

<b>Course Title:</b>	Math III
<b>Department:</b>	Mathematics
<b>Course Number:</b>	2218
<b>Grade Level(s):</b>	10-11
<b>Length of Course:</b>	Year
<b>Prerequisite(s):</b>	By placement (C or better in Math II, teacher recommendation and/or district assessment results)
<b>UC/CSU (A-G) Req:</b>	(C) Mathematics

**Brief Course Description:** The purpose of Math III is to develop students' ability to think mathematically and develop their conceptual understanding of mathematics and procedural fluency in mathematics. Math III will extend the mathematics students learned in earlier grades and apply concepts in Number and Quantity, Algebra, Functions, Modeling, Geometry, Probability and Statistics. The critical topics of this course are: Inverse Functions, Logarithmic, Polynomial, Rational and Radical Functions, Modeling with Geometry and Functions, Trigonometric Functions, Statistics. Extensive use of models (or real-world situations), manipulatives, graphs and diagrams will help students view how mathematics is a set of related topics as opposed to a set of discrete topics. In addition, students will solve problems graphically, numerically, algebraically, and verbally and make connections between these representations. Students routinely use the standards for mathematical practice to make sense of problems, justify solutions and conclusions, model with mathematics, and strategically use technology to analyze and solve real-world problems.

## I. GOALS

The students will:

- A. Use complex numbers in polynomial identities and equations
- B. Interpret the structure of expressions
- C. Write expressions in equivalent forms to solve problems
- D. Perform arithmetic operations on polynomials
- E. Understand the relationship between zeros and factors of polynomials

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- F. Use polynomial identities to solve problems
- G. Rewrite rational expressions
- H. Create equations that describe numbers or relationships
- I. Understand solving equations as a process of reasoning and explain the reasoning
- J. Represent and solve equations and inequalities graphically
- K. Interpret functions that arise in applications in terms of the context
- L. Analyze functions using different representations
- M. Build a function that models a relationship between two quantities
- N. Build new functions from existing functions
- O. Construct and compare linear, quadratic, and exponential models and solve problems
- P. Extend the domain of trigonometric functions using the unit circle
- Q. Model periodic phenomena with trigonometric functions
- R. Apply trigonometry to general triangles
- S. Translate between the geometric description and the equation for a conic section
- T. Visualize relationships between two-dimensional and three-dimensional objects
- U. Apply geometric concepts in modeling situations
- V. Summarize, represent, and interpret data on a single count or measurement variable
- W. Understand and evaluate random processes underlying statistical experiments
- X. Make inferences and justify conclusions from sample surveys, experiments, and observations
- Y. Use probability to evaluate outcomes of decisions

These goals are aligned with the California State Standards, including the Standards of Mathematical Practices.

**II. OUTLINE OF CONTENT FOR MAJOR AREAS OF STUDY**

**Semester 1**

- A. Functions and their Inverses
  - 1. Develop the concept of inverse functions in a linear modeling context using tables, graphs, and equations. (F.BF.1, F.BF.4, F.BF.4a)

