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| **Course Title:** | Math II |
| **Department:** | Mathematics |
| **Course #:** | 2216 |
| **Grade Level/s:** | 10 |
| **Length of Course:** | Year |
| **Prerequisite/s:** | By placement: Grade C or better in Math I, teacher recommendation, and/or District Assessment Results |
| **UC/CSU (A-G) Req:** | (C) Mathematics |

**Brief Course Description:** The purpose of Math II is to develop students' ability to think mathematically and develop their conceptual understanding of mathematics and procedural fluency in mathematics. Math II will extend the mathematics students learned in earlier grades and apply concepts in Number and Quantity, Algebra, Functions, Modeling, Geometry, and Probability and Statistics. The critical topics of this course are:  Linear, Quadratic, Exponential, and Trigonometric Functions; Geometric Properties of Congruence; Similarity; Right Triangles and Trigonometry; Circles and Volume; Probability; and Modeling Data. Extensive use of models (or real-world situations), manipulatives, graphs and diagrams will help students view how mathematics is a set of related topics as opposed to a set of discrete topics. In addition, students will solve problems graphically, numerically, algebraically, and make verbal connections between these representations. Students routinely use the Standards for Mathematical Practice to make sense of problems, justify solutions and conclusions, model with mathematics, and strategically use technology to analyze and solve real-world problems.

## I.  GOALS

The students will:
A.    Reason quantitatively

B.    Extend the properties of exponents to rational exponents

C.    Use properties of rational and irrational numbers

D.    Perform arithmetic operations with complex numbers [$i^2$ as the highest power of $i$]

E. Use complex numbers in polynomial identities and equations [Quadratics with real coefficients]

F. Interpret the structure of expressions [Quadratic and exponential]

G. Write expressions in equivalent forms to solve problems [Quadratic and exponential]

H. Perform arithmetic operations on polynomials [Polynomials that simplify to quadratics]

I. Create equations that describe numbers or relationships

J. Solve equations and inequalities in one variable [Quadratics with real coefficients]

K. Solve systems of equations [Linear-quadratic systems]

L. Interpret functions that arise in applications in terms of the context [Quadratic]

M. Analyze functions using different representations [Linear, exponential, quadratic, absolute value, step and piecewise-defined]

N. Build a function that models a relationship between two quantities [Quadratic and exponential]

O. Build new functions from existing functions [Quadratic and absolute value]

P. Construct and compare linear, quadratic and exponential models and solve problems

Q. Interpret expressions for functions in terms of the situation they model

R. Prove and apply trigonometric identities

S. Prove geometric theorems [Focus on validity of underlying reasoning while using variety of ways to write proofs.]

T. Understand similarity in terms of similarity transformations

U. Prove theorems involving similarity [Focus on validity of underlying reasoning while using variety of formats.]

V. Define trigonometric ratios and solve problems involving right triangles

W. Understand and apply theorems about circles

X. Find arc lengths and areas of sectors of circles [Radian introduced only as unit of measure.]

Y. Translate between the geometric description and the equation for a conic section

Z. Use coordinates to prove simple geometric theorems algebraically

AA.  Explain volume formulas and use them to solve problems

BB.  Understand independence and conditional probability and use them to interpret data [Link to data from simulations or experiments]

CC.  Use the rules of probability to compute probabilities of compound events in a uniform probability model

DD.  Use probability to evaluate outcomes of decisions

These goals are aligned with the California State Standards, including the Standards of Mathematical Practices.

