



# SMARTER Balanced Assessment

## 8th Grade Mathematics

### Sample Items



#### Item Types

SBAC assessments are made up of four item types: Selected-Response, Constructed-Response, Technology-Enhanced, and Performance Task. A description of those items follows.

##### **Selected-Response Items (SR)**

Traditionally known as multiple choice, selected-response items include a stimulus and stem followed by three to five options from which a student is directed to choose only one.

##### **Constructed-Response Items (CR)**

The main purpose of a constructed-response item is to address targets and claims that are of greater complexity. They ask students to develop answers without suggested answer choices.

##### **Technology-enhanced Items/Tasks (TE)**

Technology-enhanced items can provide evidence for mathematics practices that could not be as reliably obtained from traditional SRs and CRs. Technology-enhanced items may stand alone or may be a tool used as part of the Performance Task and/or Constructed-Response items.

##### **Performance Tasks (PT)**

Performance tasks, the most complex of all items, include the following elements:

- Integrate knowledge and skills across multiple claims.
- Measure capacities such as depth of understanding, research skills, and/or complex analysis with relevant evidence.
- Require student-initiated planning, management of information/data and ideas, and/or interaction with other materials.
- Reflect a real-world task and/or scenario-based problem..
- Allow for multiple approaches.
- Represent content that is relevant and meaningful to students.
- Allow for demonstration of important knowledge and skills.
- Require scoring that focuses on the essence of the Claim(s) for which the task was written.
- Seem feasible for the school/classroom environment.

## Claims

The Smarter Balanced summative assessments in mathematics are designed to measure the full range of student abilities in the Common Core State Standards or Core Academic Standards (CAS). Evidence will be gathered in support of four major claims: (1) Concepts and Procedures, (2) Problem Solving, (3) Communicating Reasoning, and (4) Modeling and Data Analysis. Students will receive an overall mathematics composite score. For the enhanced assessment, students will receive a score for each of three major claim areas. (Math claims 2 and 4 are combined for the purposes of score reporting.)

**Claim 1** — Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

**Claim 2** — Students can solve a range of complex, well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

**Claim 3** — Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

**Claim 4** — Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

## Glossary

**Distracter:** the incorrect response options to an SR item.

**Distracter Analysis:** the item writer's analysis of the options or rationale for inclusion of specific options.

**Item:** the entire item, including the stimulus, question/prompt, answer/options, scoring criteria, and metadata.

**Key:** the correct response(s) to an item.

**Options:** the responses to a selected-response (SR) item from which the student selects one or more answers.

**Scoring Rubric:** the descriptions for each score point for an item/task that scores more than one point for a correct response.

**Stem:** the statement of the question or prompt to which the student responds.

**Stimulus:** the text, source (e.g., video clip), and/or graphic about which the item is written. The stimulus provides the context of the item/task to which the student must respond.

**Task:** similar to an item, yet typically more involved and usually associated with constructed response, extended-response, and performance tasks.

**Top-Score Response:** one example of a complete and correct response to an item/task.

## MAT.08.CR.1.0000F.E.135 C1 TE

Sample Item ID:	MAT.08.CR.1.0000F.E.135
Grade:	08
Claim(s):	<b>Claim 1: Concepts and Procedures</b> Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Assessment Target(s):	1 E: Define, evaluate, and compare functions.
Content Domain:	Functions
Standard(s):	8.F.1
Mathematical Practice(s):	1, 2
DOK:	1
Item Type:	CR
Score Points:	1
Difficulty:	L
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Claim-specific Attributes (e.g., accessibility issues):	
Notes:	The purpose of the item is to determine whether students understand that a function assigns exactly one output to each input in its domain.  The table consists of a response space in each blank cell. Each response space allows for a maximum of 3 digits.

Fill in each  $x$ -value and  $y$ -value in the table below to create a relation that is **not** a function.

$x$	$y$

*Sample Top-score Response:*

<b><i>x</i></b>	<b><i>y</i></b>
4	0
4	1
4	2
4	3
4	4

*Scoring Rubric:*

Responses to this item will receive 0-1 point, based on the following:

**1 point:** The student shows a thorough understanding of the definition of a function. The student creates a relation that is not a function.

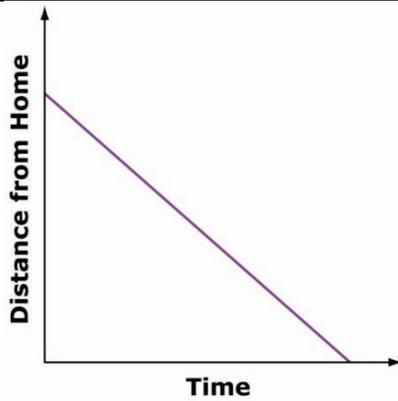
**0 points:** The student shows no understanding of the definition of a function. The student creates a relation that is a function.

## MAT.08.CR.1.0000F.F.090 C1 TF

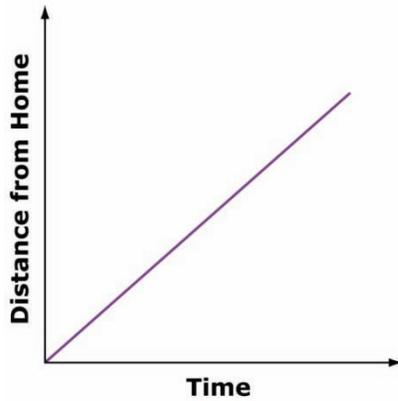
Sample Item ID:	MAT.08.CR.1.0000F.F.090
Grade:	08
Primary Claim:	<b>Claim 1: Concepts and Procedures</b> Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Secondary Claim(s):	Claim 2: Problem Solving Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.
Primary Content Domain:	Functions
Secondary Content Domain(s):	
Assessment Target(s):	1 F: Use functions to model relationships between quantities.  2 C: Interpret results in the context of a situation.  2 A: Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.  2 D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).
Standard(s):	8.F.5
Mathematical Practice(s):	1, 2, 4, 5, 6, 7, 8
DOK:	2
Item Type:	CR
Score Points:	1
Difficulty:	M
Key:	D, B, C, A
Stimulus/Source:	
Target-specific attributes (e.g., accessibility issues):	
Notes:	All four graphs must be correctly matched to its corresponding description. Response boxes only accept single alpha characters A, B, C, or D as entries.

Each day, Maria walks from home to school and then from school to home. The graphs that follow show the distance that Maria is from home at different times during the walk.

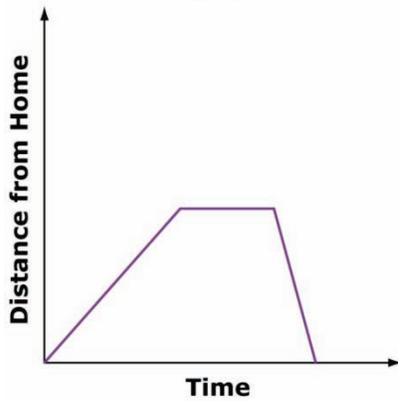
Match the graphs to the descriptions of Maria's walk shown to the right of the graphs. Next to each graph, enter the letter (A, B, C, D) of the description that best matches the graph.



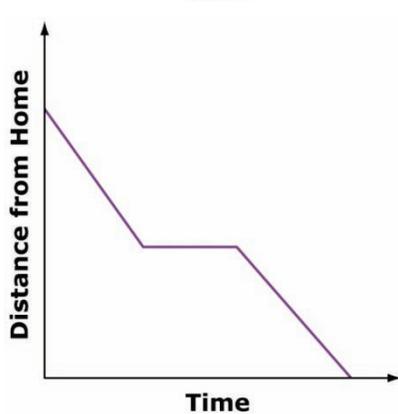
- A.** Maria walks from school to her friend's house. She visits her friend for a while. Then she walks the rest of the way home.



- B.** Maria walks from home to school at a constant rate.



- C.** Maria starts to walk from home to school. She stops to see whether she has her homework. She realizes she forgot her homework and runs back home to get it.

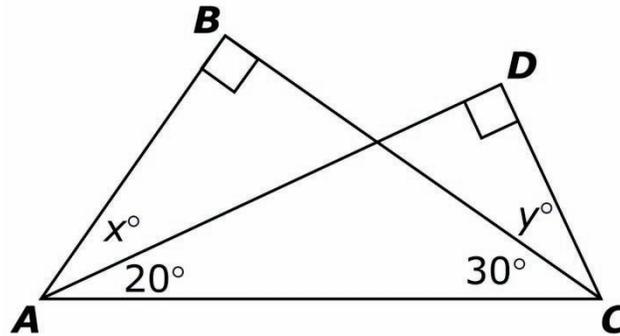


- D.** Maria walks from school to home at a constant rate.

## MAT.08.CR.1.0000G.G.129 C1 TG

Sample Item ID:	MAT.08.CR.1.0000G.G.129
Grade:	08
Claim(s):	<b>Claim 1: Concepts and Procedures</b> Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Assessment Target(s):	1 G: Understand congruence and similarity using physical models, transparencies, or geometry software.  1 D: Analyze and solve linear equations and pairs of simultaneous linear equations.
Content Domain:	Geometry
Standard(s):	8.G.5, 8.EE.7
Mathematical Practice(s):	2, 5
DOK:	2
Item Type:	CR
Score Points:	2
Difficulty:	M
Key:	$x = 40, y = 40$
Stimulus/Source:	
Target-specific attributes (e.g., accessibility issues):	
Notes:	A maximum of 3 numerical digits can be entered in the response boxes. No symbols (like - or °) will be allowed. The protractor tool will be unavailable for this item.

Right triangle  $ABC$  and right triangle  $ACD$  overlap as shown below. Angle  $DAC$  measures  $20^\circ$  and angle  $BCA$  measures  $30^\circ$ .



not drawn to scale

What are the values of  $x$  and  $y$ ?

$x =$   degrees       $y =$   degrees

*Key and Distractor Analysis:*

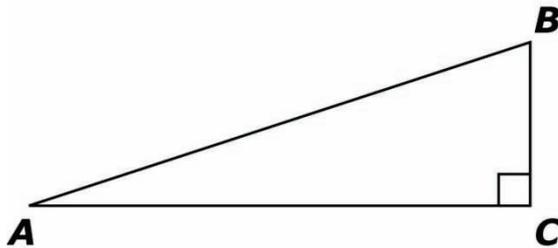
Students need to use the fact that the sum of the angles of a triangle is 180 degrees to find the correct values for  $x$  and  $y$ . Students may incorrectly assume that  $x + 20$  must be equal to  $y + 30$ .

Each part of the response will yield one point for a correct answer.  $x = 40$  and  $y = 40$ .

## MAT.08.CR.1.0000G.H.002 C1 TH

Sample Item ID:	MAT.08.CR.1.0000G.H.002
Grade:	08
Claim(s):	<b>Claim 1: Concepts and Procedures</b> Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Assessment Target(s):	1 H: Understand and apply the Pythagorean theorem.
Content Domain:	Geometry
Standard(s):	8.G.7
Mathematical Practice(s):	1, 5, 7, 8
DOK:	2
Item Type:	CR
Score Points:	2
Difficulty:	M
Key:	BC = 7, AC = 24, AB = 25 <b>OR</b> BC = 15, AC = 20, AB = 25 <b>OR</b> BC = 8, AC = 15, AB = 17
Stimulus/Source:	
Target-specific attributes (e.g., accessibility issues):	Calculators may be used for this target.
Notes:	The answer boxes will accept a maximum of 2 numerical digits.

In right triangle  $ABC$ , side  $AC$  is longer than side  $BC$ . The boxed numbers represent the possible side lengths of triangle  $ABC$ .



7	8
15	17
18	20
24	25

not drawn to scale

Identify three boxed numbers that could be the side lengths of triangle  $ABC$ . Enter the number you chose to represent the length of each side.

1a.  $BC =$

1b.  $AC =$

1c.  $AB =$

*Scoring Rubric:*

Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student shows a thorough understanding of the Pythagorean theorem. The student correctly chooses a set of 3 side lengths.

**1 point:** The student shows a partial understanding of the Pythagorean theorem. The student chooses the correct side lengths but does not enter them into the boxes correctly.

**0 points:** The student shows inconsistent or no understanding of the Pythagorean theorem.

**MAT.08.CR.1.000EE.B.494 C1 TB**

Sample Item ID:	MAT.08.CR.1.000EE.B.494
Grade:	08
Claim(s):	<b>Claim 1: Concepts and Procedures</b> Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Assessment Target(s):	1 B: Work with radicals and integer exponents.
Content Domain:	Expressions and Equations
Standard(s):	8.EE.3
Mathematical Practice(s):	1, 5
DOK:	1
Item Type:	CR
Score Points:	1
Difficulty:	L
Key:	Any number between 7 and 7.143 inclusive.
Stimulus/Source:	<a href="http://heasarc.nasa.gov/docs/cosmic/planets.html">http://heasarc.nasa.gov/docs/cosmic/planets.html</a>
Target-specific attributes (e.g., accessibility issues):	Calculators should be turned on.
Notes:	The response space allows for a maximum of 6 digits. A decimal point or comma can be used in place of one of the digits.

The average distance from Jupiter to the Sun is about  $5 \times 10^8$  miles. The average distance from Venus to the Sun is about  $7 \times 10^7$  miles.

The average distance from Jupiter to the Sun is about how many times as great as the average distance from Venus to the Sun?

times

## MAT.08.CR.2.0000G.A.001 Claim 2

Sample Item ID:	MAT.08.CR.2.0000G.A.001
Grade:	08
Primary Claim:	<b>Claim 2: Problem Solving</b> Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	Geometry
Secondary Content Domain(s):	
Assessment Target(s):	2 A: Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace. (DOK 2, 3)  1 I: Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.
Standard(s):	8.G.9
Mathematical Practice(s):	1, 2, 5, 6, 8
DOK:	2
Item Type:	CR
Score Points:	3
Difficulty:	M
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Target-specific attributes (e.g., accessibility issues):	
Notes:	Entry spaces for radius and height boxes are limited to three numeric characters.

Juan needs a right cylindrical storage tank that holds between 110 and 115 cubic feet of water.

Using whole numbers only, provide the radius and height for 3 different tanks that hold between 110 and 115 cubic feet of water.

Tank #1

radius =  ft.

height =  ft.

Tank #2

radius =  ft.

height =  ft.

Tank #3

radius =  ft.

height =  ft.

*Sample Top-Score Response:*

Tank #1:  $r = 2, h = 9$

Tank #2:  $r = 3; h = 4$

Tank #3:  $r = 6; h = 1$

*Scoring Rubric:*

Responses to this item will receive 0-3 points, based on the following:

**3 points:** The student shows a thorough understanding of the volume of cylinders. The student provides 3 correct sets of dimensions.

**2 points:** The student shows a partial understanding of the volume of cylinders. The student provides 2 correct sets of dimensions.

**1 point:** The student shows a limited understanding of the volume of cylinders. The student provides 1 correct set of dimensions.

**0 points:** The student shows inconsistent or no understanding of the volume of cylinders.

## MAT.08.CR.2.0000G.B.128 Claim 2

Sample Item ID:	MAT.08.CR.2.0000G.B.128
Grade:	08
Primary Claim:	<b>Claim 2: Problem Solving</b> Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	Geometry
Secondary Content Domain(s):	
Assessment Target(s):	2 B: Select and use appropriate tools strategically.  1 I: Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.
Standard(s):	8.G.9
Mathematical Practice(s):	1, 5, 6
DOK:	2
Item Type:	CR
Score Points:	1
Difficulty:	M
Key:	63
Stimulus/Source:	
Target-specific attributes (e.g., accessibility issues):	
Notes:	A maximum of 3 digits can be entered in the response box.

A cylindrical tank has a height of 10 feet and a radius of 4 feet. Jane fills the tank with water at a rate of 8 cubic feet per minute. At this rate, how many minutes will it take Jane to completely fill the tank without overflowing it?

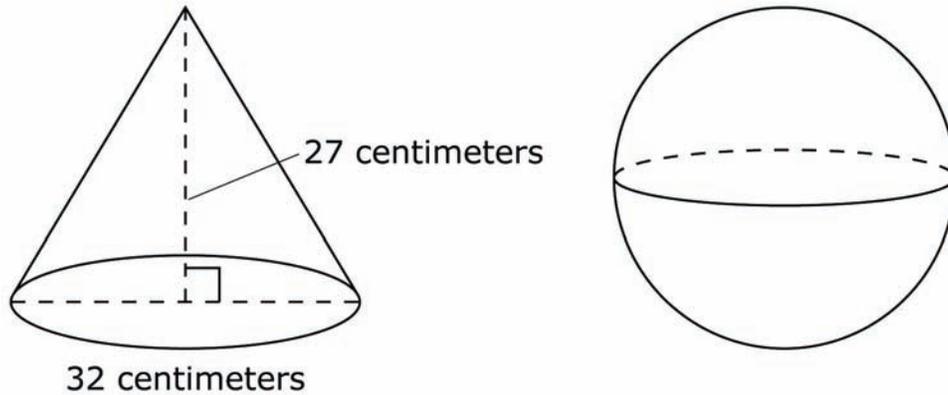
Round your answer to the nearest **minute**.

minutes

## MAT.08.CR.2.0000G.B.132 Claim 2

Sample Item ID:	MAT.08.CR.2.0000G.B.132
Grade:	08
Primary Claim:	<b>Claim 2: Problem Solving</b> Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	Geometry
Secondary Content Domain(s):	
Assessment Target(s):	2 B: Select and use appropriate tools strategically.  1 I: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.  1 B: Work with radicals and integer exponents.
Standard(s):	8.G.9, 8.EE.2
Mathematical Practice(s):	1, 5
DOK:	2
Item Type:	CR
Score Points:	1
Difficulty:	M
Key:	12
Stimulus:	Graphic of a labeled cone and an unlabeled sphere
Target-specific attributes (e.g., accessibility issues):	Calculators should be turned on.
Notes:	A maximum of 3 numeric characters and the symbol pi ( $\pi$ ) can be entered in the response box.

This cone and sphere have equal volumes.



not drawn to scale

What is the radius of the sphere?  Centimeters

## MAT.08.CR.2.000EE.A.133 Claim 2

Sample Item ID:	MAT.08.CR.2.000EE.A.133
Grade:	08
Primary Claim:	<b>Claim 2: Problem Solving</b> Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	Expressions and Equations
Secondary Content Domain(s):	The Number System
Assessment Target(s):	2 A: Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.  1 B: Work with radicals and integer exponents.  1 A: Know that there are numbers that are not rational, and approximate them by rational numbers.
Standard(s):	8.EE.3, 8.NS.2
Mathematical Practice(s):	1, 2, 4, 5, 6, 7
DOK:	2
Item Type:	CR
Score Points:	1
Difficulty:	H
Key:	187,000,000
Stimulus/Source:	<a href="http://heasarc.nasa.gov/docs/cosmic/planets.html">http://heasarc.nasa.gov/docs/cosmic/planets.html</a> ; <a href="http://neo.jpl.nasa.gov/cgi-bin/neo_elem?type=AMO;hmax=all;sort=name;sdir=ASC;max_rows=20;fmt=full;action=Display%20Table;show=1&amp;from=20">http://neo.jpl.nasa.gov/cgi-bin/neo_elem?type=AMO;hmax=all;sort=name;sdir=ASC;max_rows=20;fmt=full;action=Display%20Table;show=1&amp;from=20</a> (the semi-major axis is the average distance from the sun)
Target-specific attributes (e.g., accessibility issues):	Calculators should be turned on.
Notes:	The response space allows for a maximum of 14 digits. Commas and decimal points can be entered.



## MAT.08.ER.2.0000F.C.130 Claim 2

Sample Item ID:	MAT.08.ER.2.0000F.C.130
Grade:	08
Primary Claim:	<b>Claim 2: Problem Solving</b> Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	Functions
Secondary Content Domain(s):	
Assessment Target(s):	2 C: Interpret results in the context of a situation.  2 A: Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.  1 F: Use functions to model relationships between quantities.
Standard(s):	8.F.4
Mathematical Practice(s):	2, 4, 5
DOK:	3
Item Type:	ER
Score Points:	2
Difficulty:	M
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Target-specific attributes (e.g., accessibility issues):	Calculators may be used for this item.
Notes:	Part of PT set. The numerical entries are limited to 4 digits.

The total cost of an order of shirts from a company consists of the cost of each shirt plus a one-time design fee. The cost of each shirt is the same regardless of how many shirts are ordered.

The company provides the following examples to customers to help them estimate the total cost of an order of shirts:

- 50 shirts cost \$349.50
- 500 shirts cost \$2370

**Part A**

Based on the examples, what is the cost of each shirt, **not** including the one-time design fee?

\$

Explain how you found your answer.

**Part B**

What is the cost of the one-time design fee?

\$

Explain how you found your answer.

*Sample Top-Score Response:*

*Part A*

\$4.49; 450 more shirts cost \$2020.50 more, and 2020.50 divided by 450 is 4.49.

*Part B*

\$125; Since each shirt costs \$4.49, 50 shirts cost \$224.50, which means that the design fee is \$125.

*Scoring Rubric:*

Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student shows a thorough understanding of how to use a function to model a relationship between quantities. The numerical responses are equal to the answers given in the Sample Top-score Response and the explanations are complete and correct.

