

Course Title:	Math Applications and Interpretations IB SL
Department:	Mathematics
Course Number:	2358
Grade Level/s:	11-12
Length of course:	One Year
Prerequisite/s:	B or better in Math 3 or C or better in Advanced Math 3
UC/CSU (A-G) Req:	Pending C
Brief Course Description:	<p>This course recognizes the increasing role that mathematics and technology play in a diverse range of fields in a data-rich world. As such, it emphasizes the meaning of mathematics in context by focusing on topics that are often used as applications or in mathematical modelling. To give this understanding a firm base, this course also includes topics that are traditionally part of a pre-university mathematics course such as calculus and statistics. The course makes extensive use of technology to allow students to explore and construct mathematical models. Mathematics: applications and interpretation will develop mathematical thinking, often in the context of a practical problem and using technology to justify conjectures. Five main topics include: number and algebra, functions, geometry and trigonometry, probability and statistics, and a brief introduction to calculus.</p>

I GOALS

- The goal of Math Applications and Interpretations IB SL is to enable students to:
- A. Develop a curiosity and enjoyment of mathematics, and appreciate its elegance and power
 - B. Develop an understanding of the concepts, principles, and nature of mathematics
 - C. Communicate mathematics clearly, concisely, and confidently in a variety of contexts
 - D. Develop logical and creative thinking, and patience and persistence in problem solving to instill confidence in using mathematics
 - E. Employ and refine their powers of abstraction and generalization
 - F. Take action to apply and transfer skills to alternative situations, to other areas of knowledge and to future developments in their local and global communities
 - G. Appreciate how developments in technology and mathematics influence each other
 - H. Appreciate the moral, social, and ethical questions arising from the work of mathematicians and the applications of mathematics
 - I. Appreciate the universality of mathematics and its multicultural, international, and historical perspectives

- J. Appreciate the contribution of mathematics to other disciplines, and as a particular “area of knowledge” in the TOK course
- K. Develop the ability to reflect critically upon their own work and the work of others
- L. Independently and collaboratively extend their understanding of mathematics

II OUTLINE OF CONTENT FOR MAJOR AREAS OF STUDY

Semester 1

A. Unit I: Number and Algebra (16 hours)

1. Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables to solve mathematical problems. Concepts include:
 - a. Modelling real-life situations with the structure of arithmetic and geometric sequences and series allows for prediction, analysis, and interpretation.
 - b. Different representations of numbers enable quantities to be compared and used for computational purposes with ease and accuracy.
 - c. Numbers and formulae can appear in different, but equivalent forms, or representations, which can help us to establish identities.
 - d. Formulae are a generalization made on the basis of specific examples, which can then be extended to new examples.
 - e. Mathematical financial models such as compounded growth allow computation, evaluation and interpretation of debt and investment both approximately and accurately.
 - f. Approximation of numbers adds uncertainty or inaccuracy to calculations, leading to potential errors but can be useful when handling extremely large or small quantities.
 - g. Quantities and values can be used to describe key features and behaviors of functions and models, including quadratic functions.
 - h. The aim of the standard level (SL) content of the number and algebra topic is to introduce students to numerical concepts and techniques which combined with an introduction to arithmetic and geometric sequences and series can be used for financial and other applications.

2. Key Assignment Summary:

An example of one key assignment is as follows:

Arithmetic sequences and series. Use of the formulae for the n th term and the sum of the first n terms of the sequence. Use of sigma notation for sums of arithmetic sequences.	Spreadsheets, GDCs and graphing software may be used to generate and display sequences in several ways. If technology is used in examinations, students will be expected to identify the first term and the common difference.
Applications.	Examples include simple interest over a number of years.
Analysis, interpretation, and prediction where a model is not perfectly arithmetic in real life.	Students will need to approximate common differences.

B. Unit II: Functions (31 hours)

1. Models are depictions of real-life events using expressions, equations, or graphs while a function is defined as a relation or expression involving one or more variable. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and/or tables represents different ways to communicate mathematical ideas.

