Course Title:	Mathematical Reasoning with Connections (MRWC- Math IV)
Department:	Mathematics
Course Number:	2355
Grade Level/s:	12
Length of Course:	Year
Prerequisite/s:	By placement (C or better in Math III, teacher recommendation and state/district assessment results)
UC/CSU (A-G) Req:	(C) Mathematics

Brief Course Description: Mathematical Reasoning with Connections (MRWC) is designed as a 4th year mathematics course following Math I–III that will provide a bridge into multiple college and career options, including STEAM, CTE, and non-technical pathways. Students successfully completing MRWC will have acquired content skills and attitudes towards learning that will be expected in entry-level college mathematics, including pre-calculus, calculus, and other quantitative reasoning courses. MRWC curriculum includes standards listed in the Pre-Calculus chapter of the Mathematics Framework and combines concepts of trigonometry, geometry, and algebra that lead to the study of Calculus.

The MRWC curriculum has been developed by a consortium of mathematics professors and math educators from CSU, UC, and CCC higher education systems, together with mathematics specialists from County Offices of Education and local school districts. Curriculum has been specifically designed to address the need for stronger mathematics preparation for transitioning from high school to college and career pathways.

MRWC is structured to highlight conceptual connections in the more advanced study of topics leading to Calculus. Emphasis is given to conceptual understanding and making connections between numerical, symbolic, verbal, and graphical representations, discussion and analysis of alternative representations and multiple perspectives for approaching and understanding. The distinctiveness of MRWC lies in its unique design and topic sequencing, and in the emphasis on instructional delivery that promotes exploratory and collaborative student engagement. MRWC seamlessly interweaves the Mathematical Practice Standards throughout the curriculum and develops key Habits of Mind and a mathematical disposition required for mastering advanced, challenging college-level content knowledge.

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I   GOALS

The students will:
A. Extend their work with real and complex numbers

B. Represent complex numbers in the Cartesian plane and interpret operations on complex numbers as geometric transformations

C. Represent complex numbers in polar and trigonometric form

D. Prove trigonometric identities for compound angles to find powers and roots of complex numbers

E. Analyze commonalities across families of functions that include algebraic functions such as absolute value, root, polynomial, rational, and reciprocal, as well as transcendental functions such as exponential, logarithmic, and trigonometric
F. Extend basic trigonometric functions to reciprocal and inverse trigonometric functions

G. Determine basic ellipses and hyperbolas and use translated and rotated axes to graph all conics in non-standard positions

H. Graph advanced rational and piece-wise functions

I. Identify zeros, multiple roots, intercepts, symmetries, vertical/horizontal, and slant asymptotes, holes, and end-behavior of functions

J. Graph using parametric equations

K. Develop fluency and flexibility with both the algebraic and geometric meaning and interpretation of functional notation

L. Identify and determine formulas of functions given in tables, graphs, and real-world situations

M. Use underlying structure and the technique of u-substitution to simplify and solve advanced expressions, equations, and inequalities involving algebraic and trigonometric terms

N. Extend formulae to find distances in 3-dimensional space using algebraic and vector techniques, building on knowledge of distance as an application of the Pythagorean Theorem

O. Solve absolute value equations and inequalities by identify centers of intervals

P. Determine loci of curves and relate them to distances from foci in parabolas, ellipses, and hyperbolas

Q. Evaluate the effect of eccentricity as it relates to distance and the shapes of conics

R. Compare similarities among conics by identifying the number of parameters involved in describing the conic

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    S. Connect regression to distance of the residuals

    T. Analyze rate of change with a secant line, and investigate the derivative as the limiting case of the slope of a secant