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2. Extend the concepts of inverse functions in a quadratic modeling context with a focus on domain and range and whether a function is invertible in a given domain. (F.BF.1, F.BF.4, F.BF.4c, F.BF.4d)
  3. Solidify the concepts of inverse function in an exponential modeling context and surfaces ideas about logarithms. (F.BF.1, F.BF.4, F.BF.4c, F.BF.4d)
  4. Focus on finding inverse functions and verifying that two functions are inverses. (F.BF.4, F.BF.4a, F.BF.4b)
  5. Use tables, graphs, equations, and written descriptions of functions to match functions and their inverses together and to verify the inverse relationship between two functions. (F.BF.4a, F.BF.4b, F.BF.4c, F.BF.4d)
- B. Logarithmic Functions
1. Evaluate and compare logarithmic expressions. (F.BF.5, F.LE.4)
  2. Graph logarithmic functions with transformations (F.BF.5)
  3. Develops understanding of log properties (F.IF.8, F.LE.4)
  4. Use log properties to evaluate expressions (F.IF.8, F.LE.4)
  5. Solve exponential and logarithmic equations in base ten using technology (F.LE.4)
- C. Polynomial Functions
1. Compare growth rates of linear, quadratic, and cubic functions and recognize that cubic functions can be created from the sums of a quadratic function (F.BF.1, F.LE.3)
  2. Determine the slowest to the fastest growing functions by ordering and comparing values as  $x$  approaches infinity (F.LE.3, A.SSE.1, F.IF.4)
  3. Analyze and compare end behavior of functions in different representations (F.IF.6, F.IF.7, F.IF.9)
  4. Use graphical representations to add, subtract, and multiply polynomials (A.APR.1, F.BF.1)
  5. Determine the nature of roots and applying the Fundamental Theorem of Algebra (A.APR.3, A.APR.5, N.CN.9)
  6. Expand binomial expressions using Pascal's Triangle (A.APR.3, A.APR.5, N.CN.9)
  7. Applying the fundamental Theorem of Algebra (N.CN.8, N.CN.9, A.APR.3, F.IF.4, F.IF.7)
  8. Use the Remainder Theorem to find all linear factors and roots of a polynomial function (N.CN.9, A.APR.2, A.APR.3, F.IF.7)
  9. Graph and solve polynomial functions (N.CN.9, A.APR.2, A.APR.3, F.IF.7)
- D. Rational Expressions and Functions
1. Use context to identify inverse variation and introduce rational functions (F.IF.7d, A.CED.2, F.IF.5)
  2. Analyze the characteristics of various families of functions to assist in identifying characteristics of rational functions (A.SSE.1, F.IF.4, F.IF.8, F.IF.7d, F.BF.3)
  3. Connect rational expressions to rational numbers (A.APR.7)
  4. Connect rational numbers by using improper fractions to rewrite improper rational (A.APR.6)
  5. Identify the end behavior of rational functions. (F.IF.4, F.IF.7d)
  6. Graph rational functions using its features (F.IF.4, F.IF.7d)
  7. Graph and solve rational functions (F.IF.4, F.IF.7d, A.REI.2, A.REI.11)

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E. Modeling with Geometry

1. Visualize two-dimensional cross sections of three-dimensional objects (G.GMD.4)
2. Visualize solids of revolution (G.GMD.4)
3. Estimate volumes of solids of revolution with cylinders and frustums (G.MG.1, G.GMD.4).
4. Solve problems using geometric modeling (G.MG.1, G.MG.2, G.MG.3)
5. Examine the relationship of sides in special right triangles (G.SRT.11)
6. Develop strategies for solving non-right triangles (G.SRT.11)
7. Examine the Law of Cosines and the Law of Sines (G.SRT.10, G.SRT.11)
8. Find the missing sides, angles and area of general triangles (F.IF.8, F.IE.4)

F. Trigonometric Functions

1. Use reference triangles, right triangle trigonometry and the symmetry of a circle to find the  $y$ -coordinates of points on a circular path (F.TF.5)
2. Use reference triangles, right triangle trigonometry, angular speed and the symmetry of a circle to find the  $y$ -coordinates of points on a circular path at given instances in time an— introduction to circular trigonometric functions (F.TF.5)
3. Extend the definition of sine from a right triangle trigonometry ratio to a function of an angle of rotation (F.TF.2)
4. Graph a sine function to model circular motion and relating features of the graph to the parameters of the function (F.TF.5, F.IF.4, F.BF.3)
5. Extend the definition of cosine from a right triangle trigonometry ratio to a function of an angle of rotation (F.TF.5, F.TF.2)
6. Introduce radians a unit for measuring angles on concentric circles (F.TF.1, F.TF.2)
7. Apply the proportionality relationship of radian measure to locate points on concentric circles (F.TF.1, F.TF.2)
8. Redefine radian measure of an angle as the length of the intercepted arc on a unit circle (F.TF.1, F.TF.2)
9. Define sine and cosine on the unit circle in terms of angles of rotation measured in radians (F.TF.1, F.TF.2)
10. Introduce the horizontal shift of a trigonometric function in terms of modeling context (F.TF.5, F.BF.3)
11. Apply trigonometric graphs and inverse trigonometric functions to model periodic behavior (F.TF.5, F.BF.4)
12. Model periodic behavior using transformations of trigonometric graphs and inverse trigonometric functions to (F.TF.5, F.BF.3, F.BF.4)
13. Extend the definition of tangent from a right triangle trigonometry ratio to a function of an angle of rotation, including angles of rotation measured in radians on the unit circle (F.TF.2, F.TF.3, F.TF.4)
14. Classify sine, cosine, and tangent functions as even or odd (F.TF.2, F.TF.3, F.TF.4)

G. Modeling with Functions

1. Examining the transformations of a variety of familiar functions using tables (F.BF.3, G.CO.2)
2. Predicting the shape of a graph that is the sum or product of familiar functions (F.BF.1b)
3. Combining a variety of functions using arithmetic operations to model complex behavior (F.BF.1b)
4. Combining a variety of functions using function composition to model complex behavior (F.BF.1c)