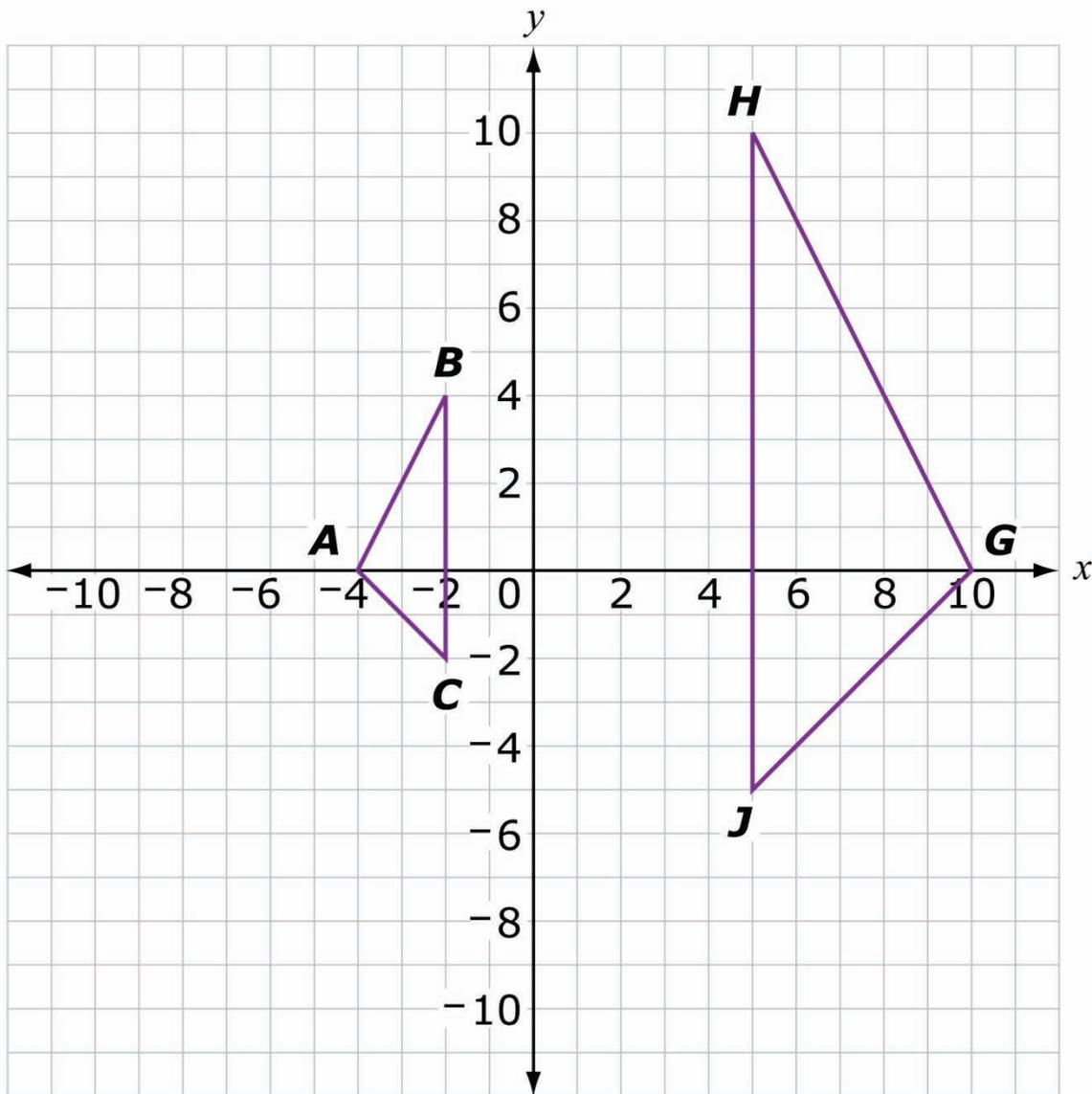
**1 point:** The student shows a partial understanding of how to use a function to model a relationship between quantities. One numerical response and the corresponding explanation are correct.

**0 points:** The student shows inconsistent or no understanding of how to use a function to model a relationship between quantities.

## MAT.08.ER.2.0000G.G.321 Claim 2

Sample Item ID:	MAT.08.ER.2.0000G.G.321
Grade:	08
Primary Claim:	<b>Claim 2: Problem Solving</b> Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	Geometry
Secondary Content Domain(s):	
Assessment Target(s):	2 A: Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.  1 G: Understand congruence and similarity using physical models, transparencies, or geometry software.  2 D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).
Standard(s):	8.G.4
Mathematical Practice(s):	1, 3, 5, 7
DOK:	2
Item Type:	ER
Score Points:	3
Difficulty:	H
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Claim-specific Attributes (e.g., accessibility issues):	
Notes:	

A transformation is applied to  $\triangle ABC$  to form  $\triangle DEF$  (not shown). Then, a transformation is applied to  $\triangle DEF$  to form  $\triangle GHJ$ .



**Part A**

Graph  $\triangle DEF$  on the  $xy$ -coordinate plane.

**Part B**

Describe the transformation applied to  $\triangle ABC$  to form  $\triangle DEF$

**Part C**

Describe the transformation applied to  $\triangle DEF$  to form  $\triangle GHJ$

**Part D**

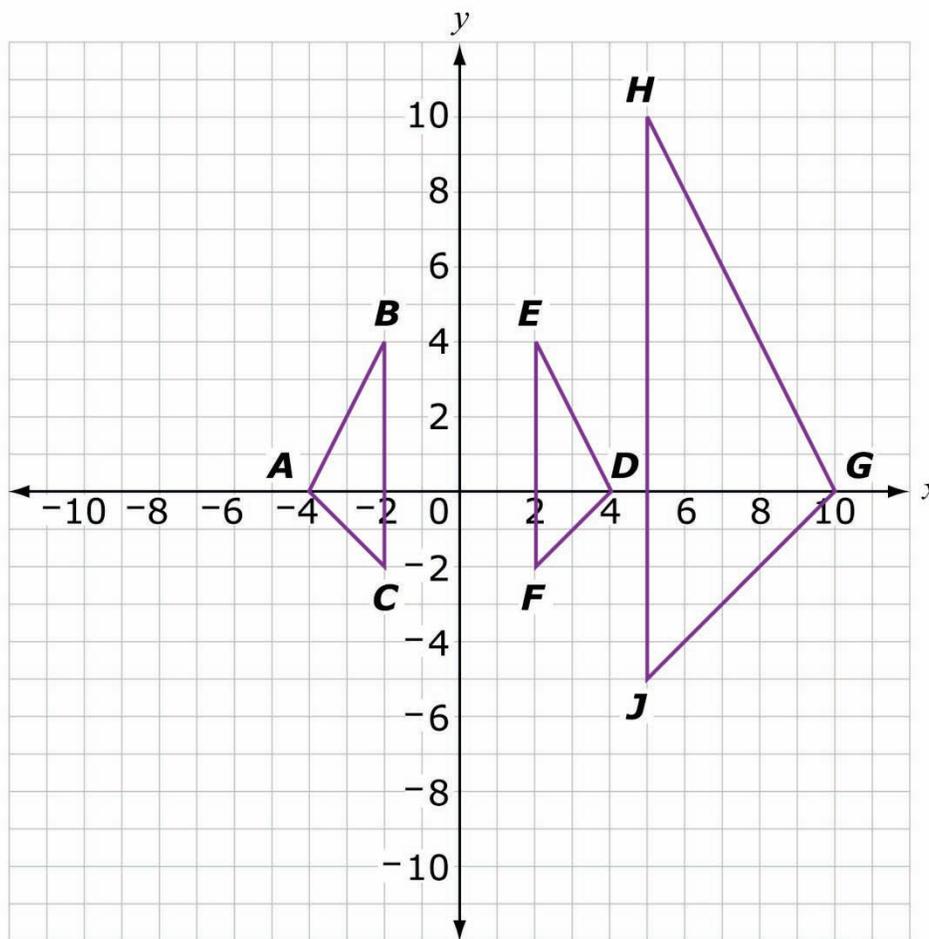
Select one statement that applies to the relationship between  $\triangle GHJ$  and  $\triangle ABC$ .

- $\triangle GHJ$  is congruent to  $\triangle ABC$ .
- $\triangle GHJ$  is similar to  $\triangle ABC$ .
- $\triangle GHJ$  is neither congruent nor similar to  $\triangle ABC$ .

Explain your reasoning.

Sample Top-Score Response:

**Part A**



**Part B**

A reflection over the  $y$ -axis

**Part C**

A dilation with a scale factor of 2.5 about the origin

**Part D**

$\triangle GHJ$  is similar to  $\triangle ABC$ .

A dilation followed by a congruence, or a congruence followed by a dilation, is a similarity. So,  $\triangle GHJ$  is similar to  $\triangle ABC$ .

*Scoring Rubric:*

The item will score 0-3 points, based on the following:

**3 points:** The student shows a thorough understanding of how to apply the concepts of transformations, congruence, and similarity to solve problems. The student draws a correct triangle in *Part A*, states correct transformations in *Parts B* and *C*, and shows that triangles *GHJ* and *ABC* are similar in *Part D*.

**2 points:** The student shows a partial understanding of how to apply the concepts of transformations, congruence, and similarity to solve problems. The student draws a correct triangle in *Part A* and states correct transformations in *Parts B* and *C* but does not show that triangles *GHJ* and *ABC* are similar in *Part D*. **OR** The student shows that triangles *GHJ* and *ABC* are similar in *Part D* but does not draw a correct triangle in *Part A*, although the responses to *Parts B* and *C* indicate that that correct triangle was intended.

**1 point:** The student shows a limited understanding of how to apply the concepts of transformations, congruence, and similarity to solve problems. The student shows that triangles *GHJ* and *ABC* are similar in *Part D* but does not draw a correct triangle in *Part A*, and only one of the transformations in *Parts B* and *C* is correct.

**0 points:** The student shows inconsistent or no understanding of how to apply the concepts of transformations, congruence, and similarity to solve problems.

## MAT.08.ER.2.000SP.D.150 Claim 2

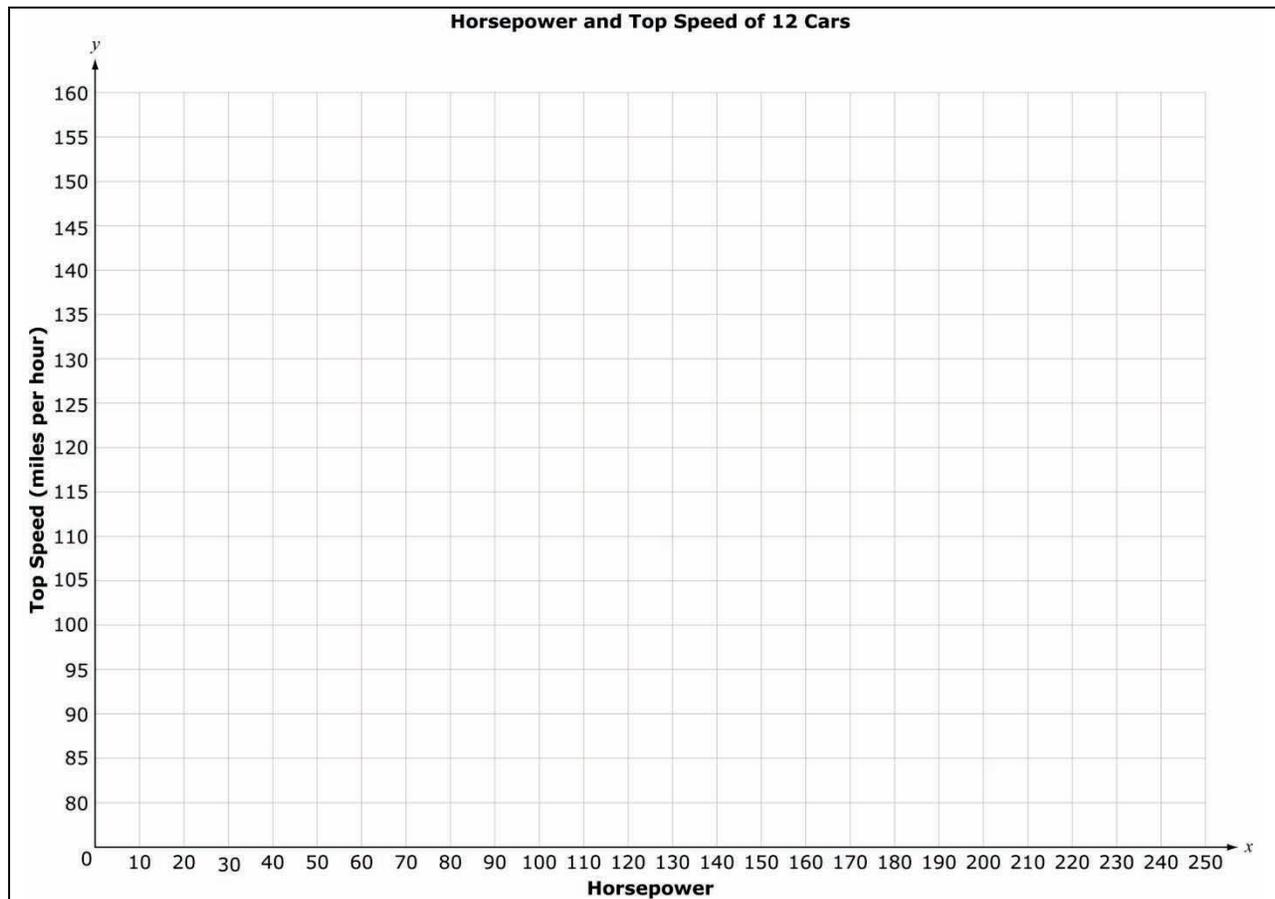
Sample Item ID:	MAT.08.ER.2.000SP.D.150
Grade:	08
Primary Claim:	<b>Claim 2: Problem Solving</b> Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.  Claim 3: Communicating Reasoning Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
Primary Content Domain:	Statistics and Probability
Secondary Content Domain(s):	
Assessment Target(s):	2 D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).  1 J: Investigate patterns of association in bivariate data.  2 A: Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.  2 B: Select and use appropriate tools strategically.  2 C: Interpret results in the context of a situation.  3 F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions.
Standard(s):	8.SP.3
Practice(s):	1, 2, 3, 4, 5, 6
DOK:	3
Item Type:	ER
Score Points:	6
Difficulty:	M
Key:	See Sample Top-Score Response.
Stimulus/Source:	<a href="http://lib.stat.cmu.edu/DASL/Datafiles/carmpgdat.html">http://lib.stat.cmu.edu/DASL/Datafiles/carmpgdat.html</a> This kind of information may be public domain. I do not think we would need a copyright for it.
Target-specific attributes (e.g., accessibility issues):	Calculators may be used for this item.
Notes:	Part of PT set.

This table shows the horsepower and top speed of 12 cars.

Horsepower	Top Speed (miles per hour)
165	122
150	117
90	109
49	96
70	105
62	98
245	148
140	114
103	112
180	133
130	115
145	120

**Part A**

Construct a scatter plot of the data in the table on the graph below.

**Part B**

Draw a line of best fit on the graph for the data points graphed.

**Part C**

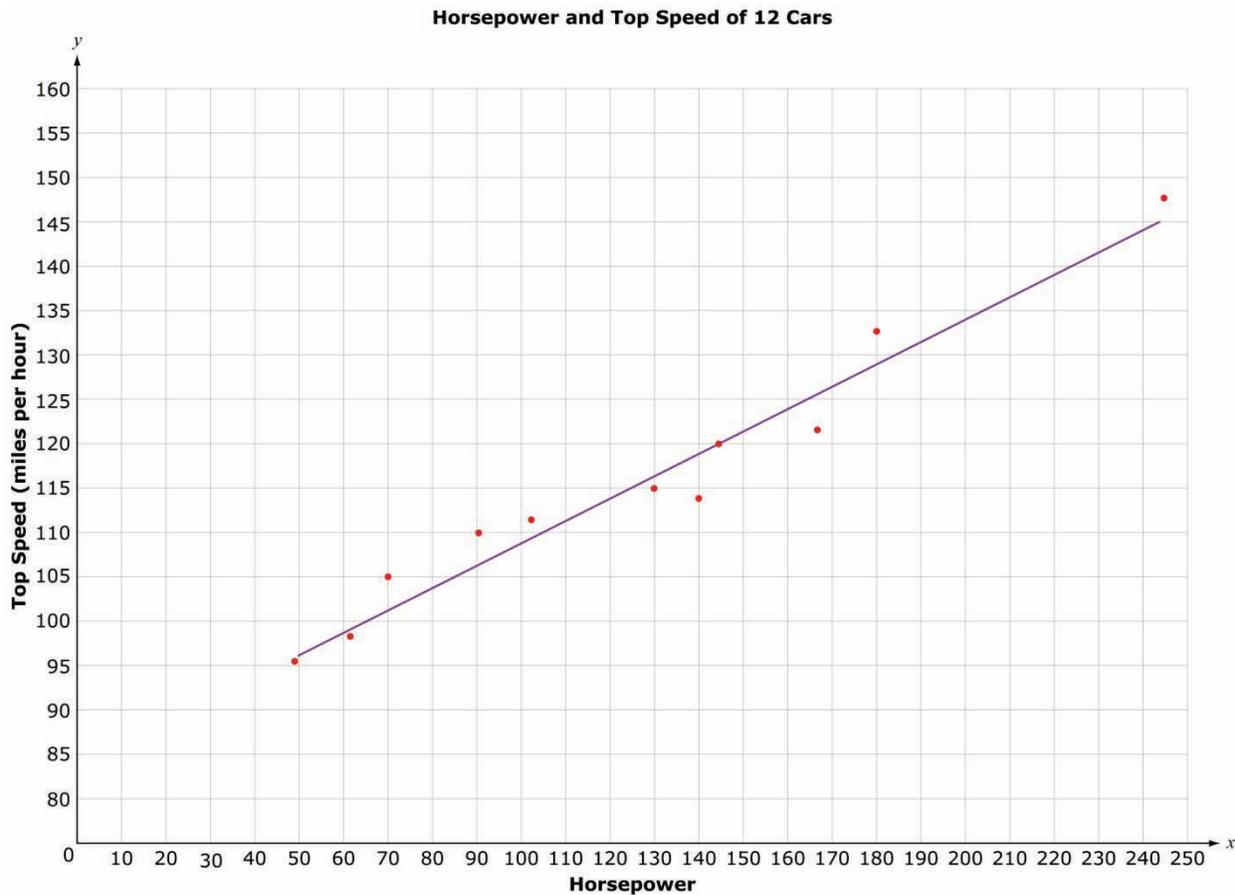
Based on the graph you drew, how much more horsepower is needed to increase the top speed of a car by 5 miles per hour?

more horsepower

Explain how you found your answer.

*Sample Top-Score Response:*

*Part A and Part B:*



*Part C:*

20 more horsepower. The slope of the line is about 10 miles per hour for 40 horsepower so it takes about 20 more horsepower for the top speed of the car to increase by 5 miles per hour.

*Scoring Rubric:*

The item will score 0-6 points, based on up to 2 points available in each part:

*Part A:*

**2 points:** The student shows a thorough understanding of how to construct a scatter plot. The student plots each point very closely to where it should be located on the graph.

**1 point:** The student shows a partial understanding of how to construct a scatter plot. The

student makes minor errors when plotting individual points.

**0 points:** The student shows inconsistent or no understanding of how to construct a scatter plot. The student makes major errors when plotting individual points, such as reversing  $x$ - and  $y$ -values.

*Part B:*

**2 points:** The student shows a thorough understanding of how to construct a line of best fit. The line of best fit is within a preset range.

**1 point:** The student shows a partial understanding of how to construct a line of best fit. The line is constructed so that parts of the line are outside of the preset range.

**0 points:** The student shows inconsistent or no understanding of how to construct a line of best fit.

*Part C:*

**2 points:** The student shows a thorough understanding of how to interpret the slope of a line of best fit. The answer of 20 and the explanation are both complete and correct.

**1 point:** The student shows a partial understanding of how to interpret the slope of a line of best fit. Minor errors may be made in calculating the slope, or minor errors may be made in interpreting the slope.

**0 points:** The student shows inconsistent or no understanding of how to interpret the slope of a line of best fit.

## MAT.08.ER.3.0000F.E.205 Claim 3

Sample Item ID:	MAT.08.ER.3.0000F.E.205
Grade:	08
Claim(s):	<b>Claim 3: Communicating Reasoning</b> Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	Functions
Secondary Content Domain(s):	
Assessment Target(s):	3 E: Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument—explain what it is.  1 E: Define, evaluate, and compare functions.  1 A (Gr 7): Analyze proportional relationships and use them to solve real-world and mathematical problems.  3 F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions.
Standard(s):	8.F.3, 7.RP.3
Mathematical Practice(s):	1, 2, 4, 7
DOK:	3
Item Type:	ER
Score Points:	2
Difficulty:	M
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Target-Specific Attributes (e.g., accessibility issues):	Calculators may be used on this item.
Notes:	Part of PT set.

Samir was assigned to write an example of a linear functional relationship. He wrote this example for the assignment.

The relationship between the year and the population of a county when the population increases by 10% each year

**Part A**

Complete the table below to create an example of the population of a certain county that is increasing by 10% each year.

Year	Population of a Certain County
0	
1	
2	
3	
4	

**Part B**

State whether Samir's example represents a linear functional relationship. Explain your reasoning.

*Sample Top-Score Response:**Part A*

Year	Population of a Certain County
0	100,000
1	110,000
2	121,000
3	133,100
4	146,410

*Part B*

Samir's example is not a linear functional relationship. The population does not increase by the same amount each year, so the relationship is not linear.

*Scoring Rubric:*

Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student shows a thorough understanding of how to distinguish between linear and nonlinear relationships. The student correctly completes the table showing a 10% increase in population each year, states that the relationship is not linear, and provides a clear and complete explanation of why the relationship is not linear. Rounding to the nearest whole number for the population is permitted.

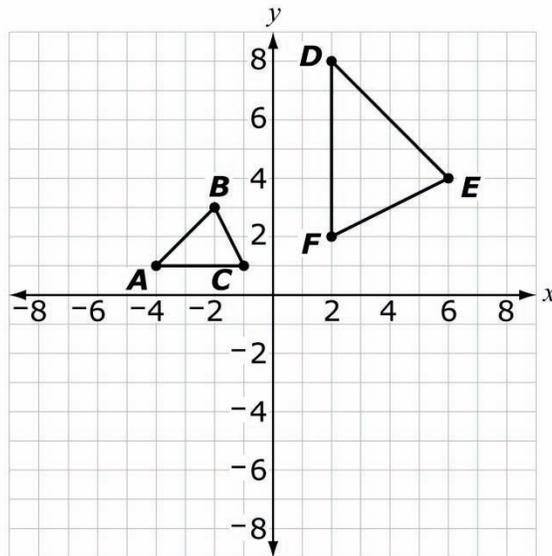
**1 point:** The student shows a partial understanding of how to distinguish between linear and nonlinear relationships. The student makes calculation errors in the table but gives a response in *Part B* that corresponds with the numbers in the table OR the student completes the table correctly but provides an incorrect explanation in *Part A*.

