Murrieta Valley Unified School District **Board Submission: June 2020**
HIGH SCHOOL COURSE OUTLINE

Course Title: Math Analysis and Approaches IB SL

Department: Mathematics

Course Number: 2359

Grade Level/s: 11-12

Length of course: One Year

Prerequisite/s: B or better in Advanced Math 3

UC/CSU (A-G) Req: Pending C

Brief Course Description: This course recognizes the increasing role that mathematics and technology play in a diverse range of fields in a data-rich world. As such, it emphasizes the meaning of mathematics in context by focusing on topics that are often used as applications or in mathematical modelling. To give this understanding a firm base, this course also includes topics that are traditionally part of a pre-university mathematics course such as calculus and statistics. Students who choose Mathematics: Analysis and Approaches at SL should be comfortable in the manipulation of algebraic expressions and enjoy the recognition of patterns and understand the mathematical generalization of these patterns. In addition, students will have strong algebraic skills and the ability to understand simple proof. They will be students who enjoy spending time with problems and get pleasure and satisfaction from solving challenging problems. Five main topics include: number and algebra, functions, geometry and trigonometry, probability and statistics, and calculus.

I GOALS

The goal of Math Analysis and Approaches IB SL is to enable students to:

A. Develop a curiosity and enjoyment of mathematics, and appreciate its elegance and power
B. Develop an understanding of the concepts, principles, and nature of mathematics
C. Communicate mathematics clearly, concisely, and confidently in a variety of contexts
D. Develop logical and creative thinking, and patience and persistence in problem solving to instill confidence in using mathematics
E. Employ and refine their powers of abstraction and generalization
F. Take action to apply and transfer skills to alternative situations, to other areas of knowledge and to future developments in their local and global communities
G. Appreciate how developments in technology and mathematics influence each other

H. Appreciate the moral, social and ethical questions arising from the work of mathematicians and the applications of mathematics
I. Appreciate the universality of mathematics and its multicultural, international, and historical perspectives
J. Appreciate the contribution of mathematics to other disciplines, and as a particular "area of knowledge" in the TOK course
K. Develop the ability to reflect critically upon their own work and the work of others
L. Independently and collaboratively extend their understanding of mathematics

II OUTLINE OF CONTENT OF MAJOR AREAS OF STUDY

Semester 1

A. Unit I: Number and Algebra (19 hours)
   1. Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables to solve mathematical problems. Concepts include:
      a. Modelling real-life situations with the structure of arithmetic and geometric sequences and series allows for prediction, analysis, and interpretation.
      b. Different representations of numbers enable equivalent quantities to be compared and used in calculations with ease to an appropriate degree of accuracy.
      c. Numbers and formulae can appear in different, but equivalent, forms, or representations, which can help us to establish identities.
      d. Formulae are a generalization made on the basis of specific examples, which can then be extended to new examples.
      e. Logarithm laws provide the means to find inverses of exponential functions which model real-life situations.
      f. Patterns in numbers inform the development of algebraic tools that can be applied to find unknowns.
      g. The binomial theorem is a generalization which provides an efficient method for expanding binomial expressions.
   2. The aim of the SL content of the number and algebra topic is to introduce students to numerical concepts and techniques which, combined with an introduction to arithmetic and geometric sequences and series, can be used for financial and other applications. Students will also be introduced to the formal concept of proof.
   3. Key Assignment Summary:
      An example of one key assignment is as follows:

Arithmetic sequences and series. Use of the formulae for the nth term and the sum of the first n terms of the sequence. Use of sigma notation for sums of arithmetic sequences.	Spreadsheets, GDCs and graphing software may be used to generate and display sequences in several ways. If technology is used in examinations, students will be expected to identify the first term and the common difference.
Applications.	Examples include simple interest over a number of years.

Analysis, interpretation, and prediction where a model is not perfectly arithmetic in real life.	Students will need to approximate common differences.