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5. Examining function transformations by composing and decomposing functions (F.BF.1c, F.BF.3)
6. Combining functions defined by tables, graphs or equations using function composition and/or arithmetic operations (F.BF.1b, F.BF.1c)

H. Statistics

1. Understand normal distributions and identify their features (S.ID.4)
2. Use the features of a normal distribution to make decisions (S.ID.4)
3. Compare normal distributions using z scores (S.ID.4)
4. Compare normal distributions using z scores and understanding of mean and standard deviation (S.ID.4)
5. Understand and identify different methods of sampling (S.IC.1)
6. Uses tables, graphs, equations, and written descriptions of functions to match functions and their inverses together and to verify the inverse relationship between two functions (S.IC.2)
7. Identify the difference between survey, observational studies, and experiments (S.IC.1, S.IC.2)
8. Use simulation to estimate the likelihood of an event (S.IC.2, S.IC.3)

**III. ACCOUNTABILITY DETERMINANTS**

A. Key Assignments

1. In the task “Tracking the Tortoise” from Functions and Their Inverses unit, students will describe the exponential graph, graph the inverse function, state the domain and range of both the function and the inverse function, and compare the exponential form of an equation to the logarithmic form of the equation.
2. In the task “Log Logic” from Logarithmic Functions unit, students develop understanding of logarithmic expressions and to make sense of the notation. In addition to evaluating log expressions, student will compare expressions that they cannot evaluate explicitly. They will also use patterns that they have seen in the task and the definition of a logarithm to justify some properties of logarithms.
3. In the task “Seeing Structure” from Polynomial Functions unit, students identify the end behavior and roots of the function. Students will graph a polynomial in factored or standard form. Students will find the other two roots of a cubic function given one of the roots of the function.
4. In the task “Features of Rational Functions” from Rational Functions unit, students determine the features of rational functions such as intercepts, domain, asymptotes, and end behavior to sketch the graph.
5. In the task “Take another Spin” from Modeling with Geometry unit, students sketch a three-dimensional object by rotating about the y-axis and find the volume of the object. Students approximate the volume of an object with curve edges. In the task “Triangle Areas by Trig” from the same unit students use right triangle trigonometry, Pythagorean Theorem and/or law of sines and cosines to find needed information to calculate the area of triangles.
6. In the task “High Noon and Sunset Shadows” from Trigonometric Functions unit, students develop strategies for transforming vertical translations, horizontal and vertical dilations of the graph by changing the radius of the Ferris wheel or the speed of rotation. Students will vary the amplitude or the period of the graph. Students may also note that sine and cosine graphs are interchangeable if shifted horizontally by an appropriate amount.

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7. In the task “Translating My Composition” from Modeling with Functions unit, students understand composition of functions by considering the roles of the individual functions of which each composition function is composed. Students will observe that function composition is not commutative. They will also notice that functions created by composition can often be interpreted as transformations of standard (or parent) functions.
8. In the task “Slacker’s Simulation” from Statistics unit, students decide if a particular result from a model is likely using a simulation. Students simulate the results of a true-false test using a coin flip. The results are collected for the entire class and discussed with the idea that the particular result is possible, but unlikely.

**B. Assessment Methods**

1. Daily Student Observation of Classroom Participation, Effort and Achievement
2. Classwork/Homework
3. Performance Tasks
4. Projects
5. Quizzes
6. End of Unit Tests
7. Semester Final Exams
8. District-wide Benchmark Exams

**IV. INSTRUCTIONAL MATERIALS AND METHODOLOGIES**

**A. Required Textbook(s)**

1. Bellman, Allan, et al. California Algebra 2. Boston: Pearson Prentice Hall, 2009

**B. Supplementary Materials**

1. Hendrickson, Honey, et al. Secondary Mathematics Three: An Integrated Approach, Mathematics Vision Project, 2013. This is an e-book located at <http://www.mathematicsvisionproject.org>
2. Holt McDougal, Explorations in CORE Math for Common Core Algebra 2, Houghton Mifflin Harcourt Publishing Company, 2010
3. [www.mathalicious.com](http://www.mathalicious.com)
4. [www.geogebra.org](http://www.geogebra.org)
5. [www.pearsonsuccessnet.com](http://www.pearsonsuccessnet.com)

**C. Instructional Methodologies**

1. Guided Inquiry
2. Direct Instruction
3. Cooperative Learning
4. Discourse
5. Problem-Based Learning
6. Visual Representations and Concrete Models