Concepts include:

- a. Different representations of functions, symbolically and visually as graphs, equations and tables provide different ways to communicate mathematical relationships.
 - b. The parameters in a function or equation may correspond to notable geometrical features of a graph and can represent physical quantities in spatial dimensions.
 - c. Moving between different forms to represent functions allows for deeper understanding and provides different approaches to problem solving.
 - d. Our spatial frame of reference affects the visible part of a function and by changing this “window” can show more or less of the function to best suit our needs.
 - e. Changing the parameters of a trigonometric function changes the position, orientation, and shape of the corresponding graph.
 - f. Different representations facilitate modelling and interpretation of physical, social, economic, and mathematical phenomena, which support solving real-life problems.
 - g. Technology plays a key role in allowing humans to represent the real world as a model and to quantify the appropriateness of the model.
 - h. The aim of the standard level content in the functions topic is to introduce students to the important unifying theme of a function in mathematics and the skills needed to model and interpret practical situations with a variety of key functions.
2. Key Assignment Summary:
An example of one key assignment is as follows:

Different forms of the equation of a straight line. Gradient; intercepts. Lines with gradients m_1 and m_2 Parallel lines $m_1=m_2$. Perpendicular lines $m_1 \times m_2 = \dots 1$.	$y=mx+c$ (gradient-intercept form). $ax+by+d=0$ (general form). $y \dots y_1=m(x \dots x_1)$ (point-gradient form). Calculate gradients of inclines such as mountain roads, bridges, etc.
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C. Unit III: Geometry and Trigonometry (18 hours)

1. Geometry and trigonometry allow us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This branch provides us with the tools for analysis, measurement and transformation of quantities, movements, and relationships. Concepts include:
 - a. The properties of shapes are highly dependent on the dimension they occupy in space.
 - b. Volume and surface area of shapes are determined by formulae, or general mathematical relationships or rules expressed using symbols or variables.
 - c. The relationships between the length of the sides and the size of the angles in a triangle can be used to solve many problems involving position, distance, angles, and area.
 - d. Different representations of trigonometric expressions help to simplify calculations.

- e. Systems of equations often, but not always, lead to intersection points.
 - f. In two dimensions, the Voronoi diagram allows us to navigate, path-find or establish an optimum position.
2. The aim of the standard level content of the geometry and trigonometry topic is to introduce students to appropriate skills and techniques for practical problem solving in two and three dimensions.
 3. Throughout this topic students should be given the opportunity to use technology such as graphing packages, graphing calculators, and dynamic geometry software to develop and apply their knowledge of geometry and trigonometry.
 4. Key Assignment Summary:
An example of one key assignment is as follows:

<p>The distance between two points in three-dimensional space, and their midpoint.</p> <p>Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere, and combinations of these solids.</p> <p>The size of an angle between two intersecting lines or between a line and a plane.</p>	<p>In SL examinations, only right-angled trigonometry questions will be set in reference to three-dimensional shapes.</p> <p>In problems related to these topics, students should be able to identify relevant right-angled triangles in three-dimensional objects and use them to find unknown lengths and angles.</p>
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Semester 2

- A. Unit I: Statistics and Probability (40 hours)
 1. Statistics is concerned with the collection, analysis and interpretation of quantitative data and uses the theory of probability to estimate parameters, discover empirical laws, test hypotheses, and predict the occurrence of events. Statistical representations and measures allow us to represent data in many different forms to aid interpretation.
 2. Probability enables us to quantify the likelihood of events occurring and so evaluate risk. Both statistics and probability provide important representations which enable us to make predictions, valid comparisons, and informed decisions. These fields have power and limitations and should be applied with care and critically questioned, in detail, to differentiate between the theoretical and the empirical/observed. Probability theory allows us to make informed choices, to evaluate risk and to make predictions about seemingly random events. Concepts include:
 - a. Organizing, representing, analyzing, and interpreting data, and utilizing different statistical tools facilitates prediction and drawing of conclusions.
 - b. Different statistical techniques require justification and the identification of their limitations and validity.
 - c. Approximation in data can approach the truth but may not always achieve it.
 - d. Correlation and regression are powerful tools for identifying patterns and equivalence of systems.