B. Unit II: Functions (21 hours)
   1. Models are depictions of real-life events using expressions, equations, or graphs while a function is defined as a relation or expression involving one or more variable. Creating different representations of functions to model the relationships between variables, visually and symbolically as graphs, equations and/or tables represents different ways to communicate mathematical ideas. Concepts include:
      a. Different representations of functions, symbolically and visually as graphs, equations and tables provide different ways to communicate mathematical relationships.
      b. The parameters in a function or equation correspond to geometrical features of a graph and can represent physical quantities in spatial dimensions.
      c. Moving between different forms to represent functions allows for deeper understanding and provides different approaches to problem solving.
      d. Our spatial frame of reference affects the visible part of a function and by changing this “window” can show more or less of the function to best suit our needs.
      e. Equivalent representations of quadratic functions can reveal different characteristics of the same relationship.
      f. Functions represent mappings that assign to each value of the independent variable (input) one and only one dependent variable (output).
   2. The aim of the SL content in the functions topic is to introduce students to the important unifying theme of a function in mathematics and to apply functional methods to a variety of mathematical situations. Throughout this topic students should be given the opportunity to use technology, such as graphing packages and graphing calculators to develop and apply their knowledge of functions, rather than using elaborate analytic techniques.
   3. Key Assignment Summary:
      An example of one key assignment is as follows:

Concept of a function, domain, range, and graph. Function notation, for example f(x), v(t), C(n). The concept of a function as a mathematical model.	Example: $f(x)=2-x$, the domain is $x\leq 2$, range is $f(x)\geq 0$. A graph is helpful in visualizing the range.
Informal concept that an inverse function reverses or undoes the effect of a function. Inverse function as a reflection in the line $y=x$, and the notation $f^{-1}(x)$.	Example: Solving $f(x)=10$ is equivalent to finding $f^{-1}(10)$. Students should be aware that inverse functions exist for one to one functions; the domain of $f^{-1}(x)$is equal to the range of f(x).

C. Unit III: Geometry and Trigonometry (25 hours)
   1. Geometry and trigonometry allow us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This branch provides us

with the tools for analysis, measurement and transformation of quantities, movements, and relationships. Concepts include:

a. The properties of shapes depend on the dimension they occupy in space.
b. Volume and surface area of shapes are determined by formulae, or general mathematical relationships or rules expressed using symbols or variables.
c. The relationships between the length of the sides and the size of the angles in a triangle can be used to solve many problems involving position, distance, angles, and area.
d. Equivalent measurement systems, such as degrees and radians, can be used for angles to facilitate ease of calculation.
e. Different representations of the values of trigonometric relationships, such as exact or approximate, may not be equivalent to one another.
f. The trigonometric functions of angles may be defined on the unit circle, which can visually and algebraically represent the periodic or symmetric nature of their values.

2. The aim of the SL content of the geometry and trigonometry topic is to introduce students to geometry in three dimensions and to non-right-angled trigonometry. Students will explore the circular functions and use properties and identities to solve problems in abstract and real-life contexts. Throughout this topic students should be given the opportunity to use technology such as graphing packages, graphing calculators, and dynamic geometry software to develop and apply their knowledge of geometry and trigonometry.
3. Key Assignment Summary:
   An example of one key assignment is as follows:

Use of sine, cosine, and tangent ratios to find the sides and angles of right-angled triangles.	In all areas of this topic, students should be encouraged to sketch well-labelled diagrams to support their solutions. Link to: inverse functions (SL2.2) when finding angles.
The sine rule: $a\sin A = b\sin B = c\sin C$. The cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$. Area of a triangle as $\frac{1}{2}ab\sin C$.	This section does not include the ambiguous case of the sine rule.

<u>Semester 2</u>

A. Unit I: Statistics and Probability (27 hours)
   1. Statistics is concerned with the collection, analysis and interpretation of quantitative data and uses the theory of probability to estimate parameters, discover empirical laws, test hypotheses, and predict the occurrence of events. Statistical representations and measures allow us to represent data in many different forms to aid interpretation.
   2. Probability enables us to quantify the likelihood of events occurring and so evaluate risk. Both statistics and probability provide important representations which enable us to make predictions, valid comparisons, and informed decisions. These fields have power and limitations and should be applied with care and critically questioned, in detail, to differentiate between the theoretical

and the empirical/observed. Probability theory allows us to make informed choices, to evaluate risk and to make predictions about seemingly random events. Concepts include:

a. Organizing, representing, analyzing, and interpreting data and utilizing different statistical tools facilitates prediction and drawing of conclusions.
b. Different statistical techniques require justification and the identification of their limitations and validity.
c. Approximation in data can approach the truth but may not always achieve it.
d. Some techniques of statistical analysis, such as regression, standardization, or formulae, can be applied in a practical context to apply to general cases.
e. Modelling through statistics can be reliable but may have limitations.