**0 points:** The student shows inconsistent or no understanding of how to distinguish between linear and nonlinear relationships. The student makes major errors when completing the table and gives a response in *Part B* that does not correspond to the values entered in the table.

## MAT.08.ER.3.0000G.B.010 Claim 3

Sample Item ID:	MAT.08.ER.3.0000G.B.010
Grade:	08
Primary Claim:	<b>Claim 3: Communicating Reasoning</b> Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	Geometry
Secondary Content Domain(s):	
Assessment Target(s):	3 B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.  1 G: Understand congruence and similarity using physical models, transparencies, or geometry software.
Standard(s):	8.G.4
Mathematical Practice(s):	1, 3, 7
DOK:	3
Item Type:	ER
Score Points:	2
Difficulty:	M
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Target-Specific Attributes (e.g., accessibility issues):	Rotations are only of multiples of 90 degrees about the origin. Reflections are only over the x- and y- axes. Dilations are only with the origin as the center. There are at most three transformations in a sequence of transformations. Limited to the coordinate plane.
Notes:	Part of PT set

Triangles  $ABC$  and  $DEF$  are shown on this coordinate grid.



Describe a sequence of transformations that verify that these triangles are geometrically similar.

*Sample Top-Score Response:*

Triangle  $ABC$  is rotated 90 degrees clockwise about the origin and then dilated by a scale factor of 2 to make triangle  $DEF$ .

*Scoring Rubric:*

Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student shows a thorough understanding of transformations. The student correct describes the transformations that take triangle  $ABC$  to triangle  $DEF$ .

**1 point:** The student shows a partial understanding of transformations. The student identifies either the rotation or dilation.

**0 points:** The student shows inconsistent or no understanding of transformations.

## MAT.08.ER.3.0000G.B.059 Claim 3

Sample Item ID:	MAT.08.ER.3.0000G.B.059
Grade:	08
Primary Claim:	<b>Claim 3: Communicating Reasoning</b> Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.  Claim 2: Problem Solving Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.
Primary Content Domain:	Geometry
Secondary Content Domain(s):	
Assessment Target(s):	3 B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.  1 G: Understand congruence and similarity using physical models, transparencies, or geometry software.  2 B: Select and use appropriate tools strategically.  3 F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions.
Standard(s):	8.G.3
Mathematical Practice(s):	1, 3, 5, 7
DOK:	3
Item Type:	ER
Score Points:	3
Difficulty:	H
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Target-Specific Attributes (e.g., accessibility issues):	This item will need a graphing feature that can be used multiple times (if needed) to give illustrations of the reasoning being given. The points graphed should be labeled with their coordinates.
Notes:	Students should be able to plot points on the coordinate plane to help them answer the question. Requires AI scoring.

A student made this conjecture about reflections on an  $xy$  coordinate plane.

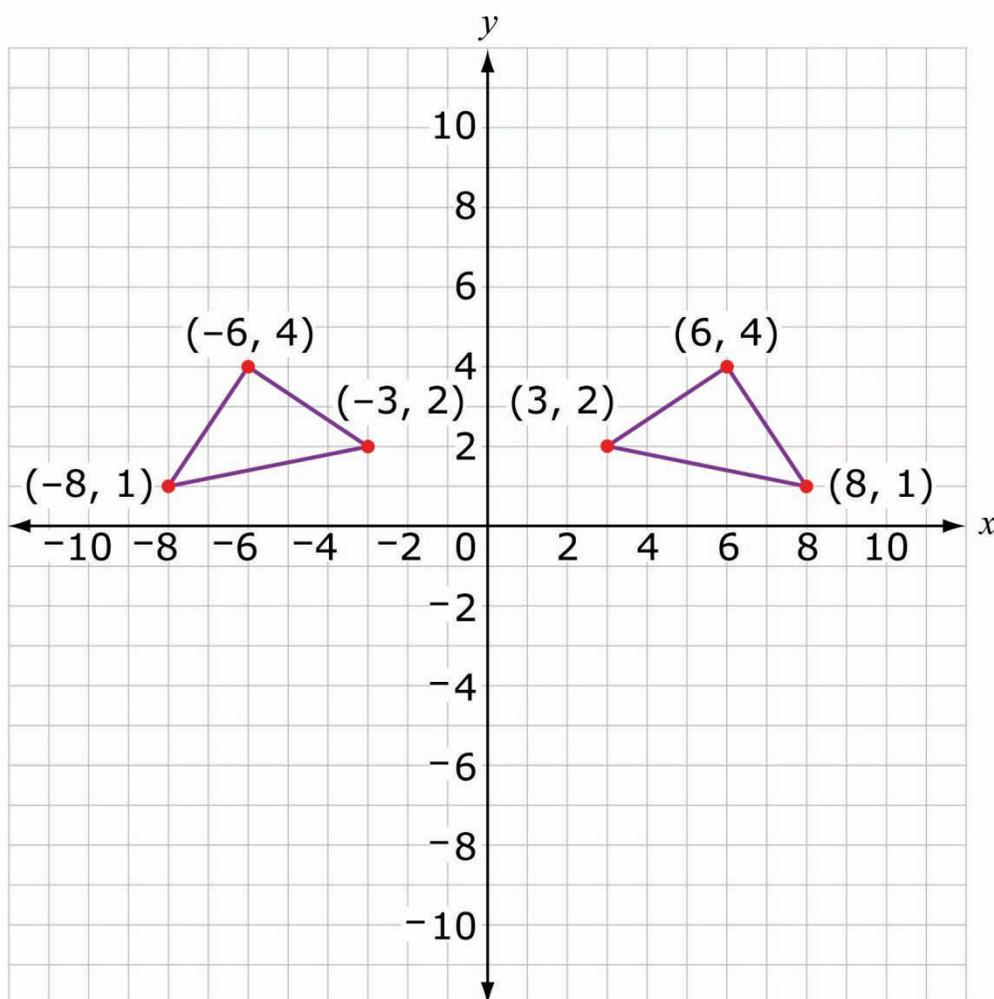
When a polygon is reflected over the  $y$ -axis, the  $x$ -coordinates of the corresponding vertices of the polygon and its image are opposite, but the  $y$ -coordinates are the same.

Develop a chain of reasoning to justify or refute the conjecture. You must demonstrate that the conjecture is always true or that there is at least one example in which the conjecture is not true. You may include one or more graphs in your response.

To include a graph to support your reasoning, put your cursor where you want the graph to be, and then click the Graph icon to insert a graph.

*Sample Top-Score Response:*

When a polygon is reflected over the  $y$ -axis, each vertex of the reflected polygon will end up on the opposite side of the  $y$ -axis but the same distance from the  $y$ -axis. So, the  $x$ -coordinates of the vertices will change from positive to negative or negative to positive, but the absolute value of the number will stay the same, so the  $x$ -coordinates of the corresponding vertices of the polygon and its image are opposites. Since the polygon is being reflected over the  $y$ -axis, the image is in a different place horizontally but it does not move up or down, which means the  $y$ -coordinates of the vertices of the image will be the same as the  $y$ -coordinates of the corresponding vertices of the original polygon. As an example, look at the graph below, and notice that the  $x$ -coordinates of the corresponding vertices of the polygon and its image are opposites but the  $y$ -coordinates are the same. This means the conjecture is correct.



*Scoring Rubric:*

Responses to this item will receive 0-3 points, based on the following:

**3 points:** The student shows a thorough understanding of how to construct a chain of reasoning to justify a statement regarding reflections. The student provides a chain of reasoning that demonstrates that it is ALWAYS true that the  $x$ -coordinates of the corresponding vertices of the polygon and its image are opposites but the  $y$ -coordinates are the same when a polygon is reflected over the  $y$ -axis.

**2 points:** The student shows a partial understanding of how to construct a chain of reasoning to justify a statement regarding reflections. The student makes progress in providing a chain of reasoning that demonstrates that it is always true that the  $x$ -coordinates of the corresponding vertices of the polygon and its image are opposites but the  $y$ -coordinates are the same when a polygon is reflected over the  $y$ -axis. However, there are gaps in the student's reasoning, such as not fully addressing the distance of the  $x$ -coordinates from the  $y$ -axis or the opposite signs of the  $x$ -coordinates on either side of the  $y$ -axis or why the  $y$ -coordinates are the same. **OR** The student provides a thorough justification that shows that the conjecture is not true for reflections over the  $x$ -axis.

**1 point:** The student shows a limited understanding of how to construct a chain of reasoning to justify a statement regarding reflections. The student only uses specific examples to show that the  $x$ -coordinates of the corresponding vertices of the polygon and its image are opposites but the  $y$ -coordinates are always the same when a polygon is reflected over the  $y$ -axis. **OR** The student makes progress in providing a justification that shows that the conjecture is not true for reflections over the  $x$ -axis. **OR** The student provides a thorough justification that shows that the conjecture is not true for a transformation other than a reflection.

**0 points:** The student shows inconsistent or no understanding of how to construct a chain of reasoning to justify a statement regarding reflections.

## MAT.08.ER.3.0000G.B.433 Claim 3

Sample Item ID:	MAT.08.ER.3.0000G.B.433
Grade:	08
Primary Claim:	<b>Claim 3: Communicating Reasoning</b> Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	Geometry
Secondary Content Domain(s):	
Assessment Target(s):	3 B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.  1 H: Understand and apply the Pythagorean theorem.  3 F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions.
Standard(s):	8.G.7
Mathematical Practice(s):	2, 3, 6, 7
DOK:	2
Item Type:	ER
Score Points:	2
Difficulty:	H
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Target-Specific Attributes (e.g., accessibility issues):	
Notes:	Part of PT set; drawing tools are turned off for this item.

Students in a class are using their knowledge of the Pythagorean theorem to make conjectures about triangles. A student makes the conjecture shown below.

A triangle has side lengths  $x$ ,  $y$ , and  $z$ . If  $x < y < z$  and  $x^2 + y^2 < z^2$ , the triangle is an obtuse triangle.

Use the Pythagorean theorem to develop a chain of reasoning to justify or refute the conjecture. You must demonstrate that the conjecture is always true or that there is at least one example in which the conjecture is not true.

*Sample Top-Score Response:*

Picture the triangle with the side of length  $x$  on the bottom, the side of length  $y$  on the left, and the side of length  $z$  on the top. If  $x^2 + y^2 = z^2$  the triangle is a right triangle. Since  $x^2 + y^2 < z^2$  if the sides of length  $x$  and  $y$  were left so they made a right angle and the side of length  $z$  started at the other end of the side of length  $x$ , it would extend past the other end of the side of length  $y$ . So the end of the side of length  $y$  has to swing out to the left so the ends of all the segments can connect to form a triangle. When the side of length  $y$  swings out to the left, the measure of the angle between that side and the side of length  $x$  increases, so the triangle is an obtuse triangle. The conjecture is true.

*Scoring Rubric:*

Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student shows a thorough understanding of how to use the Pythagorean theorem to construct a chain of reasoning to justify a statement about a triangle. The student justifies the conjecture and makes use of the Pythagorean theorem while doing so.

**1 point:** The student shows a partial understanding of how to use the Pythagorean theorem to construct a chain of reasoning to justify a statement about a triangle. The student either demonstrates knowledge of the Pythagorean theorem but does not provide a clear justification of the conjecture OR the student provides a justification of the conjecture without making use of the Pythagorean theorem.

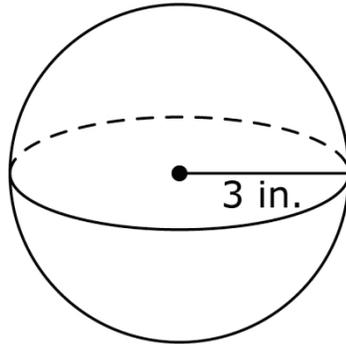
**0 points:** The student shows inconsistent or no understanding of how to use the Pythagorean theorem to construct a chain of reasoning to justify a statement about a triangle. Merely stating that the conjecture is true is not enough to earn any points.

## MAT.08.ER.3.0000G.F.011 Claim 3

Sample Item ID:	MAT.08.ER.3.0000G.F.011
Grade:	08
Primary Claim:	<b>Claim 3: Communicating Reasoning</b> Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.  Claim 2: Problem Solving Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.
Primary Content Domain:	Geometry
Secondary Content Domain(s):	
Assessment Target(s):	3 F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions.  1 I: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.  2 A: Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.
Standard(s):	8.G.9
Mathematical Practice(s):	1, 5, 7
DOK:	3
Item Type:	ER
Score Points:	3
Difficulty:	M
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Target-specific attributes (e.g., accessibility issues):	
Notes:	Part of PT set

**Part A**

This sphere has a 3-inch radius.

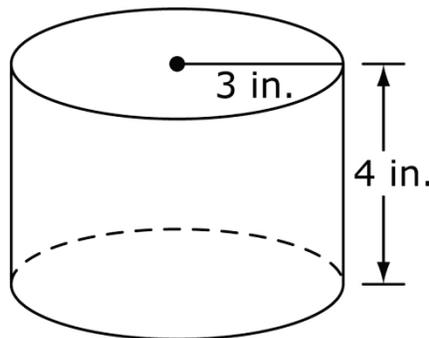


What is the volume, in cubic inches, of the sphere?

Volume =  cubic inches

**Part B**

This right cylinder has a radius of 3 inches and a height of 4 inches.



What is the volume, in cubic inches, of the cylinder?

Volume =  cubic inches

[Directions to students on the computer before progressing to Part C]

You will not be allowed to return to Part A or Part B of this task after submitting your answers and clicking Next.

**Part C**

Lin claims that the volume of any sphere with a radius of  $r$  inches is always **equal** to the volume of a cylinder with a radius of  $r$  inches and a height of  $h$  inches, when  $h = \frac{4}{3}r$ .

Show all work necessary to justify Lin's claim.

*Sample Top-Score Response:*

*Part A:*  $36\pi$  cu in. (or any number between 113 and 113.1)

*Part B:*  $36\pi$  cu in. (or any number between 113 and 113.1)

*Part C:* I can create the following equation if the volume of the sphere and cylinder are equal,

$$\frac{4}{3}\pi r^3 = \pi r^2 h$$

I can divide both sides of the equation by  $(\pi r^2)$  as shown below.

$$\left(\frac{4}{3}\pi r^3\right) \div \pi r^2 = \pi r^2 h \div \pi r^2$$

This justifies Lin's claim.

$$\frac{4}{3}r = h$$

*Scoring Rubric:*

Responses to this item will receive 0-3 points, based on the following:

**3 points:** The student shows a thorough understanding of the volume of spheres and cylinders. The student correctly determines the volume of both the sphere ( $36\pi$ ) and the cylinder ( $36\pi$ ) and provides a clear justification in *Part C*.

**2 points:** The student shows a partial understanding of the volume of spheres and cylinders. The student either gets both volumes correct but does not provide a clear justification for *Part C* OR the student gets only *Part A* or *Part B* correct AND provides a clear justification in *Part C*.

**1 point:** The student shows a limited understanding of the volume of spheres and cylinders. The student answers only one part of the task correctly.

**0 points:** The student shows inconsistent or no understanding of the volume of spheres and cylinders.

## MAT.08.ER.3.0000G.F.016 Claim 3

Sample Item ID:	MAT.08.ER.3.0000G.F.016
Grade:	08
Primary Claim:	<b>Claim 3: Communicating Reasoning</b> Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	Geometry
Secondary Content Domain(s):	
Assessment Target(s):	3 F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions.  1 H: Understand and apply the Pythagorean theorem.  3 B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.
Standard(s):	8.G.7
Mathematical Practice(s):	3, 5, 6
DOK:	3
Item Type:	CR
Score Points:	2
Difficulty:	M
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Target-specific attributes (e.g., accessibility issues):	
Notes:	Part of PT set; drawing tools are turned off for this item.

**Part A**

Triangle  $STV$  has sides with lengths of 7, 11, and 14 units. Determine whether this triangle is a right triangle.

Show all work necessary to justify your answer.

**Part B**

A right triangle has a hypotenuse with a length of 15. The lengths of the legs are whole numbers. What are the lengths of the legs?

*Sample Top-Score Response:**Part A*

$7^2 + 11^2$  does not equal  $14^2$  because  $49 + 121 = 170$ , not 196.

Therefore, it is not a right triangle because the side lengths do not satisfy the Pythagorean theorem.

*Part B*

9, 12

*Scoring Rubric:*

The item will score 0-2 points, based on the following:

**2 points:** The student shows a thorough understanding of the Pythagorean theorem and its converse. The student correctly explains that the given triangle is not a right triangle and correctly provides legs that are whole numbers for a right triangle with a hypotenuse of length 15.

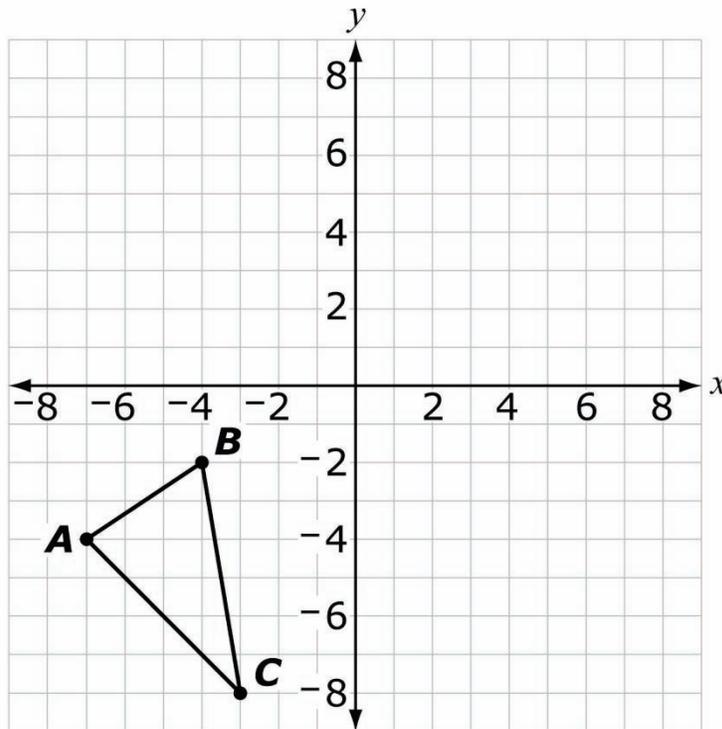
**1 point:** The student shows a partial understanding of the Pythagorean theorem and its converse. The student either correctly explains that the given triangle is not a right triangle or correctly provides legs that are whole numbers for a right triangle with a hypotenuse of length 15.

**0 points:** The student shows inconsistent or no understanding of the Pythagorean theorem and its converse.

## MAT.08.ER.3.0000G.F.559 Claim 3

Sample Item ID:	MAT.08.ER.3.0000G.F.559
Grade:	08
Primary Claim:	<b>Claim 3: Communicating Reasoning</b> Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.  Claim 2: Problem Solving Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.
Primary Content Domain:	Geometry
Secondary Content Domain(s):	
Assessment Target(s):	3 F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions.  1 G: Understand congruence and similarity using physical models, transparencies, or geometry software.  2 D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).
Standard(s):	8.G.3
Mathematical Practice(s):	1, 3, 5, 7
DOK:	3
Item Type:	ER
Score Points:	4
Difficulty:	H
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Claim-Specific Attributes (e.g., accessibility issues):	
Notes:	Students should be able to plot points on the coordinate plane to help them answer the question. Requires AI scoring. The response boxes will accept up to 2 numeric characters including negative (-) and positive (+) symbols.

Triangle  $ABC$  is shown on this coordinate grid.



**Part A**

$\triangle ABC$  is rotated 180 degrees clockwise about the origin to form  $\triangle DEF$ . What are the coordinates of the vertices of  $\triangle DEF$ ?

$D$ (, )

$E$ (, )

$F$ (, )

**Part B**

What conjecture can be made about the relationship between the coordinates of the vertices of an original shape and the coordinates of the vertices of the image of the shape when it is rotated 180 degrees clockwise about the origin?

You must demonstrate that the conjecture is always true or that there is at least one example in which the conjecture is not true.

*Sample Top-Score Response:*

**Part A**

$D(7, 4)$ ,  $E(4, 2)$ ,  $F(3, 8)$

**Part B**

The conjecture is that the coordinates of the vertices of the image will have the opposite sign of the coordinates of the vertices of the original shape. When a point is rotated 180 degrees clockwise about the origin, if a line is drawn through the original point and the origin, the image of the point will also be on the line, and it will be the same distance from the origin that the original point was, but on the opposite side of the origin. When two points on the same line are the same distance from the origin and on opposite sides of the origin, the coordinates of the points have opposite signs, because the slope from each coordinate to the origin is the same, but to move from the origin to each point to get its coordinates, you must move in opposite directions. So if you move right from the origin to get to one point, you will move left to get to the other, and if you move up from the origin to get to one point, you will move down to get to the other. So the coordinates of the vertices of the image will have the opposite sign of the coordinates of the vertices of the original shape.

*Scoring Rubric:*

Responses to this item will receive 0-4 points, based on the following:

**4 points:** The student shows thorough understanding of how to rotate figures on the plane and make and justify conjectures regarding rotated figures. The student gives the correct coordinates of the triangles in *Part A* and provides a correct conjecture and justification in *Part B*.

**3 points:** The student shows good but incomplete understanding of how to rotate figures on the plane and make and justify conjectures regarding rotated figures. The student gives the correct coordinates of the triangles in *Part A* and provides a correct conjecture in *Part B*, but the justification of the conjecture is incomplete (e.g., only specific examples are given) although it demonstrates progress. **OR** *Part B* is complete and correct but the student makes a minor error when listing the coordinates of the triangles in *Part A*.

**2 points:** The student shows partial understanding of how to rotate figures on the plane

and make and justify conjectures regarding rotated figures. The student gives the correct coordinates of the triangles in *Part A* and provides a correct conjecture in *Part B*, but the justification of the conjecture is incorrect. **OR** The student makes 1 or 2 minor mistakes when listing the coordinates of the triangles in *Part A*, provides a correct conjecture in *Part B*, but the justification of the conjecture is incomplete (e.g., only specific examples are given) although it demonstrates progress.

**1 point:** The student shows limited understanding of how to rotate figures on the plane and make and justify conjectures regarding rotated figures. The student gives the correct coordinates of the triangles in *Part A* but *Part B* is incorrect. **OR** The student provides a correct conjecture in *Part B*, but makes 1 or 2 minor mistakes when listing the coordinates of the triangles in *Part A* and the justification of the conjecture in *Part B* is incorrect.