- e. Modelling and finding structure in seemingly random events facilitate prediction.
 - f. Different probability distributions provide a representation of the relationship between the theory and reality, allowing us to make predictions about what might happen.
3. The aim of the standard level content in the statistics and probability topic is to introduce students to important concepts, techniques and representations used in statistics and probability and their meaningful application in the real world. Students should be given the opportunity to approach this topic in a practical way, to understand why certain techniques are used and to interpret the results. The use of technology such as simulations, spreadsheets, statistics software, and statistics apps can greatly enhance this topic.

4. Key Assignment Summary:

An example of one key assignment is as follows:

Concepts of population, sample, random sample, discrete and continuous data.	This is designed to cover the key questions that students should ask when they see a data set/analysis.
Reliability of data sources and bias in sampling.	Dealing with missing data, errors in the recording of data.
Interpretation of outliers.	Outlier is defined as a data item which is more than $1.5 \times$ interquartile range (IQR) from the nearest quartile. Awareness that, in context, some outliers are a valid part of the sample but some outlying data items may be an error in the sample. Link to: box and whisker diagrams (SL4.2) and measures of dispersion (SL4.3).
Sampling techniques and their effectiveness.	Simple random, convenience, systematic, quota and stratified sampling methods.

B. Unit II: Calculus (20 hours)

1. Calculus describes rates of change between two variables and the accumulation of limiting areas. Understanding these rates of change allows us to model, interpret and analyze real-world problems and situations. Calculus helps us understand the behavior of functions and allows us to interpret the features of their graphs. Concepts include:
- a. Students will understand the links between the derivative and the rate of change and interpret the meaning of this in context.
 - b. Students will understand the relationship between the integral and area and interpret the meaning of this in context.
 - c. Finding patterns in the derivatives of polynomials and their behavior, such as increasing or decreasing, allows a deeper appreciation of the properties of the function at any given point or instant.
 - d. Calculus is a concise form of communication used to approximate nature.
 - e. Numerical integration can be used to approximate areas in the physical world.

- f. Optimization of a function allows us to find the largest or smallest value that a function can take in general and can be applied to a specific set of conditions to solve problems.
 - g. Maximum and minimum points help to solve optimization problems.
 - h. The area under a function on a graph has a meaning and has applications in space and time.
2. The aim of the standard level content in the calculus topic is to introduce students to the key concepts and techniques of differential and integral calculus and their use to approach practical problems.
 3. Throughout this topic students should be given the opportunity to use technology such as graphing packages and graphing calculators to develop and apply their knowledge of calculus.
 4. Key Assignment Summary:
An example of one key assignment is as follows:

Introduction to the concept of a limit.	Estimation of the value of a limit from a table or graph. Not required: Formal analytic methods of calculating limits.
Derivative interpreted as gradient function and as rate of change.	Forms of notation: dy/dx , $f'(x)$, dV/dr or ds/dt for the first derivative. Informal understanding of the gradient of a curve as a limit.

III ACCOUNTABILITY DETERMINANTS

- A. Internal Assessment (15 hours):
 1. In addition to the five core topics, students will be required to produce an internal assessment project that incorporates the topics covered in this course. This will happen during the second semester and require approximately 15 hours.
 2. The internally-assessed component in this course is a mathematical exploration. This is a short report written by the student based on a topic chosen by him or her, and it should focus on the mathematics of that particular area. The emphasis is on mathematical communication (including formulae, diagrams, graphs, tables and so on), with his or her own focus, with the teacher providing feedback via, for example, discussion and interview. This will allow the students to develop areas of interest to them without a time constraint as in an examination and allow all students to experience a feeling of success.
 3. The final report should be approximately 12-20 pages long with double line spacing. It can be either word processed or handwritten. Students should be able to explain all stages of their work in such a way that demonstrates clear understanding. While there is no requirement that students present their work in class, it should be written in such a way that their peers would be able to follow it easily. The report should include a detailed bibliography, and sources need to be referenced in line with the IB academic honesty policy. Direct quotes must be acknowledged.
 4. The specific purposes of the exploration are to:
 - a. Develop students' personal insight into the nature of mathematics and to develop their ability to ask their own questions about mathematics