## II.  OUTLINE OF CONTENT FOR MAJOR AREAS OF STUDY

### Semester 1

A.  Quadratic Functions
1.  Build a function that models a relationship between two quantities that are quadratic and exponential (F.BF.1)
2.  Construct and compare linear, quadratic and exponential models and solve problems  (F.LE.1-3)
3.  Interpret the structure of expressions with focus on situations that exhibit a quadratic or exponential relationship (A.SSE.1)
4.  Create equations that describe numbers or relationships (A.CED.1, A.CED.2)

B.  Structures of Expressions
1.  Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases  (F.IF.7)
2.  Graph linear and quadratic functions and show intercepts, maxima and minima (F.IF.7)
3.  Use factoring and completing the square in a quadratic function to show zeros, extreme values, symmetry of the graph, and interpret these in terms of a context (A.SSE.3, F.IF.8)
4.  Write a function that describes a relationship between two quantities (F.BF.1)
5.  Determine an explicit expression, a recursive process, and steps for calculation from a context (F.BF.1)
6.  Combine standard functions types using arithmetic operations (F.BF.3)
7.  Identify the effect on the graph of replacing f($x$) by f($x$)+$k$, $k$f($x$), f($kx$) and f($x$+$k$) for specific values of $k$ (both positive and negative) using technology and find the value of $k$ given the graphs (F.BF.3)
8.  Analyze and determine an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression (A.SSE.3)

C.  Quadratic Equations
1.  Extend the properties of exponents to rational exponents (N.RN.1-3)
2.  Perform arithmetic operations with complex number (N.CN.1, 2)
3.  Use complex numbers in polynomial identities and equations (N.CN.7-9)
4.  Analyze functions using different representations (F.IF.8)
5.  Write expressions in equivalent forms to solve problems (A.SSE.3c)

6. Solve equations and inequalities in one variable [quadratics with real coefficients] (A.REI.4)
7. Solve systems of linear and quadratic equations (A.REI.7)
8. Create equations that describe numbers or relationships (A.CED.4)
9. Perform arithmetic operations on polynomials that simplify to quadratics (A.APR.1)

D. More Functions, More Features
   1. Interpret functions that arise in applications in terms of the context (F.IF.4, 5, 7b)
   2. Incorporate absolute value as piecewise-defined functions (F.IF.7b)
   3. Build new functions, such as inverse and piecewise functions, from existing functions [quadratic and absolute value] **(**F.BF.4)
   4. Apply features of functions to create and graph new functions (F.IF.4)

## Semester 2

A. Geometric Figures
   1. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are equidistant from the segment's endpoints. (G.CO.9)
   2. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. (G.CO.10)
   3. Prove theorems about parallelograms. Theorems include: opposite sides are congruent; opposite angles are congruent; the diagonals of a parallelogram bisect each other; rectangles are parallelograms with congruent diagonals. (G.CO.11)

B. Similarity and Right Triangle Trigonometry
   1. Understand similarity in terms of similarity transformations (G.SRT.1, 2, 3, 4, 5, 6, 7, 8)
   2. Prove geometric theorems with focus on validity of underlying reasoning while using variety of ways of writing proofs (G.CO.9-11)
   3. Use coordinates to prove simple geometric theorems algebraically (G.GPE.6)
   4. Prove and apply trigonometric identities (F.TF.8)

C. Circles from a Geometric Perspective
   1. Understand and apply theorems about circles (G.C.1-4)
   2. Find arc lengths and areas of sectors of circles (G.C.5)
   3. Explain volume formulas and use them to solve problems (G.GMD.1, 3)
   4. Prove geometric theorems (G.CO.9)

D. Circles and Other Conics
   1. Derive the equation of the circle of given center and radius using the Pythagorean Theorem (G.GPE.1)
   2. Complete the square to find the center and radius of a circle given by an equation (G.GPE.1)
   3. Derive the equation of a parabola given a focus and directrix (G.GPE.2)

E. Probability
   1. Explain that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent (S.CP.2)
   2. Explain the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B (S.CP.3)
   3. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified and use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities (S.CP.4)
   4. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations (S.CP.5)
   5. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model (S.CP.6)
   6. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model (S.CP.7)
   7. Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game) (S.MD.7+)