**0 points:** The student shows inconsistent or no understanding of how to rotate figures on the plane and make and justify conjectures regarding rotated figures.

## MAT.08.ER.3.000EE.A.139 Claim 3

Sample Item ID:	MAT.08.ER.3.000EE.A.139
Grade:	08
Primary Claim:	<b>Claim 3: Communicating Reasoning</b> Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	Expressions and Equations
Secondary Content Domain(s):	
Assessment Target(s):	3 A: Test propositions or conjectures with specific examples.  1 C: Understand the connections between proportional relationships, lines, and linear equations.
Standard(s):	8.EE.6
Mathematical Practice(s):	1, 2, 3, 5, 7
DOK:	2
Item Type:	ER
Score Points:	2
Difficulty:	M
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Claim-Specific Attributes (e.g., accessibility issues):	Calculators may be used for this item.
Notes:	Part of PT set

Mr. Perry's students used pairs of points to find the slopes of lines. Mr. Perry asked Avery how she used the pairs of points listed in this table to find the slope of a line.

<b>x</b>	<b>y</b>
8	18
20	45

Avery said, "The easiest way to find the slope is to divide  $y$  by  $x$ . The slope of this line is  $\frac{18}{8}$ , or  $\frac{9}{4}$ ."

**Part A**

Show another way to find the slope of the line that passes through the points listed in the table. Your way must be different from Avery's way.

**Part B**

Write an example that shows that Avery's "divide  $y$  by  $x$ " method will not work to find the slope of **any** line.

*Sample Top-Score Response:**Part A:*

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{45 - 18}{20 - 8} = \frac{27}{12} = \frac{9}{4}$$

*Part B:*

If Mr. Perry asked the class to find the slope of the line through (1, 1) and (2, 3), you can find the actual slope by using the formula and get  $\frac{3-1}{2-1} = \frac{2}{1} = 2$ , but Avery's method will not work because she would either say the slope is  $\frac{1}{1} = 1$  or  $\frac{3}{2} = 1.5$ .

*Scoring Rubric:*

Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student shows thorough understanding of the slope of lines. The student correctly finds the slope in a different way and provides an example where Avery's method does not work.

**1 point:** The student shows partial understanding of the slope of lines. The student makes one or two mathematical errors when finding the slope in a different way but provides an example where Avery's method does not work. **OR** The student correctly finds the slope in a different way but is unable to provide an example where Avery's method does not work.

**0 points:** The student shows inconsistent or no understanding of the slope of lines. If the student demonstrates a conceptual misunderstanding of slope in *Part A*, the score is 0 regardless of the example given in *Part B*.

## MAT.08.ER.3.000EE.D.137 Claim 3

Sample Item ID:	MAT.08.ER.3.000EE.D.137
Grade:	08
Primary Claim:	<b>Claim 3: Communicating Reasoning</b> Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	Expressions and Equations
Secondary Content Domain(s):	
Assessment Target(s):	3 D: Use the technique of breaking an argument into cases.  3 A: Test propositions or conjectures with specific examples.  3 E: Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument—explain what it is.  3 G: At later grades, determine conditions under which an argument does and does not apply. (For example, area increases with perimeter for squares, but not for all plane figures.)  1 D: Analyze and solve linear equations and pairs of simultaneous linear equations.
Standard(s):	8.EE.7
Mathematical Practice(s):	1, 3, 7
DOK:	2
Item Type:	ER
Score Points:	3
Difficulty:	M
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Claim-Specific Attributes (e.g., accessibility issues):	
Notes:	Part of PT set  The purpose of this item is to determine students' familiarity with the different numbers of solutions that are possible when solving a linear equation.

Consider the equation  $3(2x + 5) = ax + b$ .

**Part A**

Find one value for  $a$  and one value for  $b$  so that there is exactly one value of  $x$  that makes the equation true.

$$a = \boxed{\phantom{000}}$$

$$b = \boxed{\phantom{000}}$$

Explain your reasoning.

**Part B**

Find one value for  $a$  and one value for  $b$  so that there are infinitely many values of  $x$  that make the equation true.

$$a = \boxed{\phantom{000}}$$

$$b = \boxed{\phantom{000}}$$

Explain your reasoning.

*Sample Top-Score Response:**Part A*

$A = 5$ ;  $b = 16$  When you put these numbers in for  $a$  and  $b$ , you get a solution of  $x = 1$ .

*Part B*

$a=6$ ;  $b = 15$ ; When you put these numbers in for  $a$  and  $b$ , you get a solution of  $0 = 0$ , so there are infinitely many solutions, not just one.

*Scoring Rubric:*

Responses to this item will receive 0-3 points, based on the following:

**3 points:** The student shows a thorough understanding of the number of possible solutions of linear equations. The student gives acceptable values for  $a$  and  $b$  in both parts and provides complete and correct explanations.

**2 points:** The student shows a partial understanding of the number of possible solutions of linear equations. The student must show at least some understanding in *Part B* to earn 2 points (e.g., *Part A* is incorrect and *Part B* is correct, or *Part A* and the values for  $a$  and  $b$  in *Part B* are correct but the explanation in *Part B* is incorrect).

**1 point:** The student shows a limited understanding of the number of possible solutions of linear equations. *Part A* is correct, but inconsistent or no understanding is shown in *Part B*.

**0 points:** The student shows inconsistent or no understanding of the number of possible solutions of linear equations.

## MAT.08.ER.3.000EE.E.138 Claim 3

Sample Item ID:	MAT.08.ER.3.000EE.E.138
Grade:	08
Primary Claim:	<b>Claim 3: Communicating Reasoning</b> Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	The Number System
Secondary Content Domain(s):	Expressions and Equations
Assessment Target(s):	3 E: Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument—explain what it is.  3 A: Test propositions or conjectures with specific examples.  1 B: Work with radicals and integer exponents.
Standard(s):	8.EE.2
Mathematical Practice(s):	3
DOK:	3
Item Type:	ER
Score Points:	2
Difficulty:	L
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Claim-specific Attributes (e.g., accessibility issues):	
Notes:	Part of PT set

Ashley and Brandon have different methods for finding square roots.

### Ashley's Method

To find the square root of  $x$ , find a number so that the product of the number and itself is  $x$ . For example,  $2 \cdot 2 = 4$ , so the square root of 4 is 2.

### Brandon's Method

To find the square root of  $x$ , multiply  $x$  by  $\frac{1}{2}$ . For example,  $4 \cdot \frac{1}{2} = 2$ , so the square root of 4 is 2.

Which student's method is **not** correct?

- Ashley's method  
 Brandon's method

Explain why the method you selected is **not** correct.

*Sample Top-Score Response:*

Brandon's method is not correct.

Brandon's method works for the square root of 4, but it wouldn't work for the square root of 36. Half of 36 is 18, but the square root of 36 is 6 since 6 times 6 equals 36. Ashley describes the correct way to find the square root of a number.

*Scoring Rubric:*

Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student shows a thorough understanding of how to identify correct reasoning regarding square roots. Each part of the response is complete and correct.

**1 point:** The student shows a partial understanding of how to identify correct reasoning regarding square roots. The student recognizes that Brandon's method is not correct but attempts to explain why Ashley's method is correct instead of showing why Brandon's method is not correct.

**0 points:** The student shows inconsistent or no understanding of how to identify correct reasoning regarding square roots. Responding only that Brandon's method is not correct is not sufficient to earn any points.

## MAT.08.ER.3.000NS.A.136 Claim 3

Sample Item ID:	MAT.08.ER.3.000NS.A.136
Grade:	08
Primary Claim:	<b>Claim 3: Communicating Reasoning</b> Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	The Number System
Secondary Content Domain(s):	
Assessment Target(s):	3 A: Test propositions or conjectures with specific examples.  1 A: Know that there are numbers that are not rational, and approximate them by rational numbers.
Standard(s):	8.NS.1
Mathematical Practice(s):	1, 5, 7, 8
DOK:	2
Item Type:	ER
Score Points:	2
Difficulty:	M
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Claim-Specific Attributes (e.g., accessibility issues):	Calculators may be used for this item.
Notes:	The purpose of this item is to test whether a student understands rational and irrational numbers.

A student made this conjecture and found two examples to support the conjecture.

If a rational number is not an integer, then the square root of the rational number is irrational. For example,  $\sqrt{3.6}$  is irrational and  $\sqrt{\frac{1}{2}}$  is irrational.

Provide two examples of non-integer rational numbers that show that the conjecture is **false**.

Example 1:

Example 2:

*Sample Top-score Response:*

Example 1: 2.25    Example 2:  $\frac{1}{4}$

This item will require handscoring and verification of each response, independently.

Responses to this item will receive 1 point for each correct example.

## MAT.08.ER.3.000SP.B.134 Claim 3

Sample Item ID:	MAT.08.ER.3.000SP.B.134
Grade:	08
Primary Claim:	<b>Claim 3: Communicating Reasoning</b> Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.  Claim 2: Problem Solving Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.
Primary Content Domain:	Statistics and Probability
Secondary Content Domain(s):	
Assessment Target(s):	3 B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.  1 J: Investigate patterns of association in bivariate data.  2 A: Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.  2 D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).
Standard(s):	8.SP.4
Mathematical Practice(s):	1, 2, 3, 5, 6, 7
DOK:	3
Item Type:	ER
Score Points:	4
Difficulty:	M
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Target-Specific Attributes (e.g., accessibility issues):	Calculators may be used for this item.
Notes:	The table consists of a response space in each blank cell. Each response space allows for a maximum of 2 digits.

Jacob surveyed 25 adults to ask whether they had at least one child under the age of 18 and whether they had at least one pet. This table shows the results of the survey.

Adult	At Least One Child Under the Age of 18		At Least One Pet	
	Yes	No	Yes	No
A	✓		✓	
B		✓		✓
C	✓			✓
D		✓		✓
E		✓	✓	
F		✓		✓
G		✓	✓	
H	✓		✓	
I	✓		✓	
J	✓			✓
K		✓		✓
L	✓		✓	
M		✓	✓	
N	✓		✓	
O		✓		✓
P	✓		✓	
Q		✓		✓
R	✓		✓	
S		✓		✓
T	✓		✓	
U		✓	✓	
V	✓		✓	
W		✓		✓
X		✓	✓	
Y		✓	✓	

**Part A**

Use the results of the survey to complete this table.

At Least One Pet	At Least One Child Under the Age of 18	
	Yes	No
Yes		
No		

**Part B**

Jacob made the conjecture that there is a possible association between whether an adult has at least one child under the age of 18 and whether the adult has at least one pet.

State whether the results of the survey provide evidence that adults who have at least one child under the age of 18 also tend to have at least one pet. Explain your answer.

*Sample Top-Score Response:*

*Part A:*

At Least One Pet	At Least One Child Under the Age of 18	
	Yes	No
Yes	9	6
No	2	8

*Part B:*

Yes, there is evidence that the adults who have at least one child under the age of 18 also tend to have at least one pet. I found the relative frequencies for whether the adult had at least one pet or not given that the adult had at least one child and then given that the adult did not have any children, and 82% of the adults who had at least one child also had at least one pet. My work is shown below.

At Least One Pet	At Least One Child Under the Age of 18		Totals
	Yes	No	
Yes	$\frac{9}{11} \approx 82\%$	$\frac{6}{14} \approx 43\%$	$\frac{15}{25} \approx 60\%$
No	$\frac{2}{11} \approx 18\%$	$\frac{8}{14} \approx 57\%$	$\frac{10}{25} \approx 40\%$

*Scoring Rubric:*

Responses to this item will receive 0-4 points, based on the following:

*Part A*

**2 points:** The student thoroughly understands how to construct two-way tables. The student's response in *Part A* matches the sample top-score response.

**1 point:** The student shows a partial understanding of how to construct two-way tables. The student makes one or two minor mathematical errors when constructing the table.

**0 points:** The student shows inconsistent or no understanding of how to construct two-way tables. The student makes several mathematical errors when constructing the table.

or one or more conceptual errors.

*Part B*

**2 points:** The student thoroughly understands how to interpret two-way tables. The student uses relative frequencies or a similar method in an explanation that corresponds with the table the student provided in *Part A*.

**1 point:** The student shows a partial understanding of how to interpret two-way tables. The student uses the numbers in the table without using relative frequencies or a similar method in an explanation that corresponds with the table the student provided in *Part A*.

**0 points:** The student shows inconsistent or no understanding of how to interpret two-way tables. The student does not use the table to help construct the explanation OR the student writes an explanation that does not correspond with the table the student provided in *Part A*.

## MAT.08.PT.4.APARK.A.270

Sample Item ID:	MAT.08.PT.4.APARK.A.270
Title:	Designing a Park
Grade:	08
Primary Claim:	<b>Claim 4: Modeling and Data Analysis</b> Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.
Secondary Claim(s):	<p>Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.</p> <p>Claim 2: Problem Solving Students can solve a range of complex, well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.</p> <p>Claim 3: Communicating Reasoning Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.</p>
Primary Content Domain	<b>Geometry</b>
Secondary Content Domain(s):	Statistics and Probability, Ratios, and Proportions
Assessment Target(s):	<p>4 A: Apply mathematics to solve problems arising in everyday life, society, and the workplace.</p> <p>1 H: Understand and apply the Pythagorean theorem.</p> <p>1 J: Investigate patterns of association in bivariate data.</p> <p>1 A (Gr 6): Understand ratio concepts and use ratio reasoning to solve problems.</p> <p>2 B: Select and use appropriate tools strategically.</p> <p>3 F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions.</p> <p>4 B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.</p> <p>4 G: Identify, analyze, and synthesize relevant external resources to pose or solve problems.</p>
Standard(s):	8.G.7, 8.SP.4, 6.RP.3
Mathematical Practice(s):	1, 2, 3, 5, 6, 7
DOK:	4
Item Type:	PT
Score Points:	15
Difficulty:	H
How this task addresses the "sufficient evidence" for this claim:	The student uses concepts of geometry, number and operations, and statistics to determine the best solution to a problem where all constraints cannot be satisfied at the same time. Additionally, the student must provide justifications to support reasoning.
Target-Specific Attributes (e.g., accessibility issues):	Accommodations may be necessary for students who have visual or psychomotor challenges. Calculator and spreadsheet needed.
Stimulus/Source:	Various websites giving estimates for the cost of constructing different facilities
Notes:	Multi-part task with separate scoring rubrics for each part.
Task Overview:	During the task, the student assumes the role of an architect who is responsible for designing the best plan for a park with area and financial restraints. The student completes tasks in which he/she compares the costs of different bids, determines what facilities should be given priority in the park, and then

Grade 8 Mathematics Sample PT Form



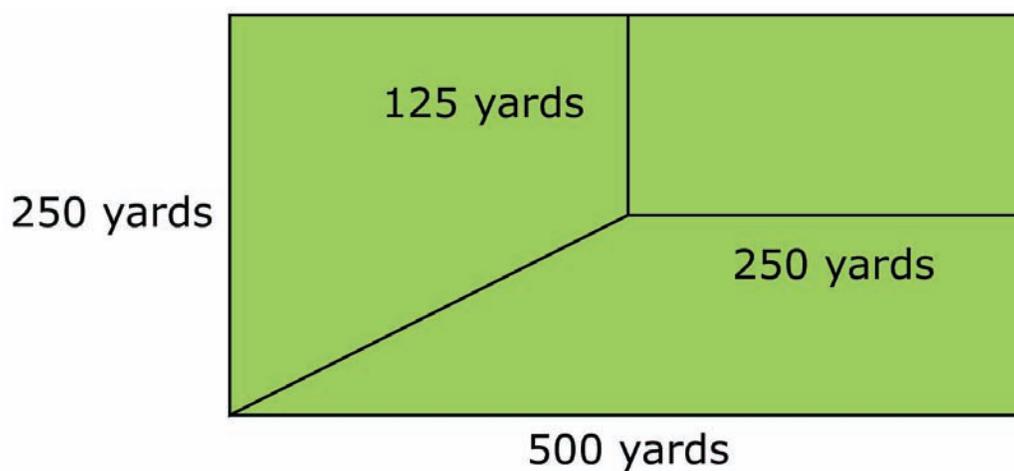
	develops a scale drawing of the best design for the park and an explanation of the choices made. This investigation is done in class using a calculator, an applet to construct the scale drawing, and a spreadsheet.
Teacher Preparation / Resource Requirements:	Teacher preparation: During Session 1, students will be expected to make a number of calculations that will require the use of calculators, as well as to evaluate the results of a survey. During Session 2, students will be expected to develop a scale drawing of the design of a park and explain the reasons for their choices in the park.
Teacher Responsibilities During Administration:	Monitor individual student work; provide resources as necessary.
Time Requirements:	Two sessions of scored individual work totaling no more than 120 minutes.
Prework: none	

**Session 1****Simpson Park**

The town of Simpson received a grant of \$1,550,000 to develop a park.

- The park will be developed on an open rectangular plot of land that is 500 yards long by 250 yards wide.
- Three paths will be built in the park that all connect at the center of the park.

The layout of the park and the paths are shown in the diagram below. The lengths of only two of the paths are given.



The grant includes the following requirements:

- Three benches must be placed where the three paths intersect in the center of the park. Additional benches must be placed 25 yards apart along the paths in the park (or as close to 25 yards as possible).
  - A light (lamppost) must be installed next to each bench in the park.
  - Two buildings that each contain men's and women's bathrooms must be built in the park.
  - At least 45% of the park must remain undeveloped.
1. Three construction companies sent in bids for the required work in the park. (A bid explains what a company would charge for the work to be completed.) The bids are shown below.

### AGUILAR CONSTRUCTION

Based on the information given, our cost estimates for the proposed new park in Simpson are as follows.

Item	Price per Unit
Path construction	\$30 per yard of the path
Lighting	\$225 per lamppost installed
Benches	\$275 per bench installed
Bathroom	\$120,750 per bathroom

Thank you for your consideration. We look forward to working with you on this project.

### BENEDETTO & SON, INC.

Thank you for contacting us for a quote regarding the upcoming work on the new park in Simpson. Our highly qualified staff has assembled the following quote for you.

- \$20 per yard of path constructed
- \$335 per lamppost installed
- \$365 per bench installed
- \$108,300 per bathroom built

Please do not hesitate to contact us if we can be of further assistance. We look forward to getting construction underway.

**COLEMAN CONTRACTORS**

We appreciate the opportunity to present you the following estimate. For the sake of simplicity, we have provided the total for each aspect of the project to enable you to compare our bid with the bids of others.

<b>Item</b>	<b>Total Price</b>
Path construction	\$17,500
Lighting	\$9000
Benches	\$9000
Bathrooms	\$225,500

We trust that our bid will be competitive, and we anticipate your quick response.

Determine the total cost of each bid and identify which company sent in the bid that will cost the least amount of money. Show your work and explain how you found your answer.

The park planning committee conducted a random survey of 200 people to find out how often they would use different facilities if they were available at the park. The results of the survey are shown below.

	<b>3 or More Times per Week</b>	<b>Once or Twice per Week</b>	<b>Once or Twice per Month</b>	<b>A Few Times per Year</b>	<b>Never</b>
<b>Basketball Courts</b>	40	47	22	21	70
<b>Volleyball Courts</b>	11	9	11	40	129
<b>Turf Field</b>	37	50	45	15	53
<b>Running Track</b>	35	63	42	33	27
<b>Tennis Courts</b>	9	19	36	34	102
<b>Baseball Field</b>	12	31	24	50	83
<b>Softball Field</b>	27	43	52	14	64
<b>Playground</b>	36	81	43	21	19
<b>Dog Playground</b>	28	18	19	18	117
<b>Picnic Pavilion</b>	17	16	22	53	92
<b>Pond</b>	7	9	9	19	156
<b>Community Garden</b>	5	8	14	12	161

Based on the results of the survey, you need to decide which facilities would be used the most and which facilities would be used the least when the park is developed.

2. Based on the results of the survey, which 4 facilities would be used the most? Explain your reasoning.

3. Based on the results of the survey, which 4 facilities would be used the least? Explain your reasoning.

During the next session, you will use the costs of the different facilities and the areas they occupy to help you make a final decision about the best way to develop Simpson Park.

## This is the end of Session 1

### Session 2



#### Home Page

Today you will use the costs of the different facilities and the areas they occupy to help you make a final decision about the best way to develop the park so it will get used as much as possible.

At the top of the screen, you will see tabs<sup>1</sup> for 6 pages. You are currently on the **Home** page.

On the **Survey** page, you will find the results of the survey conducted to find how often people would use different facilities if they were available at the park.

On the **Info** page, you will find the costs and dimensions of the different facilities, as well as reminders of what needs to be done.

On the **Spreadsheet** page, you will record the facilities you add to the park and the cost of each. You may want to decide on all

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<sup>1</sup> The display for Session 2 will show the tabs at the top of the page. When students click on a tab, they will be taken to the descriptions for each page laid out in this session.

the facilities that should be included in the park before constructing a scale drawing on the **Model** page.

On the **Model** page, you will add facilities to a scale drawing of the park to show what the final developed park will look like. The bathrooms have already been added to the park.

On the **Conclusion** page, you will write an explanation of why you chose to develop the park as you did so it will get used as much as possible.

The score you receive for this task will be based on the work you enter on the **Model**, **Spreadsheet**, and **Conclusion** pages. There are multiple correct answers to this task, so do your best and have fun!



## Survey Page

The park planning committee surveyed 200 people at random to find out how often they would use different facilities if they were available at the park. The results of the survey are shown below.

	3 or More Times per Week	Once or Twice per Week	Once or Twice per Month	A Few Times per Year	Never
<b>Basketball Courts</b>	40	47	22	21	70
<b>Volleyball Courts</b>	11	9	11	40	129
<b>Turf Field</b>	37	50	45	15	53
<b>Running Track</b>	35	63	42	33	27
<b>Tennis Courts</b>	9	19	36	34	102
<b>Baseball Field</b>	12	31	24	50	83
<b>Softball Field</b>	27	43	52	14	64
<b>Playground</b>	36	81	43	21	19
<b>Dog Playground</b>	28	18	19	18	117
<b>Picnic Pavilion</b>	17	16	22	53	92
<b>Pond</b>	7	9	9	19	156
<b>Community Garden</b>	5	8	14	12	161

Home

Survey

Info

Spreadsheet

Model

Conclusion

## Info Page

Things to keep in mind as you develop the park:

- You have \$1,300,000 remaining to spend on developing the park. Your decisions should bring the total cost as close to \$1,300,000 as possible without going over that amount.
- The paths, benches, and bathrooms have taken up 3750 square yards of the park already.
- At least 45% of the park must remain undeveloped.
- The results of the survey must be used to help you determine the facilities to build in the park so it will get used as much as possible.