3. The aim of the SL content in the statistics and probability topic is to introduce students to the important concepts, techniques and representations used in statistics and probability. Students should be given the opportunity to approach this topic in a practical way, to understand why certain techniques are used and to interpret the results. The use of technology such as simulations, spreadsheets, statistics software, and statistics apps can greatly enhance this topic. It is expected that most of the calculations required will be carried out using technology, but explanations of calculations by hand may enhance understanding. The emphasis is on understanding and interpreting the results obtained, in context.
4. Key Assignment Summary:
   An example of one key assignment is as follows:

Linear correlation of bivariate data. Pearson's product-moment correlation coefficient, r.	Technology should be used to calculate r. However, hand calculations of r may enhance understanding. Critical values of r will be given where appropriate. Students should be aware that Pearson's product moment correlation coefficient (r) is only meaningful for linear relationships.
Scatter diagrams; lines of best fit, by eye, passing through the mean point.	Positive, zero, negative; strong, weak, no correlation. Students should be able to make the distinction between correlation and causation and know that correlation does not imply causation.
Equation of the regression line of y on x.	Technology should be used to find the equation.

B. Unit II: Calculus (30 hours)
   1. Calculus describes rates of change between two variables and the accumulation of limiting areas. Understanding these rates of change allows us to model, interpret and analyze real-world problems and situations. Calculus helps us understand the behavior of functions and allows us to interpret the features of their graphs. Concepts include:
      a. The derivative may be represented physically as a rate of change and geometrically as the gradient or slope function.

b. Areas under curves can be approximated by the sum of the areas of rectangles which may be calculated even more accurately using integration.
c. Examining rates of change close to turning points helps to identify intervals where the function increases/decreases and identify the concavity of the function.
d. Numerical integration can be used to approximate areas in the physical world.
e. Mathematical modelling can provide effective solutions to real-life problems in optimization by maximizing or minimizing a quantity, such as cost or profit.
f. Derivatives and integrals describe real-world kinematics problems in two and three-dimensional space by examining displacement, velocity and acceleration.

2. The aim of the SL content in the calculus topic is to introduce students to the concepts and techniques of differential and integral calculus and their applications. Throughout this topic students should be given the opportunity to use technology such as graphing packages and graphing calculators to develop and apply their knowledge of calculus.
3. Key Assignment Summary:
   An example of one key assignment is as follows:

Introduction to integration as anti-differentiation of functions of the form $f(x)=axn+bxn-1+....$, where $n\in\mathbb{Z}$, $n\neq-1$	Students should be aware of the link between anti-derivatives, definite integrals, and area.
Anti-differentiation with a boundary condition to determine the constant term.	Example: If $dydx=3x2+x$ and $y=10$ when $x=1$, then $y=x3+12x2+8.5$.
Definite integrals using technology. Area of a region enclosed by a curve $y=f(x)$ and the x -axis, where $f(x)>0$.	Students are expected to first write a correct expression before calculating the area, for example $\int26(3x2+4)dx$. The use of dynamic geometry or graphing software is encouraged in the development of this concept.

III ACCOUNTABILITY DETERMINANTS

A. Internal Assessment (15 hours):
   1. In addition to the five core topics, students will be required to produce an internal assessment project that incorporates the topics covered in this course. This will happen during the second semester and require approximately 15 hours.
   2. The internally-assessed component in this course is a mathematical exploration. This is a short report written by the student based on a topic chosen by him or her, and it should focus on the mathematics of that particular area. The emphasis is on mathematical communication (including formulae, diagrams, graphs, tables and so on), with his or her own focus, with the teacher providing feedback via, for example, discussion and interview. This will allow the students to develop areas of interest to them without a time constraint as in an examination and allow all students to experience a feeling of success.