II   OUTLINE OF CONTENT FOR MAJOR AREAS OF STUDY

<u>Semester 1</u>

A. Reasoning with Numbers
1. Deepen conceptual understanding of the relationships between and the structures of various subsets of numbers that make up the Complex Number System. (N.CN.3)
2. Find conjugates of complex numbers and use them to divide complex numbers. (N.CN.3)
3. Find moduli of complex numbers and relate them to distance and absolute value. (N.CN.6)
4. Represent complex numbers in the Cartesian (Argand) plane and describe the algebraic operations on complex numbers in terms of geometric transformations of translation, dilation, and rotation. (N.CN.5)
5. Connect complex numbers to vectors in the plane, and relate modulus of a complex number to length of a vector. (N.CN.6, N.VM.1)
6. Relate scalar and vector multiplication to multiplication of complex numbers. (N.VM.4-5)
7. Represent complex numbers in polar and trigonometric forms and prove de Moivre's formula for multiplying and finding rational roots of complex numbers in trigonometric form. (N.VM.4-5)
8. Explore symmetries in the multiple roots of complex numbers and use the symmetries to explore infinite geometric sequences of complex numbers. (N.VM.3-5)
9. Prove that various subsets of the real and complex numbers are closed under different operations (including division, powers, and rooting). (N.VM.5-6)
10. Identify irrational numbers as limiting values of infinite sequences, including nested radicals and continued fractions. (N.VM.5-6)
11. Prove the existence and magnitude of numbers of irrational and complex numbers through geometric construction and algebraic proof. (N.VM.5-6)

B. Reasoning with Functions
1. Link patterns of real numbers to discrete functions, including arithmetic and geometric sequences and series. (F.IF.4-5)
2. Identify anomalies in the domain of continuous functions, including vertical asymptotes and removable points of discontinuity. (F.IF.4, F.IF.7d)
3. Use numeric limits and algebraic procedures to identify whether a number that is excluded from the domain is a removable point of discontinuity or a vertical asymptote. (F.IF.4-5, F.IF.7d)
4. Use numeric limits to explore function behavior on either side of a vertical asymptote. (F.IF.4-5, F.IF.7d)
5. Identify key concepts related to functions including advanced study of domain and range, roots, symmetries and periodicity, positive/negative and increasing/decreasing. (F.IF.4-5)

6. Use algebraic factoring to predict function behavior based on multiplicity of roots and to find intervals on which functions are increasing/decreasing and positive/negative/constant. (F.IF.4, F.IF.7d)
7. Create functions given information about function features and behaviors. (F.IF.4, F.IF.7d, F.BF.3)
8. Create functions in two or more variables that represent relationships between quantities expressed in verbal, numeric or graphical form. (F.IF.4-5, F.BF.3)
9. Use numeric limits and algebraic procedures to identify and describe the end behavior of a function, including limits at infinity, horizontal and slant asymptotes. (F.IF.4-5)
10. Study the graphs and features of reciprocal and inverse functions. (F.IF.7e, F.BF.3, F.BF.4b-d)
11. Relate features of reciprocal and inverse functions to understand trigonometric functions of cosecant, secant, cotangent and the inverse trigonometric functions. (F.BF.4b-d)
12. Use completing the square techniques to graph ellipses and hyperbolas in standard and non-standard positions. (G.GPE.3.1, CA)
13. Use trigonometric techniques to draw rotated conics. (G.GPE.3)
14. Study parametric forms of equations and relate them to transformations. (F.BF.10,CA)
15. Make connections between geometrical transformations (such as translation, rotation, reflections, dilations and stretches of graphs) and the algebraic process of function composition. (F.BF.3-4)
16. Expand composition to include composition of three or more functions. (F.BF.4)
17. Create new function graphs by composing functions given in graphical or tabular representations. (F.IF.4, F.IF. 7, F.BF.3-4)
18. Decompose complicated functions into component functions, both graphically and algebraically. (F.IF.4, F.IF. 7, F.BF.3-4)
19. Study basic properties of matrices and vectors. (N.VM.4-8)
20. Use vectors and matrices as a means to represent function transformations. (N.VM.9-12)
21. Use parametric equations to graph advanced functions. (F.IF.10, CA)
22. Interpret function notation and function composition graphically, verbally, numerically and algebraically. (A.SSE.1-2, F.IF.4, F.IF.7)
23. Use function notation to prove features of functions such as odd/even, increasing/decreasing, the existence of symmetry lines in parabolas and other conics. (F.BF.3, F.IF.10, CA)