Park Facility	Estimated Cost	Length (yards)	Width (yards)
4 Basketball Courts	\$200,000	100	35
2 Volleyball Courts	\$75,000	35	25
Turf Field	\$900,000	130	90
Running Track	\$380,000	200	130
Turf Field with Running Track	\$1,280,000	200	130
4 Tennis Courts	\$200,000	100	40
Baseball Field	\$100,000	120	120
Softball Field	\$75,000	100	100
Playground	\$70,000	50	25
Dog Playground	\$30,000	100	25
Picnic Pavilion	\$70,000	20	15
Pond	\$30,000	20	20
Community Garden	\$10,000	50	50

Home

Survey

Info

Spreadsheet

Model

Conclusion

## Spreadsheet Page

This page is to help you keep track of the total area and cost of the facilities. Remember to take into account the area that must be taken up by the paths, benches, bathrooms, and the area that must remain undeveloped. You may want to decide on all the facilities that should be included in the park before constructing the model.

	A	B	C	D	E	F	G
1							
2			Estimated Cost	Length (yards)	Width (yards)		
3		Open Areas					
4		Paths, Benches, and Bathrooms					
5		4 Basketball Courts	\$200,000	100	35		
6		2 Volleyball Courts	\$75,000	35	25		
7		Turf Field	\$900,000	130	90		
8		Running Track	\$380,000	200	130		
9		Turf Field with Running Track	\$1,280,000	200	130		
10		4 Tennis Courts	\$200,000	100	40		
11		Baseball Field	\$100,000	120	120		
12		Softball Field	\$75,000	100	100		
13		Playground	\$70,000	50	25		
14		Dog Playground	\$30,000	100	25		
15		Picnic Pavilion	\$70,000	20	15		
16		Pond	\$30,000	20	20		
17		Community Garden	\$10,000	50	50		
18							
19							
20							
21							
22							

Note: Items may be added to and removed from the **Spreadsheet** at any time until you click **“Submit.”**

Before you continue to the **Model** page, think about the following questions.

- Have you spent more than the \$1,300,000 you had remaining?
- Will your choices of facilities lead to the park getting used as much as possible (according to the survey)?

You may need to return to this page several times to adjust your choices as you try to make all the facilities fit on the scale model.

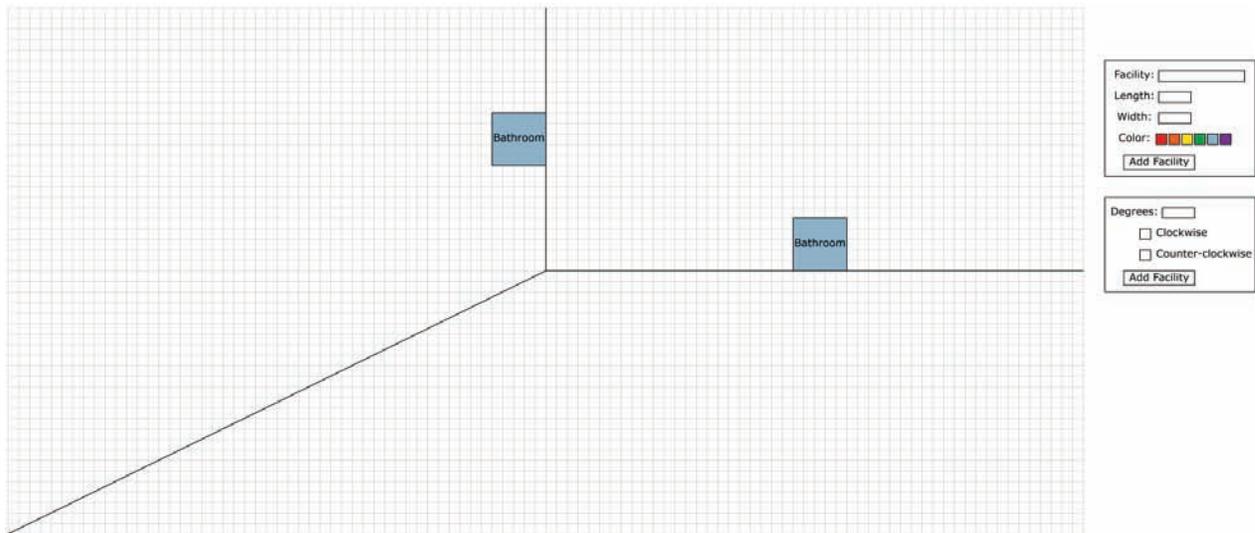


### Model Page

When you decide to add an item to the park, you must take the following actions.

- When prompted, enter the name of the facility. Then enter its length and width. Then choose a color for the facility. (For example, the bathrooms are blue.)

- Click on the “Add Facility” button.
- The facility will appear below the “Rotate” menu. Use the arrow keys to move the facility to the desired location in the park.
- Use the “Rotate” menu to turn a facility clockwise or counterclockwise, as needed.



Note: Items may be added to and removed from the **Model** at any time until you click “**Submit.**”

Remember to think about the requirements of the grant. You may go back to any page in this session to review the information.



## Conclusion Page

Explain why you chose to develop the Simpson Park in the way you did. Be sure to include both the information and the mathematics used to support your decisions in your response. To be considered complete, your explanation must address the following:

- Reasons for including and excluding certain facilities (related to space, costs, and survey results)
- Mathematics that shows that the requirements of the grant were met in the design of the park

*Sample Top-Score Response:***Question 1**

First I need to determine the total length of the path and the number of benches needed.

For the length of the path, one section is  $250 \div 2 = 125$  yards long, and another section is  $500 \div 2 = 250$  yards long. I need to use the Pythagorean theorem to find the length of the third section.

$$a^2 + b^2 = c^2$$

$$125^2 + 250^2 = c^2$$

$$15625 + 62500 = c^2$$

$$78125 = c^2$$

$$c \approx 280$$

The length of the third section is about 280 yards, so the total length of the path is 655 yards.

The benches are located about every 25 yards, so the first section needs 5 benches, the second section needs 10, and the third section needs 11, since  $280 \div 25 \approx 11.2$ . There are also 3 benches in the middle at the intersection of the paths for a total of 29 benches.

Also, a light is needed by each bench, so that is 29 lights, and there will be 2 bathroom facilities.

I will multiply the cost per yard of the path by the number of yards in the path, I will add the cost per light and the cost per bench and multiply the sum by 29 because there are 29 of each, and I will multiply the cost per bathroom by the number of bathrooms, and then add that all up together. Or, in the case of Coleman Contractors, I will just add the numbers given.

So, for Aguilar Construction, the cost is:

$$30 \times 655 = \$19,650$$

$$225 + 275 = 500$$

$$500 \times 29 = \$14,500$$

$$2 \times 120750 = \$241,500$$

$$19650 + 14500 + 241500 = \$275,650$$

For Benedetto & Son, the cost is:

$$20 \times 655 = \$13,100$$

$$335 + 365 = 700$$

$$700 \times 29 = \$20,300$$

$$2 \times 108,300 = \$216,600$$

$$13100 + 20300 + 216600 = \$250,000$$

For Coleman Contractors, the cost is:

$$17500 + 9000 + 9000 + 225,500 = \$261,000$$

So Benedetto & Son costs the least at \$250,000. This is the bid that should be accepted.

**Question 2**

The four facilities that should have the highest priority are the playground, the track, the basketball courts, and the turf field, in that order. These 4 facilities had the 4 highest numbers of people who said they would use them 3 or more times per week, and they also had the 4 highest numbers of people who said they would use them once or twice per week. They also had 4 of the 5 lowest numbers of people who said they would never use them. So it seems like these are the facilities that people would use most.

**Question 3**

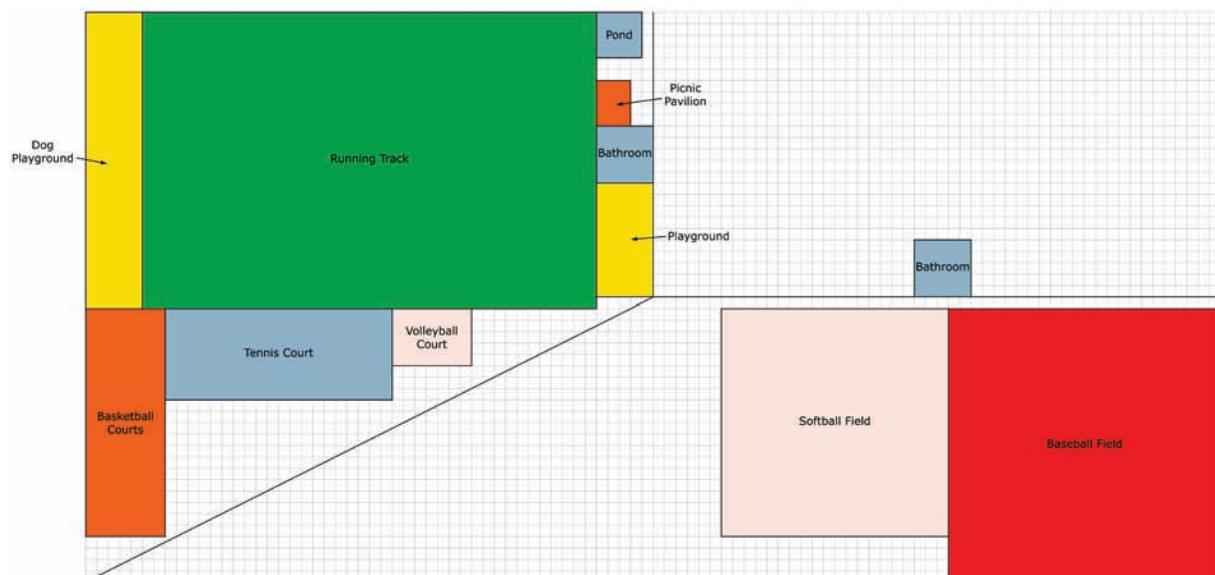
The four facilities that should have the lowest priority are the community garden, the pond, the volleyball courts, and the tennis courts. These facilities had the 4 lowest numbers of people who said they would use them 3 or more times per week, and they also had 4 of the

6 lowest numbers of people who said they would use them once or twice per week. For the people who said never, they had 4 of the 5 highest numbers. This is a little harder than #2 because in some ways the baseball field, the dog playground, and the picnic pavilion are close, but I think the ones I mentioned in the beginning are wanted a little bit less than these ones.

**Spreadsheet**

	A	B	C	D	E	F	G
1							
2			<b>Estimated Cost</b>	<b>Length (yards)</b>	<b>Width (yards)</b>	<b>Area</b>	
3		Open Areas				56,250	
4		Paths, Benches, and Bathrooms				3750	
5		4 Basketball Courts	\$200,000	100	35	3500	
6		2 Volleyball Courts	\$75,000	35	25	875	
7							
8		Running Track	\$380,000	200	130	26000	
9							
10		4 Tennis Courts	\$200,000	100	40	4000	
11		Baseball Field	\$100,000	120	120	14400	
12		Softball Field	\$75,000	100	100	10000	
13		Playground	\$70,000	50	25	1250	
14		Dog Playground	\$30,000	100	25	2500	
15		Picnic Pavilion	\$70,000	20	15	300	
16		Pond	\$30,000	20	20	400	
17							
18		<b>Total used</b>	<b>\$1,230,000</b>			<b>123225</b>	
19							
20		<b>Amount available</b>	<b>\$1,300,000</b>			<b>125000</b>	
21							

**Model**



**Conclusion**

First I calculated the area of each facility on the spreadsheet. I also included the minimum area that needed to remain open space, and the space taken up by the paths, benches, and bathrooms. I found the total cost and the total area, and I saw that my total cost was way more than \$1,300,000, and my area was more than 125,000 square yards also.

Then I decided to make up my mind about the track and turf field, because they took up a lot of area, they very expensive, and they were confusing on the chart since things were mentioned more than once. If I included the track and turf field, I would have only \$20,000 to do anything else, which didn't seem smart. Since cost was over by a lot more than area, I thought cost should be a bigger concern than area. I decided to eliminate the turf field since it cost so much and it seemed like people wanted the track just as much. So I eliminated the turf field and turf field with running track lines from my spreadsheet, leaving me with just the running track. This helped simplify things.

To my surprise, I was already under budget, but I still had a little too much area. I noticed that the community garden took up a lot of area, was pretty cheap, and not many people wanted it, so I deleted that and saw that I was within the limits for money and area. I was not using some of the money, but I figured the foundation could give it to someone else.

Then it was time to fit everything into the park. Since there was only 125 yards on either side of the path on the right hand side of the park, I had to put the running track on the left-hand side of the park, so I did that first. Then I wanted to put as many facilities there as I could to keep the open parts of the park together. I noticed that the basketball courts just fit vertically below the track, so I put them there and then was able to fit the tennis and volleyball courts below the track as well. I saw that the dog playground was pretty long and it would fit next to the track, but I moved that out to the edge of the park so people walking their dogs could just go into the dog playground. Plus the track was then closer to the bathroom. There was room just below the bathroom for the playground, which seemed like a good idea, and then I was able to fit the picnic pavilion close to the bathroom and the pond above that.

All I had left then were the baseball and softball fields. The baseball field wouldn't fit about the path on the right hand side of the park, so I put both of them below that path to preserve the open space.

*Scoring Notes:*

Each scored portion of the task is evaluated individually. The total number of points is determined by adding the points assigned for each task.

*Scoring Rubric:*

**Question 1:** Responses to this item will receive 0-4 points, based on the following:

**4 points:** The student shows a thorough understanding of how to perform the calculations needed to find the cost of the park requirements. The student uses the Pythagorean theorem to find the length of one path and performs the other calculations correctly.

**3 points:** The student shows a strong understanding of how to perform the calculations needed to find the cost of the park requirements. The student uses the Pythagorean theorem to find the length of one path but makes one or two minor computational errors when performing the other calculations.

**2 points:** The student shows a partial understanding of how to perform the calculations needed to find the cost of the park requirements. The student shows some conceptual understanding of the calculations needed to answer the question correctly but makes one conceptual error in the process of solving the problem.

**1 point:** The student shows a limited understanding of how to perform the calculations needed to find the cost of the park requirements. The student shows some conceptual understanding of the calculations needed to answer the question correctly but makes multiple conceptual errors in the process of solving the problem.

**0 points:** The student shows inconsistent or no understanding of how to perform the calculations needed to find the cost of the park requirements. The student does not apply the Pythagorean theorem correctly and makes numerous conceptual and computational errors when performing other calculations.

**Question 2:** Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student shows a thorough understanding of how to interpret the table to determine the facilities that should be of highest priority. The student correctly designates high-priority facilities and provides a complete and correct explanation of the reasons why those facilities were of high priority.

**1 point:** The student shows a partial understanding of how to interpret the table to determine the facilities that should be of highest priority. The student provides an explanation that neglects some aspects of the information in the table.

**0 points:** The student shows inconsistent or no understanding of how to interpret the table to determine the facilities that should be of highest priority. The student identifies multiple facilities as high priority that are not and provides an explanation that does not address the key parts of the table necessary for interpreting the information shown.

**Question 3:** Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student shows a thorough understanding of how to interpret the table to determine the facilities that should be of lowest priority. The student correctly designates low-priority facilities and provides a complete and correct explanation of the reasons why those facilities were of low priority.

**1 point:** The student shows a partial understanding of how to interpret the table to determine the facilities that should be of lowest priority. The student provides an explanation that neglects some aspects of the information in the table.

**0 points:** The student shows inconsistent or no understanding of how to interpret the table to determine the facilities that should be of lowest priority. The student identifies multiple facilities as high or low priority (that are not) and provides an explanation that does not address the key parts of the table necessary for interpreting the information shown.

**Spreadsheet:** Responses to this item will receive 0-1 point, based on whether the student enters information into the spreadsheet correctly.

**Scale drawing:** The scale drawing will receive 0-3 points, based on the following:

**3 points:** The student shows a thorough understanding of how to create a scale drawing of the park. The student includes all of the facilities indicated in the spreadsheet and the conclusion and represents them correctly in the scale drawing.

**2 points:** The student shows a partial understanding of how to create a scale drawing of the park. The student creates a scale drawing that is generally correct but incorrectly includes one facility, incorrectly excludes one facility, or makes one or two errors in the representation of the facilities in the scale drawing.

**1 point:** The student shows a limited understanding of how to create a scale drawing of the park. The student creates a scale drawing that is somewhat correct but contains multiple errors, such as including extra facilities, excluding facilities that should be included, or making several errors in the representation of the facilities in the scale drawing.

**0 points:** The student shows inconsistent or no understanding of how to create a scale drawing of the park. The student creates a scale drawing that has little connection to the spreadsheet or the conclusion and contains numerous errors in the representation of the facilities in the scale drawing.

**Conclusion:** Responses to this item will receive 0-3 points, based on the following:

**3 points:** The student shows a thorough understanding of how to analyze a complex scenario and construct a chain of reasoning to justify the solution to a scenario. The student includes all the different aspects of the task in the justification and provides evidence to support the conclusion made.

**2 points:** The student shows a partial understanding of how to analyze a complex scenario and construct a chain of reasoning to justify the solution to a scenario. The student includes many of the different aspects of the task in the justification and provides evidence to support the conclusion made but utilizes nonmathematical justifications in the explanation.

**1 point:** The student shows a limited understanding of how to analyze a complex scenario and construct a chain of reasoning to justify the solution to a scenario. The student includes some of the different aspects of the task in the justification but makes a conclusion that would not have been made if all of the aspects of the task had been considered.

**0 points:** The student shows inconsistent or no understanding of how to analyze a complex scenario and construct a chain of reasoning to justify the solution to a scenario. The student makes a conclusion that is contradictory to the aspects of the task considered in the justification.

## MAT.08.PT.4.MYPET.A.415

Sample Item ID:	MAT.08.PT.4.MYPET.A.415
Title:	Cost of a Pet
Grade:	08
Primary Claim:	<p><b>Claim 4: Modeling and Data Analysis</b> Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.</p>
Secondary Claim(s):	<p>Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.</p> <p>Claim 2: Problem Solving Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.</p>
Primary Content Domain:	Statistics and Probability
Secondary Content Domain(s):	Functions
Assessment Target(s):	<p>4 A: Apply mathematics to solve problems arising in everyday life, society, and the workplace.</p> <p>1 C: Understand the connections between proportional relationships, lines, and linear equations.</p> <p>1 F: Use functions to model relationships between quantities.</p> <p>1 D (Gr 7): Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</p> <p>1 A (Gr 6): Understand ratio concepts and use ratio reasoning to solve problems.</p> <p>1 D (Gr 5): Perform operations with multi-digit whole numbers and with decimals to hundredths.</p> <p>2 B: Select and use appropriate tools strategically.</p> <p>2 C: Interpret results in the context of a situation.</p> <p>4 B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.</p> <p>4 D: Interpret results in the context of a situation.</p> <p>4 E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon.</p>

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	<p>4 F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).</p> <p>4 G: Identify, analyze and synthesize relevant external resources to pose or solve problems.</p>
Standard(s):	8.EE.6, 8.F.4, 7.EE.3, 6.RP.2, 5.NBT.7
Mathematical Practice(s):	1, 2, 4, 5, 6
DOK:	4
Item Type:	PT
Score Points:	11
Difficulty:	M
How this task addresses the “sufficient evidence” for this claim:	The student must use information derived from research to estimate the costs to adopt and maintain a pet. This work will be supported by the use of calculations, graphical representation of data, and generalizations using algebra.
Target-Specific Attributes (e.g., accessibility issues):	Accommodations may be necessary for students who have visual challenges. Technology is needed to enable students to create a circle graph, bar graph, or line graph.
Stimulus/Source:	<a href="http://www.petsmart.com">www.petsmart.com</a>
Notes:	Multi-part task
Task Overview:	<p>Prework: In groups or as a whole class, students brainstorm what items are needed to maintain a pet over time.</p> <p>Day 1: With partners, students decide which type of pet they want to adopt. Students use a set of provided “Web sites” to look up the costs of necessary items for the chosen pet.</p> <p>Day 2: Students individually estimate the cost of adopting and maintaining their chosen pet for 1 year. Students explain why their estimate is reasonable. As part of the explanation, the student must make and refer to a line graph showing the monthly increase in money spent over the year.</p>
Teacher Preparation / Resource Requirements:	Teacher preparation: The day before the administration of the task, there is prework that is designed to encourage students to begin thinking about the upcoming task. The prework involves a class brainstorming session and discussion. The remainder of the task involves a group assignment for one day followed by scored portions of the task on the second day. During Session 1, students work with partners to develop a list of items that are needed to own a pet and the costs of those items. During Session 2, students work individually to estimate the cost of raising a pet for the first and second year and the relationship between time and cost. Students provide justifications for their answers.
Teacher Responsibilities During Administration:	Monitor individual student work; provide resources as necessary.
Time Requirements:	One pretask session that requires no more than 15 minutes of class discussion. Group work follows (45 minutes), directed by

	the teacher and prior to the beginning of the scored portion of the task. The scored portion of the task starts with <i>Part A</i> . The total time involved should be 15 minutes (pretask) + 45 minutes (group work) + 60 minutes (scored portion).
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Prework: Class discussion, facilitated by teacher, no more than 15 minutes.

Students brainstorm ideas for items that are needed to maintain a pet over time. At the end of a class discussion, the teacher should elicit from students a list of items, possibly including food, toys, a cage/carrier/bed, a leash or collar, veterinary visits, litter box. The teacher should note that not all of these items are needed for all types of animals.

### Teacher Preparation:

[Note: Students will be grouped in partners or small groups. If possible, a student who owns or has owned a pet should be placed in each group.]

### Teacher says:

Today you are going to do some research about the cost of owning a pet. You will create a list of items that you will need for your pet and the cost of each item. You will need this list and associated costs in order to complete the upcoming performance task.

