthly payment formula to compute the monthly payment for a hypothetical mortgage amount over 15 and 30 years. Students compute the total payments, based on 12 monthly payments each year, and the total interest for the entire loan. Students use a mortgage calculator to assume an extra, 13th payment is made each year, so payments are made once every 4 weeks instead of once each month. Students compute the interest and new total repayment period and compare the total interest to the original conventional mortgage to see the savings in total years and interest.

9. Students are given financial information about a prospective retiree and asked to act as a financial retirement planner. The prospective retiree has also supplied the planner with desired monetary goals in retirement. Based upon information about savings plans, social security benefits, pensions, and life insurance policies, and using formulas learned in this unit, the planner is to write up a financial plan for the prospective retiree that includes at least two ways of meeting the goals and has mathematical justification for the recommendations.

10. Students are given a budget spreadsheet that contains the headings of income, fixed expenses, variable expenses, and non-monthly expenses. There are sub-headings under each of these listing specific categories relating to the heading. Students are given a full accounting of a person's financial status and asked to build a spreadsheet that calculates that person's cash flow. In addition, the students are

given information about the person's assets and liabilities and are asked to add it to the spreadsheet and determine the net worth. Finally, based upon the calculation of the debt-to-income ratio, students are asked to develop a debt reduction plan for the individual if necessary.

B. Assessment Methods
   1. Daily Student Observation of Classroom Participation, Effort and Achievement
   2. Classwork/Homework
   3. Projects
   4. Quizzes
   5. End of Unit Tests

IV. INSTRUCTIONAL MATERIALS AND METHODOLOGIES

A. Required Textbook(s)
   1. Gerver, R. & Sgroi, R. *Financial Algebra.* South-Western/Cengage Learning: Mason OH. Copyright 2011

B. Supplementary Materials
   1. Bellman, Allan, et al. California Algebra 2. Boston: Pearson Prentice Hall, 2009
   2. Larson, R., *Precalculus with Limits: A Graphing Approach (High School Edition)* Cengage Learning: Boston, MA. Copyright 2012
   3. www.practicalmoneyskills.com

IV. INSTRUCTIONAL METHODOLOGIES

A. Motivational Unit Openers/Essential Questions

B. Closed Reading

C. Guided Inquiry

D. Direct Instruction

E. Cooperative Learning

F. Discourse

G. Visual Representations and Concrete Models

H. Problem-Based Learning

I. Experiential Learning