- b. Provide opportunities for students to complete a piece of mathematical work over an extended period of time
- c. Enable students to experience the satisfaction of applying mathematical processes independently
- d. Internal assessment 84 Mathematics: applications and interpretation guide
- e. Provide students with the opportunity to experience for themselves the beauty, power, and usefulness of mathematics
- f. Encourage students, where appropriate, to discover, use and appreciate the power of technology as a mathematical tool
- g. Enable students to develop the qualities of patience and persistence, and to reflect on the significance of their work
- h. Provide opportunities for students to show, with confidence, how they have developed mathematically.

B. Assessment Methods:

1. Assessment criteria are used when the assessment task is open-ended. Each criterion concentrates on a particular skill that students are expected to demonstrate. An assessment objective describes what students should be able to do, and assessment criteria describe how well they should be able to do it. Using assessment criteria allows discrimination between different answers and encourages a variety of responses. Each criterion comprises a set of hierarchically-ordered level descriptors. Each level descriptor is worth one or more marks. Each criterion is applied independently using a best-fit model. The maximum marks for each criterion may differ according to the criterion's importance. The marks awarded for each criterion are added together to give the total mark for the piece of work. For the purpose of the IB Exam, assessments will follow the following components: Homework (20%), Internal Assessment and Projects (20%), Formal Assessments (60%).

IV INSTRUCTIONAL MATERIALS AND METHODOLOGIES

A. Required Textbook(s):

Title: Mathematics Applications and Interpretations SL 2

ISBN: 9781925489576

Format: Print

Author(s): Haese, Humphries, et. al

Publisher: Haese Publications

Year: 2019 (2nd ed.)

Additional Info: N/A

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B. Supplemental Materials:

1. The use of technology is an integral part of DP mathematics courses. Developing an appreciation of how developments in technology and mathematics have influenced each other is one of the aims of the courses and using technology accurately, appropriately, and efficiently both to explore new ideas and to solve problems is one of the assessment objectives. Learning how to use different forms of technology is an important skill in mathematics and time

has been allowed in each topic of the syllabus and through the “toolkit” in order to do this.

2. Formulae are only included in this guide document where there may be some ambiguity. All formulae required for the course are in the mathematics formula booklet.
3. From the beginning of the course it is recommended that teachers ensure students are familiar with the contents of the formula booklet by either giving students a printed copy or making an electronic copy available to them.
4. Each student is required to have access to a clean copy of the formula booklet during the examination. For each examination, it is the responsibility of the school to download a copy of the formula booklet from IBIS or the program resource center, check that there are no printing errors, and ensure that there are sufficient copies available for all students.

C. Instructional Methodologies:

1. Approaches to teaching and learning across the Diploma Program refers to deliberate strategies, skills and attitudes which permeate the teaching and learning environment. These approaches and tools, intrinsically linked with the learner profile attributes, enhance student learning and assist student preparation for the Diploma Program assessment and beyond. The aims of approaches to teaching and learning in the Diploma Program are to:
 - a. Empower teachers as teachers of learners as well as teachers of content
 - b. Empower teachers to create clearer strategies for facilitating learning experiences in which students are more meaningfully engaged in structured inquiry and greater critical and creative thinking
 - c. Promote both the aims of individual subjects (making them more than course aspirations) and linking previously isolated knowledge (concurrency of learning)
 - d. Encourage students to develop an explicit variety of skills that will equip them to continue to be actively engaged in learning after they leave school, and to help them not only obtain university admission through better grades but also prepare for success during tertiary education and beyond
 - e. Enhance further the coherence and relevance of the students' Diploma Program experience
 - f. Allow schools to identify the distinctive nature of an IB Diploma Program education, with its blend of idealism and practicality.
2. The five approaches to learning (developing thinking skills, social skills, communication skills, self-management skills and research skills) along with the six approaches to teaching (teaching that is inquiry-based, conceptually-focused, contextualized, collaborative, differentiated and informed by assessment) encompass the key values and principles that underpin IB pedagogy.