First, choose the type of pet that you [by partner or group] would like to research. The choices are a dog, a cat, a hamster, or a parakeet.

These links will direct you to Web pages that might be useful to you: [Note that all of the Web sites below are fictitious, and would need to be developed. A PowerPoint file shows some examples of what the Web sites would look like as part of the fictitious cached file available with this task.]

[PlusmaxPetSupplies.com](#) [sells pet supplies only; gives info about how much and how often to feed pets]

[AndreasPetShop.com](#) [sells pets, and some (but not all) supplies]

[PetFacts.info](#) [gives info on what items you need to have a pet]

[MeAndMyPets.com](#) [a personal blog about someone who has many pets; this site will not contain any useful information for students]

[AnimalRescueGroup.org](http://AnimalRescueGroup.org) [students can use this site to choose to adopt a pet instead of buy one]

[DowntownVeterinaryCare.com](http://DowntownVeterinaryCare.com) [a veterinarian's site that will give estimates on how often a pet should go to the veterinarian and what a typical visit might cost]

[HappyDayPettingZoo.com](http://HappyDayPettingZoo.com) [another site without any useful info; this site will have the hours and prices for visiting a local petting zoo]

Create a list of items you will need for your pet and the prices of the items.

## **Cost of Owning and Caring for a Pet**

### ***Part A***

Enter the list of pet items and costs you created yesterday, including the initial cost to buy or adopt your pet.

Based on your research, estimate the cost of adopting and maintaining a pet <dog, cat, hamster, parakeet> for one year.

You may click on the links to the Web pages you looked at yesterday to help you revise or adjust your list in order to determine the cost for an entire year.

**Part B**

What do you estimate the cost of adopting and maintaining a pet <dog, cat, hamster, parakeet> for one year is?

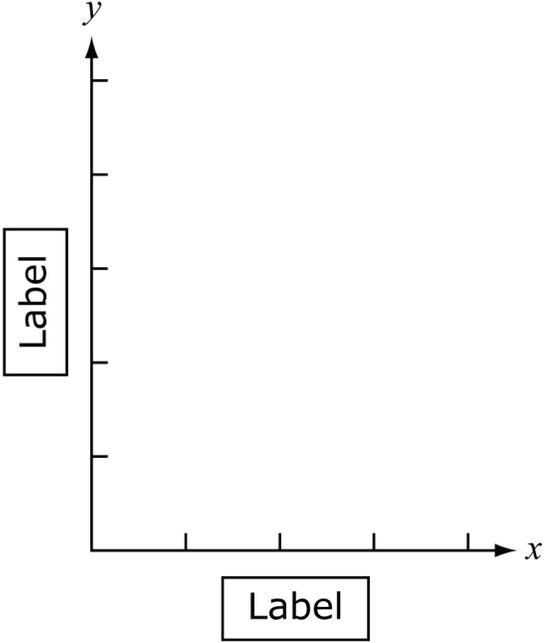
\$

Provide a thorough explanation below that proves you have thought of all the costs associated with owning your choice of pet. Be sure to account for all one-time costs, as well as recurring costs and the frequency of those costs. Adjust your total above if needed.

Next, support your answer by using the line graph tool to create a line graph that shows the months of the year and the total amount of money you have spent on your pet through the end of each month. Your graph should show how the amount of money you have spent increases over time.

x-minimum	<input type="text" value="0"/>
x-maximum	<input type="text" value="12"/>
x-scale	<input type="text" value="2"/>
y-minimum	<input type="text" value="0"/>
y-maximum	<input type="text" value="50"/>
y-scale	<input type="text" value="10"/>

Data Points:



**Part C**

Estimate the cost of maintaining your pet <dog, cat, hamster, parakeet> for *one additional year* after the first.

\$

Again, provide a thorough explanation below that proves you have thought of all the costs associated with owning your choice of pet for one *additional year*.

**Part D**

Write a linear equation that best fits the total cost of adopting and maintaining your pet for a given number of *months*. Assume that your pet will not need to go to the veterinarian throughout this time.

Equation:

Explain how you created your model equation. Be sure to describe what the variables represent and how you determined the slope and intercept for your equation.

**Sample Top-Score Response:**

[Note: This response is based on a student choosing a hamster.]

**Group work:**

My group researched the cost of owning a hamster. (Group work not scored.)

**Part A**

Items needed:

- Carefresh Basic Pet Bedding (14-liter bag) \$5.99

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- All Living Things Water Bottles for Small Animals \$3.99
- All Living Things Hamster Starter Kit \$22.94
- Great Choice Hamster & Gerbil Food (5 pounds) \$6.99
- Super Pet Small Animal Run-About Ball \$7.99
- Super Pet CritterTrail Puzzle Playgrounds \$8.99
- Winter White Hamster \$15.99

### Part B

The cost of adopting and maintaining a pet hamster for one year is \$231.62.

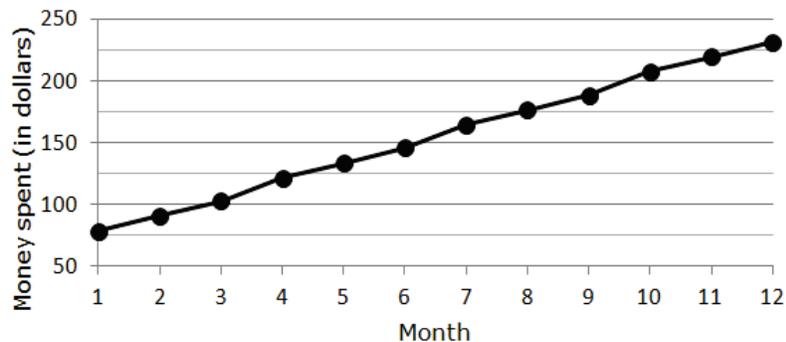
x-minimum	<input type="text" value="1"/>
x-maximum	<input type="text" value="12"/>
x-scale	<input type="text" value="1"/>
y-minimum	<input type="text" value="50"/>
y-maximum	<input type="text" value="250"/>
y-scale	<input type="text" value="25"/>

Data Points:

(1, 78.87)	(7, 164.73)
(2, 90.85)	(8, 176.71)
(3, 102.83)	(9, 188.69)
(4, 121.80)	(10, 207.66)
(5, 133.78)	(11, 219.64)
(6, 145.76)	(12, 231.62)

[Click to add another data point](#)

### Total Amount of Money Spent on Hamster after Each Month



Explanation:

I know that my estimation is accurate because it includes the costs of all items that I would need to adopt and maintain a pet hamster for one year (assuming that my hamster does not need to go to the veterinarian). I looked on a pet supply Web site and on a pet shop Web site in order to find the costs of the items needed.

I first found all of the one-time costs:

- All Living Things Water Bottles for Small Animals \$3.99
- All Living Things Hamster Starter Kit \$22.94
- Super Pet Small Animal Run-About Ball \$7.99
- Super Pet CritterTrail Puzzle Playgrounds \$8.99

## Grade 8 Mathematics Sample PT Form

- Winter White Hamster \$15.99

Then I found the annual costs for bedding and food; I calculated the number of bags of each that I would need to buy for one year, based on the pet supply Web site's information.

- Carefresh Basic Pet Bedding (14-liter bag) \$5.99
  - The directions listed on the Web site are: Replace bedding 4 times each month. Each time use 7 liters (1/2 a bag) of bedding in the bottom of your hamster or gerbil's cage.
  - So, we would need to use  $4 \times 1/2 = 2$  bags a month. This would cost  $2 \times \$5.99 = \$11.98$  a month.
  - Over one year, this would cost  $\$11.98$  per month  $\times 12$  months =  $\$143.76$ .
- Great Choice Hamster & Gerbil Food (5 pounds) \$6.99
  - According to the Web site, 1 bag feeds a hamster for approximately 90 days. This is about 3 months.
  - So every 3 months, or 4 times a year, we would need a new bag of food. This would cost  $4 \times \$6.99 = \$27.96$  a year.

To find the total cost of owning a pet hamster for one year, I added the one-time costs to the annual costs for bedding and food:

$$3.99 + 22.94 + 7.99 + 8.99 + 15.99 + 143.76 + 27.96 = 231.62$$

That is how I estimated that the cost of adopting and maintaining a pet hamster for one year would be approximately \$231.62.

### **Part C**

The cost for maintaining a pet hamster for one additional year is approximately \$171.72.  
Explanation:

I made my estimation for the cost for one additional year by adding only the annual food and bedding costs, \$27.96 and \$143.76, respectively, because all of the other expenses for the first year were one-time costs:

$$143.76 + 27.96 = 171.72$$

That is how I estimated that the cost of maintaining a pet hamster for one additional year would be approximately \$171.72.

### **Part D**

My model equation is  $y = 13.89x - 1 + 78.87$ , where  $y$  represents the total amount of money spent on the hamster after each month, and  $x$  represents the number of months since adopting the hamster.

First, I noted that my graph looks almost like a straight line, so I decided to use a linear equation to model the cost. Since the total cost was \$78.87 after the first month and \$231.62 after the twelfth month, I decided to create a line that went through the points  $(1, 78.87)$  and  $(12, 231.62)$ . Next I found the slope of the line  $m$ :

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$$m = \frac{231.62 - 78.87}{12 - 1} \approx 13.89$$

Using this slope and the point  $(1, 78.87)$ , I found that the equation of the line should be  $y = 13.89x - 1 + 78.87$ .

### Scoring Notes:

Note that in the sample response for *Part D*, the slope and  $y$ -intercept were calculated based on the graph. At this grade level, it would be considered acceptable for a student to make simply an approximation of the slope and  $y$ -intercept by looking at the graph.

### Scoring Rubric:

Each part of the task is evaluated individually. The total number of points is determined by adding the points assigned for each part. A total of 12 points is possible.

#### Part A

**2 points:** Thorough understanding of how to identify, analyze, and synthesize relevant external resources to solve problems. The student identifies all of the items necessary for the pet and gives each cost based on the information provided.

**1 point:** Partial understanding of how to identify, analyze, and synthesize relevant external resources to solve problems. The student identifies all but 1 or 2 of the items necessary for the pet and gives each cost based on the information provided. **OR** The student identifies all of the items necessary for the pet but 1 or 2 of the associated costs are not based on the information provided.

**0 points:** Limited or no understanding of how to identify, analyze, and synthesize relevant external resources to solve problems. The student has more than 2 of the pet items/costs incorrect or incomplete.

#### Part B

**3 points:** Thorough understanding of how to identify important quantities in a practical situation and map their relationships using a line graph. The student provides explanations of correct work on how the estimation was obtained. The data shown in the line graph is consistent with the explanation provided, the graph is labeled correctly, and appropriate scales are used for the axes.

**2 points:** Partial understanding of how to identify important quantities in a practical situation and map their relationships using a line graph. The student provides explanations of correct work on how the estimation was obtained. The data shown in the line graph is consistent with the explanation provided, but the graph is labeled incorrectly and/or inappropriate scales are used for the axes. **OR** The student makes a minor error in calculation that leads to an incorrect graph, but provides explanations of work that show conceptual understanding. The data shown in the line graph is consistent with the explanation provided, the graph is labeled correctly, and appropriate scales are used for the axes.

**1 point:** Limited understanding of how to identify important quantities in a practical situation and map their relationships using a line graph. The student makes a minor error in

calculation that leads to an incorrect graph, but provides explanations of work that show conceptual understanding. The data shown in the line graph is consistent with the explanation provided, but the graph is labeled incorrectly and/or inappropriate scales are used for the axes.

**0 points:** No understanding of how to identify important quantities in a practical situation and map their relationships using a line graph. The student provides explanations that do not show understanding of how the estimation was obtained. The line graph provided is incorrect or irrelevant.

**Part C**

**3 points:** Thorough understanding of how to interpret the results of *Part B* in terms of the context and how to extend those results to an additional year. The student estimates the cost for one additional year by extending only the recurring costs from the first year. One-time costs from the first year are not included. The student provides explanations of correct work on how the estimation was obtained.

**2 points:** Partial understanding of how to interpret the results of *Part B* in terms of the context and how to extend those results to an additional year. The student makes a minor error in calculating the estimation. The student extends only the recurring costs from the first year. One-time costs from the first year are not included. The student provides explanations of correct work on how the estimation was obtained.

**1 point:** Limited understanding of how to interpret the results of *Part B* in terms of the context and how to extend those results to an additional year. The student estimates the cost for one additional year by extending the recurring costs from the first year, but 1 or 2 one-time costs from the first year are included. The student provides explanations of correct work on how the estimation was obtained.

**0 points:** No understanding of how to interpret the results of *Part B* in terms of the context and how to extend those results to an additional year.

**Part D**

**3 points:** Thorough understanding of how to develop a mathematical model for the cost of a pet. The student writes a model consistent with the graph provided in *Part B*. The student states the meaning of the symbols used in the model. The student provides explanations of correct work on how the model was obtained.

**2 points:** Partial understanding of how to develop a mathematical model for the cost of a pet. The student writes a model consistent with the graph provided in *Part B*, but the student does not state the meaning of the symbols used in the model. The student provides explanations of correct work on how the model was obtained. **OR** Thorough understanding of how to develop a mathematical model for the cost of a pet. The student writes a model consistent with the graph provided in *Part B*, but the model gives the cost per year (instead of per month). The student states the meaning of the symbols used in the model. The student provides explanations of correct work on how the model was obtained.

**1 point:** Limited understanding of how to develop a mathematical model for the cost of a pet. The student writes a model consistent with the graph provided in *Part B*, but the model gives the cost per year (instead of per month) and the student does not state the meaning of the symbols used in the model. The student provides explanations of correct work on how

the model was obtained.

**0 points:** Inconsistent or no understanding of how to develop a mathematical model for the cost of a pet. The student does not write a model consistent with the graph provided in *Part B* or does not accurately provide explanations of how the model was obtained.

## MAT.08.PT.4.WTANK.A.271

Sample Item ID:	MAT.08.PT.4.WTANK.A.271
Title:	Water Tower Options
Grade:	08
Primary Claim:	<b>Claim 4: Modeling and Data Analysis</b> Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.  Claim 2: Problem Solving Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.
Primary Content Domain:	<b>Geometry</b>
Secondary Content Domain(s):	Ratios and Proportional Relationships, Statistics and Probability
Assessment Target(s):	4 A: Apply mathematics to solve problems arising in everyday life, society, and the workplace.  1 H: Understand and apply the Pythagorean theorem.  1 I: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.  1 A (Gr. 7): Analyze proportional relationships and use them to solve real-world and mathematical problems.  1 G (Gr. 7): Use random sampling to draw inferences about a population.  1 A (Gr. 6): Understand ratio concepts and use ratio reasoning to solve problems.  2 A: Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.  2 B: Select and use appropriate tools strategically.  4 B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem.  4 G: Identify, analyze, and synthesize relevant external resources to pose or solve problems.
Standard(s):	8.G.9, 8.G.7, 7.SP.2, 7.RP.3, 6.RP.3
Mathematical Practice(s):	1, 2, 3, 4, 5, 6, 7, 8
DOK:	4

Grade 8 Mathematics Sample PT Form



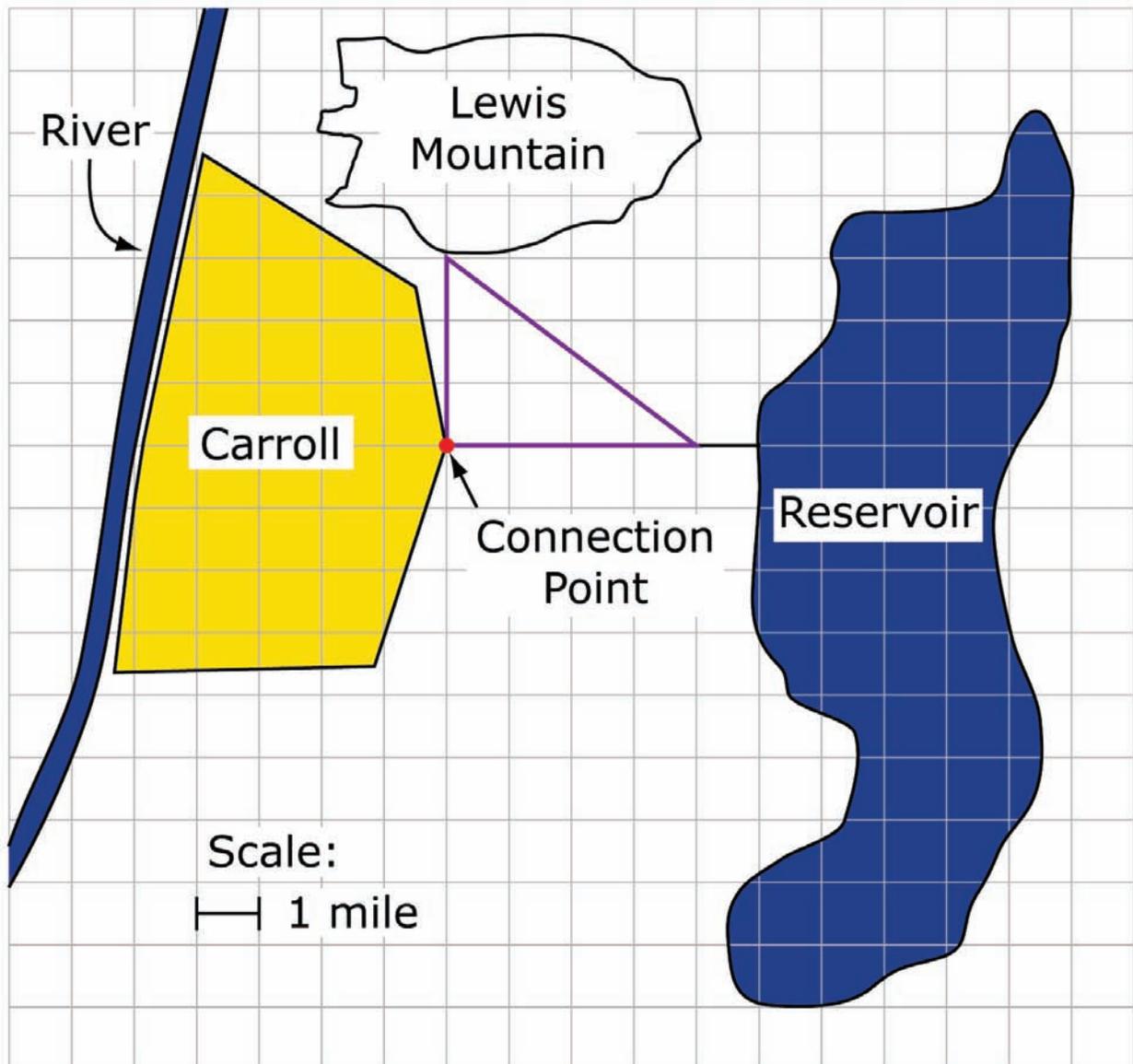
Item Type:	PT
Score Points:	16
Difficulty:	M
How this task addresses the “sufficient evidence” for this claim:	The student uses concepts of geometry, number and operations, and statistics to determine the best solution to a problem where all constraints cannot be satisfied at the same time. Additionally, the student must provide justifications to support reasoning.
Target-Specific Attributes (e.g., accessibility issues):	Accommodations may be necessary for students who have visual challenges. Calculator and spreadsheet needed.
Stimulus/Source:	<a href="http://www.lifewater.org/resources/rws5/rws5d3.pdf">http://www.lifewater.org/resources/rws5/rws5d3.pdf</a> <a href="http://www.nesc.wvu.edu/pdf/dw/publications/ontap/2009_tb/reservoirs_towers_tanks_DWFSOM15.pdf">http://www.nesc.wvu.edu/pdf/dw/publications/ontap/2009_tb/reservoirs_towers_tanks_DWFSOM15.pdf</a> <a href="http://www.howstuffworks.com/water.htm">http://www.howstuffworks.com/water.htm</a>
Notes:	Multi-part task
Task Overview:	During the task, the student assumes the role of a member of the finance committee of a town council who is given the responsibility of determining the best plan for constructing a water tank or tower for the town. The student completes tasks in which he or she investigates the costs associated with building the water tank or tower in different locations, as well as the costs associated with the different designs, and then combines this information with some survey data to make a recommendation to the council. This investigation is done in class using spreadsheets and a calculator.
Teacher Preparation/Resource Requirements:	The day before the administration of the task, there is a prework task that will be used to get students thinking about the problem. The prework involves a teacher-led class discussion. The remainder of the task should be completed individually over two 1-hour sessions. During Session 1, students will be expected to make a number of calculations that will require the use of spreadsheets and calculators. During Session 2, students may need to finish the work from the previous day, but they need to be sure to set aside time to think about the benefits and costs of the different options and write a final recommendation.
Teacher Responsibilities During Administration:	Monitor individual student work; provide resources as necessary.
Time Requirements:	One pretask session that requires no more than 30 minutes of small-group and class work. Two sessions of scored individual work totaling no more than 120 minutes.

**Prework:**

In preparation for this task, teachers need to guide a class discussion about the considerations that need to be made when constructing a water tank or water tower for a town. It is best if this discussion takes place one to two days before the performance task is administered.

During the discussion, every student needs to be able to see the map that accompanies the performance task, whether on a screen at the front of the class, on a paper copy, or by some other method.

**Teacher says:** In the next few days, you will be working on a performance task. In the task, a new water tank or water tower for a town needs to be built, and your job is to recommend the best solution for the town.



**Teacher continues:** The map shows the location of the reservoir from which the water supply will be coming, the town of Carroll, the connection point between the town water

system and the water from the reservoir, and a mountain outside of town. A structure is needed to hold the water before it is used in order to provide a constant water pressure throughout the town. The options are to build a water tank on the mountain outside of town or to build a water tower at the connection point. Today we are going to brainstorm the costs and other factors associated with building either a water tank on the mountain or a water tower at the connection point so that the best recommendation can be presented.

**Teacher asks:** First, what costs would be associated with building a water tank on the mountain outside of town? What other factors should be considered?

As students brainstorm ideas, the ideas should be recorded.

**Teacher asks:** Now, what costs would be associated with building a water tower at the connection point on the edge of town? What other factors should be considered?

As students brainstorm ideas, the ideas should be recorded.