## III. ACCOUNTABILITY DETERMINANTS

A. Key Assignments
   1. In the task "Tortoise and the Hare" from Quadratic Functions unit, students compare quadratic and exponential functions by examining tables and graphs for each. Students consider rates of change for each function type in various intervals and ultimately, see that an increasing exponential function will exceed a quadratic function.
   2. In the task "I've Got a Fill In" from Structures of Expressions unit, students build fluency in writing equivalent expressions for quadratic equations using factoring, completing the square, and the distributive property. Students will use the equations that they have constructed to analyze and graph quadratic functions.
   3. In the task "Throwing an Interception" from Quadratic Equations unit, students use the quadratic formula as a way to find x-intercepts of a quadratic functions. Students determine the quadratic formula from the perspective of visualizing the distance the x-intercepts are a way from the axis of symmetry and from the perspective of writing the general quadratic form in vertex form by completing the square.
   4. In the task "Bernie's Bikes" from More Functions, More Features unit, students solidify their understanding about inverse functions. Students will find the inverse of a function and know when to restrict the domain so that they can produce an invertible function from a non-invertible function. Students will also become familiar with square root functions as a result of this task and will connect square root functions to their domain, range and graphs.
   5. In the task "Parallelogram Conjectures and Proof?" from Geometric Figures unit, students prove theorems about the properties of parallelograms that were surfaced in Mathematics I as students explored the rotational symmetry and line symmetry of various types of quadrilaterals. Students will solidify that these properties of parallelograms are a consequence of the opposite sides of the quadrilateral being parallel to each other. Students will draw upon their theorems about parallel lines

being cut by a transversal—with the diagonals of the parallelogram forming the transversals—to prove these additional properties of parallelograms.

6. In the task "Relationships With Meaning" from Similarity and Right Triangle Trigonometry unit, students find relationships between sine and cosine using their knowledge of right triangles, complementary angles, and the Pythagorean theorem. Students will reason about conjectures through discussion related to the complementary relationship between sine and cosine as well as the Pythagorean identity.

7. In the task "Rays and Radians" from Circles from a Geometric Perspective unit, students examine and practice ideas and procedures associated with radians. Students observe that the circumference of a circle measures 2π radians, and use this fact to name many standard angles as fractions of π. They also create and use a conversion factor, π/180°, to convert degree measurements to radians.

8. In the task "Directing Our Focus" from Circles and Other Conics unit, students develop the definition of a parabola as the set of all points equidistant from a given point (the focus) and a line (the directrix). Students determine an equation for a parabola using the distance formula. Students consider the relationship between the focus and directrix and how the parabola changes as focus and directrix are moved/translated in relation to each other.

9. In the task "Fried Freddys" from Probability unit, students gain a stronger understanding regarding the law of large numbers and how this helps to estimate probable outcomes. Students also solidify their understanding around the following ideas: whether or not there is enough data to estimate outcomes; distinguish between a general probability, a conditional probability, and the addition rule; use of a Venn diagram to analyze data and to write various probability statements (unions, intersections, complements); apply the Addition Rule and interpret the answer in terms of the model; use estimated outcomes to make recommendations and decisions.

B. Assessment Methods
1. Daily Student Observation of Classroom Participation, Effort and Achievement
2. Classwork/Homework
3. Performance Tasks
4. Projects
5. Quizzes
6. End of Unit Tests
7. Semester Final Exams
8. District-wide Benchmark Exams

## IV. INSTRUCTIONAL MATERIALS AND METHODOLOGIES

A. Required Textbook(s)
1. Larson, Boswell, et al. California Geometry. McDougall Littell, 2007.

B. Supplementary Materials
1. Hendrickson, Honey, et al. Secondary Mathematics Two: An Integrated Approach
2. Mathematics Vision Project, 2013. This is an e-book located at http://www.mathematicsvisionproject.org
3. Holt McDougal, Explorations in CORE Math for Common Core Geometry. Houghton Mifflin Harcourt Publishing Company, 2010

           4.  www.mathalicious.com
           5.  www.geogebra.org

C.  Instructional Methodologies
    1.  Guided Inquiry
    2.  Direct Instruction
    3.  Cooperative Learning
    4.  Discourse
    5.  Problem-Based Learning
    6.  Visual Representations and Concrete Models