Semester 2

A. Reasoning with Equivalences
   1. Prove/disprove identities among equivalent and non-equivalent expressions involving polynomial, rational, root, exponential, and logarithmic terms. (A.SSE.1-2)
   2. Prove/disprove trigonometric identities involving co-functions, compound/double/half angle formula. (F.TF.6-9, F.TF.10, CA)
   3. Use similarity of triangles to develop sine and cosine and area formula for solving non-right triangles. (G.SRT.9-11)
   4. Develop fluency and flexibility in manipulating complicated forms of composite expressions and equations (including advanced factoring) by identifying and strategically using the idea of underlying structure. (A.SSE.1-2)

5. Use advanced factoring techniques on expressions and equations involving binomials with rational exponents and terms with logarithmic and trigonometric exponents. (A.SSE.1-2)
6. Solve advanced composite equations and inequalities involving polynomial, rational, root, absolute value, trigonometric, exponential and logarithmic expressions by identifying and strategically using underlying structure and alternative representations. (A.SSE.1-2, A.APR.6-7)
7. Identify and solve trigonometric equations and inequalities that have underlying structure of polynomial form using algebraic and graphical techniques. (F.TF.6-7, A.CED.1-3)
8. Represent and solve systems of linear equations using matrices and a vector variable and use matrix inversion to solve. (A.CED.2-3, A.REI.8-9)
9. Solve advanced systems of non-linear equations and inequalities involving roots, absolute value, exponentials, logarithms and trigonometric terms. (A.CED.2-3)
10. Create equations in two or more variables to represent relationships between quantities. (A.CED.1-2)
11. Explore the geometry of polygons, curves, perimeter and area through equivalences such as similarity and congruence and transformations that preserve perimeter and/or area. (N.VM.11-12)
12. Study parallelism as an equivalence relation through two- and three-dimensional vectors. (N.VM.11-12)

B. Reasoning with Distances
1. Extend the concept of distance as absolute value to find distances and midpoints between points in 3-dimensional space. (N.CN.6, N.VM.12)
2. Use the concept of loci to explore conics and other curves in algebraic and polar form. (G.GPE.3)
3. Use real world data sets to connect the least square method of linear regression to the measurement of residuals as distances. (S.ID.6)
4. Extend the concept of distance to study slope, rate of change and secant lines. (S.ID.7, F.IF.6, CA Math Framework: Calculus 4.1-4.2)
5. Explore the slope of a tangent line to a curve as the limiting case of the slope of a secant, and develop the concept of a derivative as a point. (S.ID.7, CA Math Framework: Calculus 4.1-4.2)

II   ACCOUNTABILITY DETERMINANTS

A. Key Assignments
As listed in the MRWC curriculum materials

B. Assessment Methods
Different forms of formative and summative assessments will be used. Students will demonstrate their ongoing conceptual understanding and procedural fluency through mathematical activities, small group discussions and explorations, personal reflection quick writes, in addition to worksheets and individual written assessments such as quizzes, tests, final summative exams. Students will also be assessed through group projects, oral and written presentations.

III   INSTRUCTIONAL MATERIALS AND METHODOLOGIES

A. Required Textbook(s)
Title: Mathematical Reasoning with Connections
ISBN: N/A
Format: Print
Author(s):
Publisher: California State University
Year: 2018
Additional Info:

B. Supplementary Materials
Title: Precalculus with Limits: A Graphing Approach (6th ed.)
ISBN: 9781111427641
Format: Print
Author(s): Ron Larson
Publisher: Brooks and Cole Publishing
Year: 2011
Additional Info: Supplemental textbook

C. Instructional Methodologies
MRWC uses a non-traditional instructional approach emphasizing collaboration and exploration through mathematical activities, problem posing, and the use of technology that will address diverse learning styles. Instruction is designed to challenge students to approach mathematics as sense-making through a focus on questioning and probing deeper.   Teacher-led instruction and student explorations will focus on discovering the conceptual basis for standard procedures. It will facilitate the development of students' ability to choose strategically among multiple solutions options, and to articulate the reasons for those decisions.   Students will use informal and formal justifications to defend their understandings and critique the reasoning of others. Instruction will emphasize the use of and fluency in the full range of the language of mathematics. Content topics will be approached through six instructional modalities i.e. verbal, numeric, symbolic, graphical, geometric and technological.