**Teacher asks:** Before we end, is there anything else that we forgot that we need to consider?

Record any final ideas that the students have. Consider asking probing questions if students have missed any of the following topics or do not understand why the following topics are important: the height of the tank/tower, the capacity of the tank/tower, the cost of the water pipes to and from the tank/tower, and the cost of the land.

Close the activity and save the lists of students' ideas for the following day.

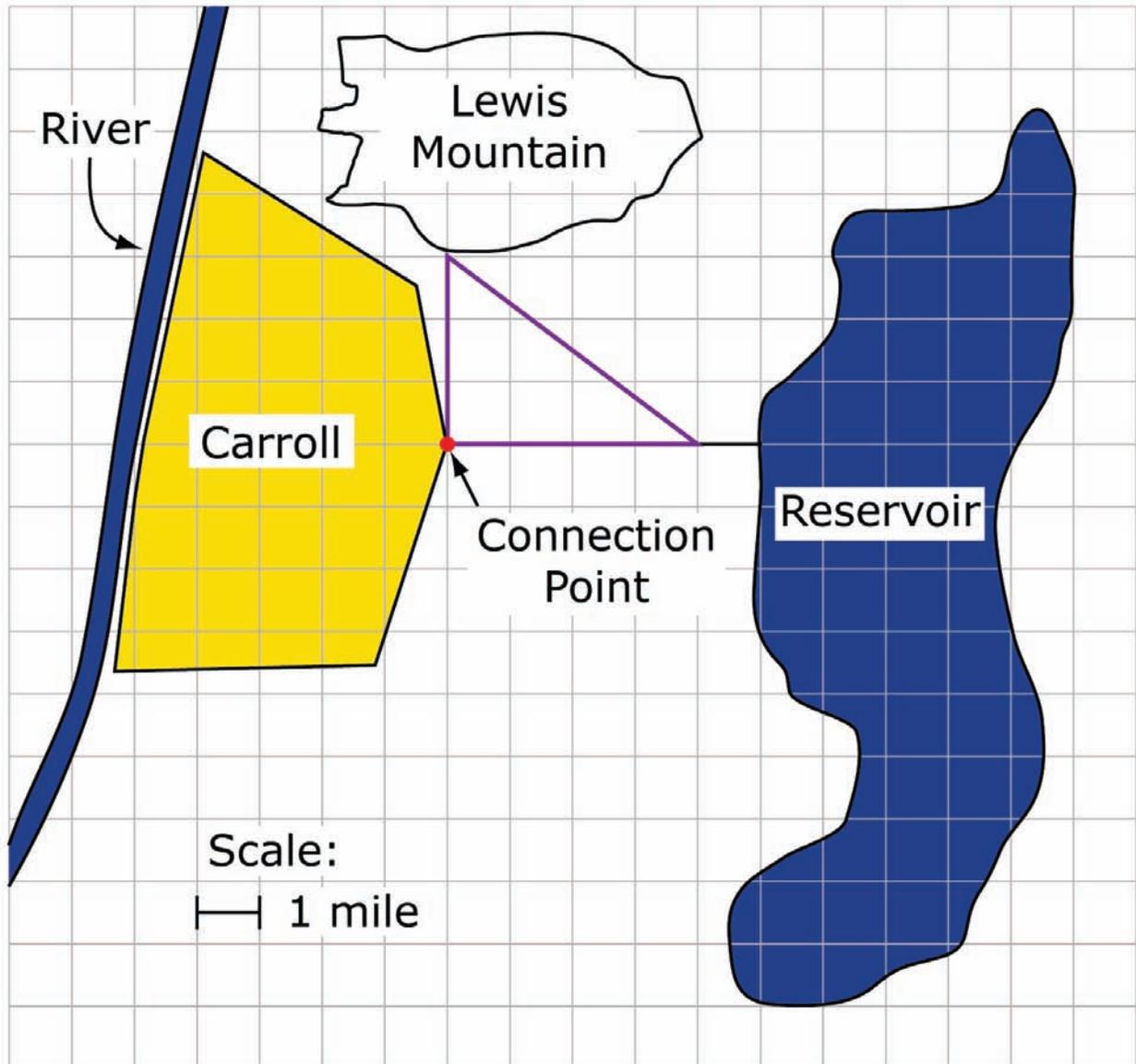
## Water Tank or Water Tower?

### Session 1

The population of Carroll is approximately 20,000 and has remained constant for the last 20 years. Carroll currently uses an outdated water supply system to get its water from a nearby river. The outdated system will be replaced by a system that draws from a reservoir.

Members of the finance committee on the Carroll town council are responsible for investigating the costs of a new water treatment plant. As one of the finance committee members, you must make a recommendation that will best meet the needs of the town at this time. The two options are as follows:

- Building a water tank on Lewis Mountain
- Building a water tower at a connection point on the edge of town



You must first determine the costs based on the packet of information provided by four companies bidding on the town's new water treatment plant. After analyzing the information provided, you will summarize your findings and write a report to the town council that includes your recommendation.

[The packet of information referred to here is attached to this item, following the rubric.]

To begin, answer the following questions.

1. How high does the water tank need to be above the highest elevation in town?
2. What is the minimum size of each design (reservoir, Hydrosphere, and Watercolumn) that could be used to meet the needs of the town?
3. What is the cost to construct the minimum size of each design, including the tower if necessary?
4. What is the cost of the pipe for each design?
5. What is the cost of the land for each design?

You have information and estimates from the following companies to help you answer the questions.

- Thomas Tanks & Towers, Inc.
- William Brown Polls
- Nguyen Construction Co.
- Ravi Patel Real Estate

Be sure to clearly label your answers for each question. Show all work necessary to support your answers using diagrams, pictures, expressions/equations, or words as appropriate.

A calculator tool<sup>1</sup> and a spreadsheet tool<sup>2</sup> are available by clicking on the appropriate icon for each in the top right corner. Answers may be written or displayed in the spreadsheet and saved as part of your response.

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<sup>1</sup> The technology platform will have to allow for at least a scientific calculator for this task.

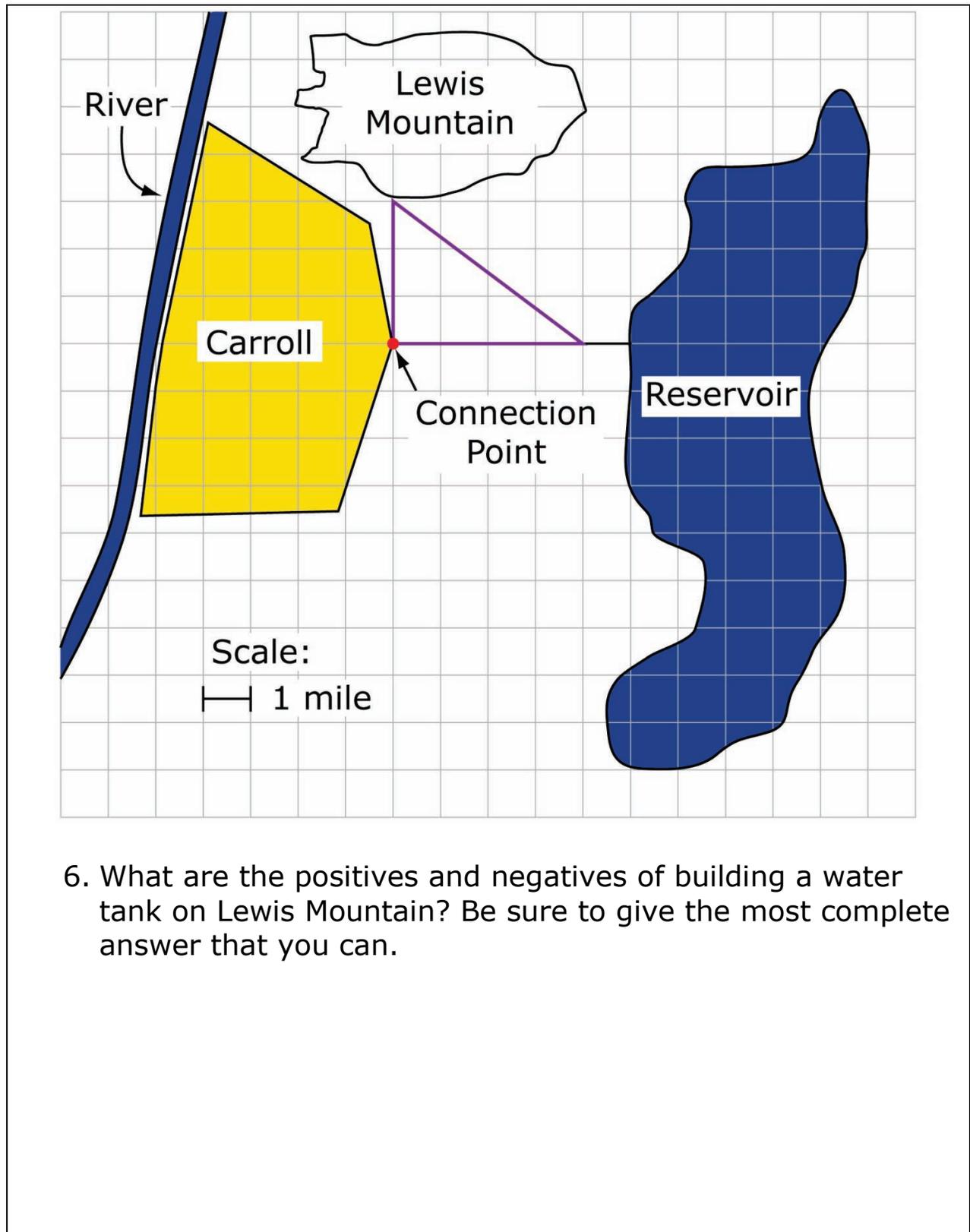
<sup>2</sup> The technology platform should allow for a basic spreadsheet interface that can be used, with the results or snapshot of the spreadsheet included as part of the student's response.

## **Session 2**

Now that you have determined the costs of the different designs, you need to write your report and make a recommendation to the town council.

The following questions will guide your report. To support your answers to these questions, you must include the following:

- Cost estimates gathered from your work completed in Session 1 of this task
- Survey information provided in the information packet



6. What are the positives and negatives of building a water tank on Lewis Mountain? Be sure to give the most complete answer that you can.

7. What are the positives and negatives of building a water tower at the connection point? Be sure to give the most complete answer that you can.

8. What is your recommendation to the other members of the town council in Carroll? Be sure to give the most complete answer that you can.

*Sample Top-Score Response:*

**Session 1:**

1. How high does the water tank need to be above the highest elevation in town?

According to Thomas Tanks & Towers, Inc., for each foot that the water tower is above the highest elevation in town, the water pressure increases by 0.43 pounds per square inch (PSI). Thomas Tanks & Towers, Inc., recommends maintaining a water pressure between 50 and 100 pounds per square inch (PSI).

In order to minimize the height that the water tank needs to be above the highest elevation in town, and thus minimize the cost associated with building a tower, I calculate the height needed to maintain a pressure of at least 50 PSI:

$$\frac{50}{0.43} \approx 116.28 \text{ feet}$$

So the height needed to maintain a pressure of at least 50 PSI is about 117 feet above the highest elevation in town.

2. What is the minimum size of each design (reservoir, Hydrosphere, and Watercolumn) that could be used to meet the needs of the town?

If the tank is built at the connection point, a tower of about 117 feet will be needed. If the tank is built on Lewis Mountain, a tower does not need to be built, because the parcels of land available on the mountain are approximately 150 feet above the highest elevation in town.

According to the information provided, the population of Carroll is approximately 20,000 and has remained constant for the last 20 years. Since the population has remained constant, I will determine what size water tank would be needed for 20,000 people.

Thomas Tanks & Towers, Inc., recommends that the tank hold about a day's worth of water for the community. The average person uses about 90 gallons of water per day. I calculate the total volume of water needed to be held in the tank:

$$20,000 \times 90 = 1,800,000 \text{ gallons of water}$$

So the tank will need to hold 1,800,000 gallons of water. Next I figure out the volume that the tank would need to have in order to hold 1,800,000 gallons of water. Since one cubic foot holds approximately 7.5 gallons of water, the tank will need to have a volume of  $1,800,000 / 7.5 = 240,000$  cubic feet.

Since the options of the water tank have different shapes and sizes, I need to determine which of these has a volume of at least 240,000 cubic feet.

Ground Level Cylindrical Reservoir Options:

Since this is a cylinder, the volume of water that can be held by the tank is  $\pi r^2 h$  cubic feet, where  $r$  is the radius and  $h$  is the height of the tank. I used a spreadsheet to calculate the volumes and rounded to the nearest cubic foot, shown below.

Diameter (feet)	Height (feet)	Volume (cubic feet)
48	40	72,382
58	40	105,683
67	40	141,026
94	40	277,591
115	40	415,476
148	40	688,134

If I were to choose a Ground Level Cylindrical Reservoir that would both minimize cost and have adequate capacity, the tank would need a diameter of 94 feet and a height of 40 feet. Since this is a ground-level tank, it would need to be built on Lewis Mountain, and thus wouldn't require a tower.

Hydrosphere Water Tower Options:

Since this is a sphere, the volume of water that can be held by the tank is  $\frac{4}{3}\pi r^3$  cubic feet, where  $r$  is the radius of the tank. I used a spreadsheet to calculate the volumes and rounded to the nearest cubic foot, shown below.

Diameter (feet)	Volume (cubic feet)
52	73,622
59	107,536
65	143,793
70	179,594
75	220,893
81	278,262

If I were to choose a Hydrosphere Water Tower that would both minimize cost and have adequate capacity, the tank would need a diameter of 81 feet. Since this is a water tower, it would need to be built at the connection point, and thus it would require a tower of approximately 117 feet.

#### Watercolumn Water Tower Options:

This tank is a cylinder and part of a cone. According to the information provided, the height of the upper cylindrical section of a Watercolumn Water Tower is half of the head range. The volume of the tank in the lower section of a Watercolumn is 70% of the volume in the upper cylindrical section. Thus, the total volume of water that can be held by the tank is  $1.7\pi r^2 h$  cubic feet, where  $r$  is the radius and  $h$  is the height of the cylindrical section of the tank. I used a spreadsheet to calculate the volumes and rounded to the nearest cubic foot, shown below.

Diameter (feet)	Head Range (feet)	Volume (cubic feet)
58	32	71,865
64	40	109,378
74	40	146,229
80	40	170,903
90	40	216,299
100	42	280,387
108	44	342,617
120	42	403,757
125	45	469,398
135	45	547,506

If I were to choose a Watercolumn Water Tower that would both minimize cost and have adequate capacity, the tank would need a diameter of 100 feet and a height of 42 feet. Since this is a water tower, it would need to be built at the connection point, and thus it would require a tower of approximately 117 feet.

3. What is the cost to construct the minimum size of each design, including the tower if necessary?

Again, I will look only at tanks that both minimize cost and have adequate capacity.

#### Ground Level Cylindrical Reservoir Option:

According to Thomas Tanks & Towers, Inc., the cost of the Ground Level Cylindrical Reservoir with a diameter of 94 feet and a height of 40 feet is \$2,500,000. Since this would be built on Lewis Mountain, no tower is needed.

#### Hydrosphere Water Tower Option:

According to Thomas Tanks & Towers, Inc., the cost of the Hydrosphere Water Tower with diameter of 81 feet is \$3,220,000. Since this would be built at the connection point, a 117-foot tower is needed. According to the information provided, the

construction costs are \$300 for each foot that the tank is raised above the ground, so the tower would cost  $117 \times \$300 = \$35,100$ . Therefore, the cost of this water tank/tower option is  $\$3,220,000 + \$35,100 = \$3,255,100$ .

**Watercolumn Water Tower Options:**

According to Thomas Tanks & Towers, Inc., the cost of the Watercolumn Water Tower with a diameter of 100 feet and a height of 42 feet is \$3,020,000. Since this would be built at the connection point, a 117-foot tower is needed. According to the information provided, the construction costs are \$300 for each foot that the tank is raised above the ground, so the tower would cost  $117 \times \$300 = \$35,100$ . Therefore, the cost of this water tank/tower option is  $\$3,020,000 + \$35,100 = \$3,055,100$ .

4. What is the cost of the pipe for each design?

For the Ground Level Cylindrical Reservoir, since this would be built on Lewis Mountain, I find the length of pipe needed from the reservoir to the mountain, the length of pipe needed from the mountain to the connection point, and the cost of building pipes from the bottom of the mountain to the tank.

According to the map, a portion of the pipe from the reservoir to the mountain appears to be the hypotenuse of a right triangle with legs of length 3 and 4. By the Pythagorean theorem, the length of this portion is  $\sqrt{3^2 + 4^2} = 5$  miles. So based on the map, there should be  $1 + 5 = 6$  miles of pipe from the reservoir to the mountain and 3 miles of pipe from the mountain to the connection point. Nguyen Construction estimates that the cost of constructing the water pipe would be \$4,300,000 per mile, and the cost of building pipes from the bottom of the mountain to the tank would be an additional \$700,000. Thus, the cost of pipe needed for this water tank would be  $9 \times \$4,300,000 + \$700,000 = \$39,400,000$  for this option.

For the Hydrosphere Water Tower and Watercolumn Water Tower, since these would be built at the connection point on the edge of town, I find the length of pipe needed from the reservoir to the connection point. According to the map, there should be a total of  $1 + 4 = 5$  miles of pipe. Nguyen Construction estimates that the cost of constructing the water pipe would be \$4,300,000 per mile. Thus, the cost of pipe needed for these water towers would be  $5 \times \$4,300,000 = \$21,500,000$  for either of these options.

5. What is the cost of the land for each design?

The Ground Level Cylindrical Reservoir would be built on Lewis Mountain, which Ravi Patel Real Estate expects to cost \$9,000 per acre. According to Thomas Tanks & Towers, Inc., 10 acres of land are required for a Ground Level Cylindrical Reservoir. Thus, the cost of land for this option is  $\$9,000 \times 10 = \$90,000$ .

The Hydrosphere Water Tower and Watercolumn Water Tower would be built at the connection point, which Ravi Patel Real Estate expects to cost \$55,000 per acre. According to Thomas Tanks & Towers, Inc., 5 acres of land are required for either water tower. Thus, the cost of land for either option is  $\$55,000 \times 5 = \$275,000$ .

**Session 2:**

6. What are the positives and negatives of building a water tank on Lewis Mountain? Be sure to give the most complete answer that you can.

Here are the costs associated with building a Ground Level Cylindrical Reservoir on

## Grade 8 Mathematics Sample PT Form

Lewis Mountain:

- Cost of water tank: \$2,500,000
- Cost of pipe: \$39,400,000
- Cost of land: \$90,000
- Total construction cost:  $\$2,500,000 + \$39,400,000 + \$90,000 = \$41,900,090$

The positive of building a water tank on Lewis Mountain is that, according to William Brown Polls, about 55% of the population of Carroll prefer that the water tank be built on Lewis Mountain. The negative is that it costs a lot more than the other two options.

7. What are the positives and negatives of building a water tower at the connection point? Be sure to give the most complete answer that you can.

Here are the costs associated with building a Hydrosphere Water Tower at the connection point:

- Cost of water tank: \$3,255,100
- Cost of pipe: \$21,500,000
- Cost of land: \$275,000
- Total construction cost:  $\$3,255,100 + \$21,500,000 + \$275,000 = \$25,030,100$

Here are the costs associated with building a Watercolumn Water Tower at the connection point:

- Cost of water tank: \$3,055,100
- Cost of pipe: \$21,500,000
- Cost of land: \$275,000
- Total construction cost:  $\$3,055,100 + \$21,500,000 + \$275,000 = \$24,830,100$

The positive of building a water tower at the connection point is that it will cost the town much less than building a water tank on Lewis Mountain. The negative is that the population of the town would prefer to have the tank built on Lewis Mountain.

8. What is your recommendation to the other members of the town council in Carroll? Be sure to give the most complete answer that you can.

I recommend that the town of Carroll build a Watercolumn Water Tower, with a diameter of 100 feet and a height of 42 feet, at the connection point. This water tower will have a water capacity and pressure that is adequate for the town's population and will minimize the total construction cost.

The costs associated with building a Watercolumn Water Tower at the connection point are as follows:

- Cost of water tank: \$3,055,100

## Grade 8 Mathematics Sample PT Form

- Cost of pipe: \$21,500,000
- Cost of land: \$275,000
- Total construction cost:  $\$3,055,100 + \$21,500,000 + \$275,000 = \$24,830,100$

I recognize that 55% of the population of Carroll did not prefer to build a water tower at the connection point, but the cost of building a water tank on Lewis Mountain (\$41,900,090) was not feasible. However, in order to appease the public and strengthen town pride, I further recommend that a design featuring the high school mascot be put on the tank.

*Scoring Notes:* Each scored portion of the task is evaluated individually. The total number of points is determined by adding the points assigned for each task.

*Scoring Rubric:*

**Question 1:** Responses to this question will receive 0–1 point, based on whether the student correctly answers the question. This question is a one-step problem, so it is straightforward.

**Question 2:** Responses to this item will receive 0–3 points, based on the following:

**3 points:** The student shows a thorough understanding of how to find the volume of cylinders and spheres. The student finds correct volumes and determines the number of gallons that each tank holds.

**2 points:** The student shows a partial understanding of how to find the volume of cylinders and spheres. The student performs many aspects of the task correctly but demonstrates some misunderstandings, such as neglecting to determine the number of gallons that each tank holds or using the diameter instead of the radius to determine the volume.

**1 point:** The student shows a limited understanding of how to find the volume of cylinders and spheres. The student makes some correct calculations but does not demonstrate understanding of the volume of both cylinders and spheres.

**0 points:** The student shows inconsistent or no understanding of how to find the volume of cylinders and spheres. The student makes numerous errors in calculating the volume of the cylinders and spheres.

**Question 3:** Responses to this item will receive 0–2 points, based on the following:

**2 points:** The student shows a thorough understanding of determining the cost of each option. The student uses the tables to find the cost of the smallest tank that could be used for each design and includes the cost for the tower where needed.

**1 point:** The student shows a partial understanding of determining the cost of each option.

The student makes minor errors when using the tables to find the cost of the smallest tank that could be used for each design and including the cost for the tower where needed.