3. The final report should be approximately 12-20 pages long with double line spacing. It can be either word processed or handwritten. Students should be able to explain all stages of their work in such a way that demonstrates clear understanding. While there is no requirement that students present their work in class, it should be written in such a way that their peers would be able to follow it easily. The report should include a detailed bibliography, and sources need to be referenced in line with the IB academic honesty policy. Direct quotes must be acknowledged.
4. The specific purposes of the exploration are to:
    a. develop students' personal insight into the nature of mathematics and to develop their ability to ask their own questions about mathematics
    b. provide opportunities for students to complete a piece of mathematical work over an extended period of time
    c. enable students to experience the satisfaction of applying mathematical processes independently
    d. Internal assessment 84 Mathematics: Analysis and Approaches guide
    e. provide students with the opportunity to experience for themselves the beauty, power, and usefulness of mathematics
    f. encourage students, where appropriate, to discover, use and appreciate the power of technology as a mathematical tool
    g. enable students to develop the qualities of patience and persistence, and to reflect on the significance of their work
    h. provide opportunities for students to show, with confidence, how they have developed mathematically.

B. Assessment Methods:
    1. Assessment criteria are used when the assessment task is open-ended. Each criterion concentrates on a particular skill that students are expected to demonstrate. An assessment objective describes what students should be able to do, and assessment criteria describe how well they should be able to do it. Using assessment criteria allows discrimination between different answers and encourages a variety of responses. Each criterion comprises a set of hierarchically-ordered level descriptors. Each level descriptor is worth one or more marks. Each criterion is applied independently using a best-fit model. The maximum marks for each criterion may differ according to the criterion's importance. The marks awarded for each criterion are added together to give the total mark for the piece of work. For the purpose of the IB Exam, assessments will follow the following components: Homework (20%), Internal Assessment and Projects (20%), Formal Assessments (60%).

IV INSTRUCTIONAL MATERIALS AND METHODOLOGIES

A. Required Textbook(s):

Title: Mathematics Analysis and Approaches SL 2019
ISBN: 9781925489569
Format: Print
Author(s): Haese, Humphries, et. al
Publisher: Haese Publications
Year: 2019 (2nd ed.)

Additional Info: N/A

B. Supplemental Materials:
   1. The use of technology is an integral part of DP mathematics courses. Developing an appreciation of how developments in technology and mathematics have influenced each other is one of the aims of the courses and using technology accurately, appropriately, and efficiently both to explore new ideas and to solve problems is one of the assessment objectives. Learning how to use different forms of technology is an important skill in mathematics and time has been allowed in each topic of the syllabus and through the "toolkit" in order to do this.
   2. Formulae are only included in this guide document where there may be some ambiguity. All formulae required for the course are in the mathematics formula booklet.
   3. From the beginning of the course it is recommended that teachers ensure students are familiar with the contents of the formula booklet by either giving students a printed copy or making an electronic copy available to them.
   4. Each student is required to have access to a clean copy of the formula booklet during the examination. For each examination, it is the responsibility of the school to download a copy of the formula booklet from IBIS or the program resource center, check that there are no printing errors, and ensure that there are sufficient copies available for all students.

C. Instructional Methodologies:
   1. Approaches to teaching and learning across the Diploma Program refers to deliberate strategies, skills and attitudes which permeate the teaching and learning environment. These approaches and tools, intrinsically linked with the learner profile attributes, enhance student learning and assist student preparation for the Diploma Program assessment and beyond. The aims of approaches to teaching and learning in the Diploma Program are to:
      a. empower teachers as teachers of learners as well as teachers of content
      b. empower teachers to create clearer strategies for facilitating learning experiences in which students are more meaningfully engaged in structured inquiry and greater critical and creative thinking
      c. promote both the aims of individual subjects (making them more than course aspirations) and linking previously isolated knowledge (concurrency of learning)
      d. encourage students to develop an explicit variety of skills that will equip them to continue to be actively engaged in learning after they leave school, and to help them not only obtain university admission through better grades but also prepare for success during tertiary education and beyond
      e. enhance further the coherence and relevance of the students' Diploma Program experience
      f. allow schools to identify the distinctive nature of an IB Diploma Program education, with its blend of idealism and practicality.
   2. The five approaches to learning (developing thinking skills, social skills, communication skills, self-management skills and research skills) along with the six approaches to teaching (teaching that is inquiry-based, conceptually-focused, contextualized, collaborative, differentiated and informed by assessment) encompass the key values and principles that underpin IB pedagogy.