**0 points:** The student shows inconsistent or no understanding of determining the cost of each option. The student does not demonstrate the ability to use the tables to determine the cost of the smallest tank that could be used for each design, nor does the student calculate the cost of the tower.

**Question 4:** Responses to this item will receive 0–2 points, based on the following:

**2 points:** The student shows a thorough understanding of determining the cost of each option. The student uses the Pythagorean theorem and the additional \$700,000 cost to find the cost of the pipes to and from the tank on Lewis Mountain, and multiplies to find the cost of the pipes to the water tower.

**1 point:** The student shows a partial understanding of determining the cost of each option. The student does not use the Pythagorean theorem or makes minor mathematical errors.

**0 points:** The student shows inconsistent or no understanding of determining the cost of each option. The student makes numerous errors in calculating the cost of the pipe for each option.

**Question 5:** Responses to this item will receive 0–1 point, based on whether the student correctly answers the question. The calculations in this question are straightforward, so no partial credit is given for this question.

**Question 6:** Responses to this item will receive 0–2 points, based on the following:

**2 points:** The student shows a thorough understanding of the benefits and costs of building a water tank on Lewis Mountain. The student includes all costs as well as the opinion of the community in the response.

**1 point:** The student shows a partial understanding of the benefits and costs of building a water tank on Lewis Mountain. The student considers only three of the four factors that need to be considered in making the decision.

**0 points:** The student shows inconsistent or no understanding of the benefits and costs of building a water tank on Lewis Mountain. The student considers less than three of the factors that need to be considered in making the decision.

**Question 7:** Responses to this item will receive 0–2 points, based on the following:

**2 points:** The student shows a thorough understanding of the benefits and costs of building a water tower on the outskirts of Carroll. The student includes all costs as well as the opinion of the community in the response.

**1 point:** The student shows a partial understanding of the benefits and costs of building a

water tower on the outskirts of Carroll. The student considers only three of the four factors that need to be considered in making the decision.

**0 points:** The student shows inconsistent or no understanding of the benefits and costs of building a water tower on the outskirts of Carroll. The student considers less than three of the factors that need to be considered in making the decision.

**Question 8:** Responses to this item will receive 0–3 points, based on the following:

**3 points:** The student shows a thorough understanding of how to analyze a complex scenario and construct a chain of reasoning to justify the solution to a scenario. The student includes all the different aspects of the task in the justification and provides evidence to support the conclusion made.

**2 points:** The student shows a partial understanding of how to analyze a complex scenario and construct a chain of reasoning to justify the solution to a scenario. The student includes many of the different aspects of the task in the justification and provides evidence to support the conclusion made, but some omissions are made.

**1 point:** The student shows a limited understanding of how to analyze a complex scenario and construct a chain of reasoning to justify the solution to a scenario. The student includes some of the different aspects of the task in the justification but makes a conclusion that would not have been made if all of the aspects of the task had been considered.

**0 points:** The student shows inconsistent or no understanding of how to analyze a complex scenario and construct a chain of reasoning to justify the solution to a scenario. The student makes a conclusion that is contradictory to the aspects of the task considered in the justification.

This packet contains information from Nguyen Construction Co., Ravi Patel Real Estate, William Brown Polls, and Thomas Tanks & Towers, Inc. to be used to determine the costs of the construction of a water tank or water tower for the town of Carroll.

## THOMAS TANKS & TOWERS, INC.

### REQUIRED TANK HEIGHT

- For each foot that the base of a water tank is above the highest elevation in town, the water pressure increases by 0.43 pounds per square inch (PSI).
- The construction costs are \$300 for each foot that the tank is raised above the ground.

### RECOMMENDED WATER PRESSURE

- We recommend maintaining a water pressure between 50 and 100 pounds per square inch (PSI).

### RECOMMENDED WATER TANK CAPACITY

- We recommend that the tank hold about a day's worth of water for the community. The average person uses about 90 gallons of water per day.

### GENERAL NOTES

- One cubic foot holds approximately 7.5 gallons.
- The cost of a reservoir, Hydrosphere, or Watercolumn does not include the cost to purchase the land, develop the land, or build the tower.

### GROUND-LEVEL CYLINDRICAL RESERVOIR OPTIONS

Diameter (feet)	Height (feet)	Cost (dollars)
48	40	\$650,000.00
58	40	\$950,000.00
67	40	\$1,270,000.00
94	40	\$2,500,000.00
115	40	\$3,730,000.00
148	40	\$6,180,000.00



- 10 acres of land are required for a ground-level reservoir.
- A ground-level cylindrical reservoir cannot be constructed on top of a tower.

### **HYDROSPHERE WATER TOWER OPTIONS**

<b>Diameter (feet)</b>	<b>Cost (dollars)</b>
52	\$1,030,000.00
59	\$1,400,000.00
65	\$1,780,000.00
70	\$2,170,000.00
75	\$2,610,000.00
81	\$3,220,000.00

Base of the  
water tank



- 5 acres of land are required for a Hydrosphere water tower.

### **WATERCOLUMN WATER TOWER OPTIONS**

#### NOTES:

- The height of the upper cylindrical section of a Watercolumn is half of the head range.
- The volume of the lower noncylindrical section of the tank in a Watercolumn is 70% of the volume in the upper cylindrical section.

Diameter (feet)	Head Range (feet)	Cost (dollars)
58	32	\$1,060,000.00
64	40	\$1,410,000.00
74	40	\$1,760,000.00
80	40	\$1,990,000.00
90	40	\$2,420,000.00
100	42	\$3,020,000.00
108	44	\$3,610,000.00
120	42	\$4,190,000.00
125	45	\$4,800,000.00
135	45	\$5,540,000.00

Head range  
Base of the water tank



- 5 acres of land are required for a Watercolumn water tower.

### **NGUYEN CONSTRUCTION CO.**

Based on your requirements, we are pleased to offer an estimate of

    \$20,000     per acre to develop a site for a water tank or water tower.

This estimate includes:

- Grading and site preparation
- Foundation construction
- Parking lot and sidewalk construction
- Planting of grass seed and native shrubs and trees at the conclusion of the project

In addition, we estimate the cost to construct the water pipe at \$4,300,000 per mile. If the water tank is constructed on Lewis Mountain, we estimate \$700,00 to build pipes from the bottom of the mountain to the tank.

### **RAVI PATEL REAL ESTATE**

We have researched the approximate costs for 10-acre parcels of land on Lewis Mountain and 5-acre parcels of land on the edge of the town of Carroll.

It is reasonable to expect to pay \$9,000 per acre for land on Lewis Mountain. The parcels of land available on the mountain are approximately 150 feet above the highest elevation in town.

It is reasonable to expect to pay \$55,000 per acre for land on the edge of the town of Carroll.

We look forward to providing assistance in the purchase of the land needed for this project.

### **WILLIAM BROWN POLLS**

Thank you for allowing us the opportunity to serve your community. The questions used in our survey and the results are shown below.

*Question 1: How strongly do you agree with the following statement? I would prefer for the new water tank to be built on a tower on the edge of town instead of on Lewis Mountain.*

<b>Response</b>	<b>Percent of the Population</b>
Strongly agree	20%
Somewhat agree	15%
Neither agree nor disagree	10%
Somewhat disagree	10%
Strongly disagree	45%

*Question 2: If the new water tank is built on a tower on the edge of town, what would you like to see in the space on the water tank? A design welcoming people to Carroll, a design featuring the high school mascot, or advertisements?*

<b>Response</b>	<b>Percent of the Population</b>
A design welcoming people to Carroll	15%
A design featuring the highschool mascot	75%
Sell the space for advertising	10%

All percents shown are estimated to be correct within  $\pm 3\%$ .

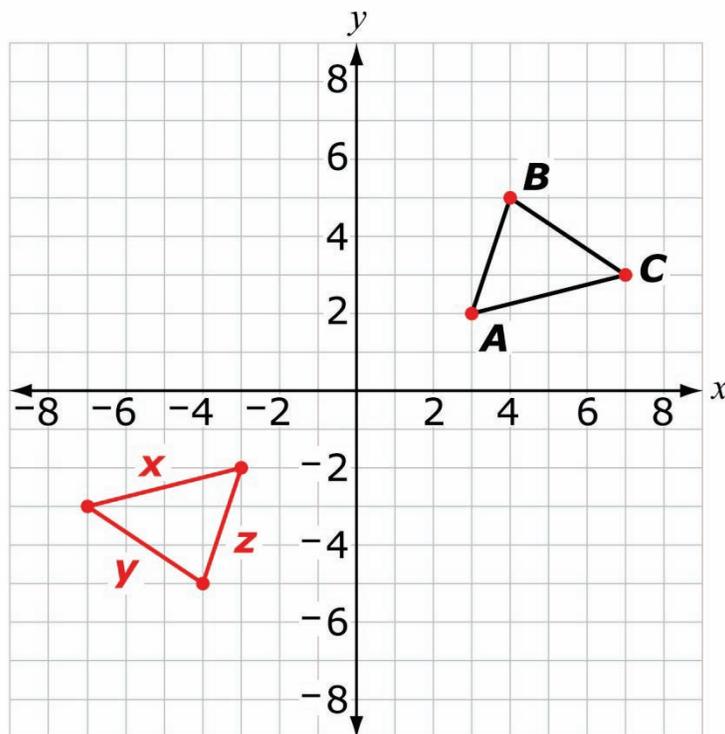
Please contact us if we can be of further service to your community.

## MAT.08.SR.1.0000G.G.141 C1 TG

Sample Item ID:	MAT.08.SR.1.0000G.G.141
Grade:	08
Claim(s):	<b>Claim 1: Concepts and Procedures</b> Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Assessment Target(s):	1 G: Understand congruence and similarity using physical models, transparencies, or geometry software.
Content Domain:	Geometry
Standard(s):	8.G.2
Mathematical Practice(s):	2, 6
DOK:	1
Item Type:	SR
Score Points:	1
Difficulty:	L
Key:	$AB = z, AC = x, BC = y$
Stimulus/Source:	
Target-Specific Attributes (e.g., accessibility issues):	
Notes:	A check should appear (or the box is highlighted) when a box is selected. Students may only select one box in each row and column.

Triangle  $ABC$  on this coordinate grid was created by joining points  $A(3, 2)$ ,  $B(4, 5)$ , and  $C(7, 3)$  with line segments.

Triangle  $ABC$  was reflected over the  $x$ -axis and then reflected over the  $y$ -axis to form the red triangle, where  $x$ ,  $y$ , and  $z$  represent the lengths of the sides of the red triangle.



Click the appropriate boxes in the table to show which sides of the triangles have equal lengths.

	$x$	$y$	$z$
$AB$			
$AC$			
$BC$			

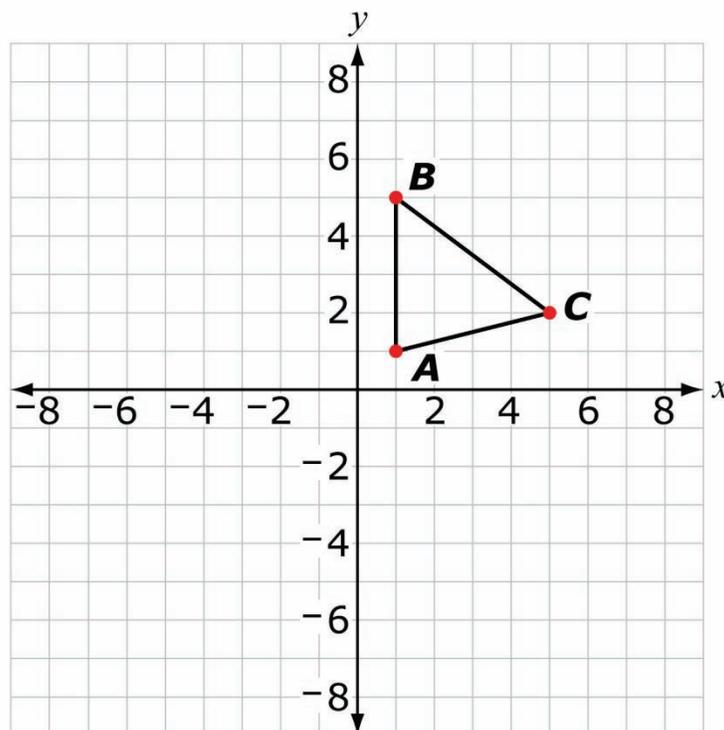
*Key and Distractor Analysis:*

- $AB = z$ . Students may work from left to right and think that  $AB = y$ .
- $AC = x$ . Most students will probably answer this one correctly.
- $BC = y$ . Students may work from left to right and think that  $BC = z$ .

## MAT.08.SR.1.0000G.G.142 C1 TG

Sample Item ID:	MAT.08.SR.1.0000G.G.142
Grade:	08
Claim(s):	<b>Claim 1: Concepts and Procedures</b> Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.
Assessment Target(s):	1 G: Understand congruence and similarity using physical models, transparencies, or geometry software.
Content Domain:	Geometry
Standard(s):	8.G.3
Mathematical Practice(s):	2, 7
DOK:	1
Item Type:	SR
Score Points:	1
Difficulty:	M
Key:	A
Stimulus/Source:	
Target-Specific Attributes (e.g., accessibility issues):	Blind or low-vision students may have difficulty with this item.
Notes:	

Triangle  $ABC$  is shown on this coordinate grid.



Triangle  $ABC$  is dilated with the origin as the center of the dilation. Which ordered pair could represent the image of point  $C (5, 2)$  after the dilation?

- (A)  $(2.5, 1)$
- (B)  $(5, -2)$
- (C)  $(7.5, 4.5)$
- (D)  $(-1, -4)$

*Key and Distractor Analysis:*

- A. Key
- B. Students may confuse dilation with reflection.
- C. Students may think that you can add the same amount from each coordinate when dilating.
- D. Students may think that you can subtract the same amount from each coordinate when dilating.

## MAT.08.SR.1.0000G.H.143 C1 TH

Sample Item ID:	MAT.08.SR.1.0000G.H.143
Grade:	08
Claim(s):	<b>Claim 1: Concepts and Procedures</b> Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.
Assessment Target(s):	1 H: Understand and apply the Pythagorean theorem.
Content Domain:	Geometry
Standard(s):	8.G.8
Mathematical Practice(s):	5, 6
DOK:	1
Item Type:	SR
Score Points:	1
Difficulty:	M
Key:	C
Stimulus/Source:	
Target-specific attributes (e.g., accessibility issues):	
Notes:	Calculator tool should be turned on for this item.

What is the distance between  $(0, 0)$  and  $(8, 15)$  on the  $xy$ -coordinate plane?

- A 7 units
- B 8 units
- C 17 units
- D 23 units

*Key and Distractor Analysis:*

- A. Students may subtract 8 from 15 to find the distance.
- B. Students may think that the distance is equal to the  $x$  value of the point that is not at the origin.
- C. Key
- D. Students may add 8 and 15 to find the distance.

## MAT.08.SR.1.000EE.B.203

Sample Item ID:	<b>MAT.08.SR.1.000EE.B.203</b>
Grade:	08
Claim(s):	<b>Claim 1: Concepts and Procedures</b> Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Assessment Target(s):	<b>1 B:</b> Work with radicals and integer exponents.
Content Domain:	Equations and Expressions
Standard(s):	8.EE.1
Mathematical Practice(s):	1, 5, 7
DOK:	1
Item Type:	SR
Score Points:	1
Difficulty:	M
Key:	A, C, D
Stimulus/Source:	
Target-Specific Attributes (e.g., accessibility issues):	Students may not use calculators for this target.
Notes:	Multiple correct keys

Select **all** of the expressions that have a value between 0 and 1.

- (A)  $8^7 \cdot 8^{-12}$
- (B)  $\frac{7^4}{7^{-3}}$
- (C)  $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^9$
- (D)  $\frac{(-5)^6}{(-5)^{10}}$

*Key and Distractor Analysis:*

- A. Key. Students may think that a negative exponent means that the number is negative, not that the number is between 0 and 1.
- B. The expression is equal to  $7^7$ , so it is greater than 1. Students may make mistakes when finding the exponent of the equivalent expression.
- C. Key. Students may think that since the base is raised to such a high exponent it will be greater than 1, but since the base is between 0 and 1, the value of the expression is between 0 and 1.
- D. Key. Students may miscalculate the exponent of the equivalent expression or forget that the expression will be positive because the exponent is even.

## MAT.08.SR.1.000EE.D.201

Sample Item ID:	<b>MAT.08.SR.1.000EE.D.201</b>
Grade:	08
Claim(s):	<b>Claim 1: Concepts and Procedures</b> Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Assessment Target(s):	<b>1 D:</b> Analyze and solve linear equations and pairs of simultaneous linear equations.
Content Domain:	Equations and Expressions
Standard(s):	8.EE.7
Mathematical Practice(s):	1, 7, 8
DOK:	2
Item Type:	SR
Score Points:	1
Difficulty:	M
Key:	B,C
Stimulus/Source:	
Target-Specific Attributes (e.g., accessibility issues):	
Notes:	Calculator tool should be turned on for this item.

Three students solved the equation  $3(5x - 14) = 18$  in different ways, but each student arrived at the correct answer. Select **all** of the solutions that show a correct method for solving the equation.

(A)

$$\begin{array}{r}
 3(5x - 14) = 18 \\
 8x - 14 = 18 \\
 + 14 \quad + 14 \\
 \hline
 8x = 32 \\
 \frac{8x}{8} = \frac{32}{8} \\
 x = 4
 \end{array}$$

(B)

$$\begin{array}{r}
 \frac{1}{3} \cdot 3(5x - 14) = 18 \cdot \frac{1}{3} \\
 5x - 14 = 6 \\
 + 14 \quad + 14 \\
 \hline
 5x = 20 \\
 \frac{5x}{5} = \frac{20}{5} \\
 x = 4
 \end{array}$$

(C)

$$\begin{array}{r}
 3(5x - 14) = 18 \\
 \frac{15x}{15} - \frac{42}{15} = \frac{18}{15} \\
 + \frac{42}{15} \quad + \frac{42}{15} \\
 \hline
 x = \frac{60}{15} \\
 x = 4
 \end{array}$$

*Key and Distractor Analysis:*

- A. This solution is the simplest to follow, but the method is incorrect.
- B. Key. Although the method in this solution is correct, it is not the most commonly used method for solving equations like this, so students may think it is incorrect.
- C. Key. Although the method in this solution is correct, it is not the most commonly used method for solving equations like this, so students may think it is incorrect.

## MAT.08.SR.1.000EE.D.204

Sample Item ID:	<b>MAT.08.SR.1.000EE.D.204</b>
Grade:	08
Claim(s):	<b>Claim 1: Concepts and Procedures</b> Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Assessment Target(s):	<b>1 D:</b> Analyze and solve linear equations and pairs of simultaneous linear equations.
Content Domain:	Equations and Expressions
Standard(s):	8.EE.7
Practice(s):	1, 7, 8
DOK:	2
Item Type:	SR
Score Points:	1
Difficulty:	M
Key:	One Solution, No Solution, One Solution
Stimulus/Source:	
Target-Specific Attributes (e.g., accessibility issues):	
Notes:	Calculator tool should be turned on.

For each linear equation in this table, indicate whether the equation has no solution, one solution, or infinitely many solutions.

Equation	No Solution	One Solution	Infinitely Many Solutions
$7x + 21 = 21$			
$12x + 15 = 12x - 15$			
$-5x - 25 = 5x + 25$			

*Key and Distractor Analysis:*

1. One solution. This is designed to be an easy equation to solve to help students enter the problem. Answering this question correctly demonstrates minimal understanding.
2. No solution. Students may think there is no difference between adding 15 on the left-hand side and subtracting 15 on the right-hand side.
3. One solution. Students may think there are infinitely many solutions because the left-hand side is the negative of the right-hand side.

This item is worth 1 point because all of the concepts used to answer it correctly are linked.

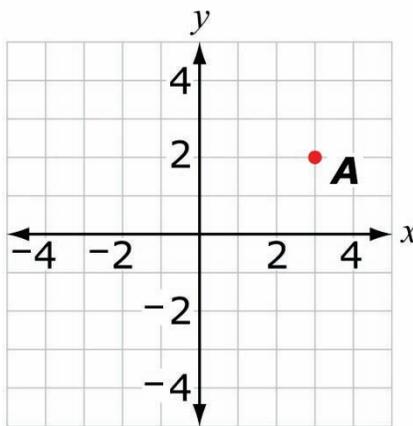
## MAT.08.TE.1.0000F.E.140

Sample Item ID:	<b>MAT.08.TE.1.0000F.E.140</b>
Grade:	08
Claim(s):	<b>Claim 1: Concepts and Procedures</b> Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Assessment Target(s):	<b>1 E:</b> Define, evaluate, and compare functions.
Content Domain:	Functions
Standard(s):	8.F.1
Mathematical Practice(s):	1, 2, 5
DOK:	1
Item Type:	TE
Score Points:	1
Difficulty:	M
Key:	$(3, 5), (3, 4), (3, 3), (3, 1), (3, 0), (3, -1),$ $(3, -2), (3, -3), (3, -4),$ or $(3, -5)$
Stimulus/Source:	
Target-Specific Attributes (e.g., accessibility issues):	
Notes:	TEI Template: Placing Points. The candidate can choose any lattice point shown on the $xy$ -plane.

Point  $A$  is plotted on the  $xy$ -coordinate plane below. You must determine the location of point  $C$  given the following criteria:

- Point  $C$  has integer coordinates.
- The graph of line  $\overline{AC}$  is **not** a function.

Click on the  $xy$ -coordinate plane below to place a point that could represent point  $C$ .



*Key and Distractor Analysis:*

For the correct answer, students must choose a point that is an ordered pair of integers along the line  $x = 3$ .

*TE Information:*

**Item Code:** MAT.08.TE.1.0000F.E.140

**Template:** Placing Points

**Interaction Space Parameters:**

- A: False
- B: (-5, -5); (5, 5); one unit; axes are labeled with  $x$  and  $y$
- C: True
- D: True
- E: The point (3, 2) labeled  $A$
- F: True
- G: One

**Scoring Data:** SumOnly

For all coordinates, tolerance = 0, correct score-points = 1, and incorrect score-points = 0

(3,5)

(3,4)

(3,3)

(3,1)

(3,0)

(3,-1)

(3,-2)

(3,-3)

(3,-4)

(3,-5)

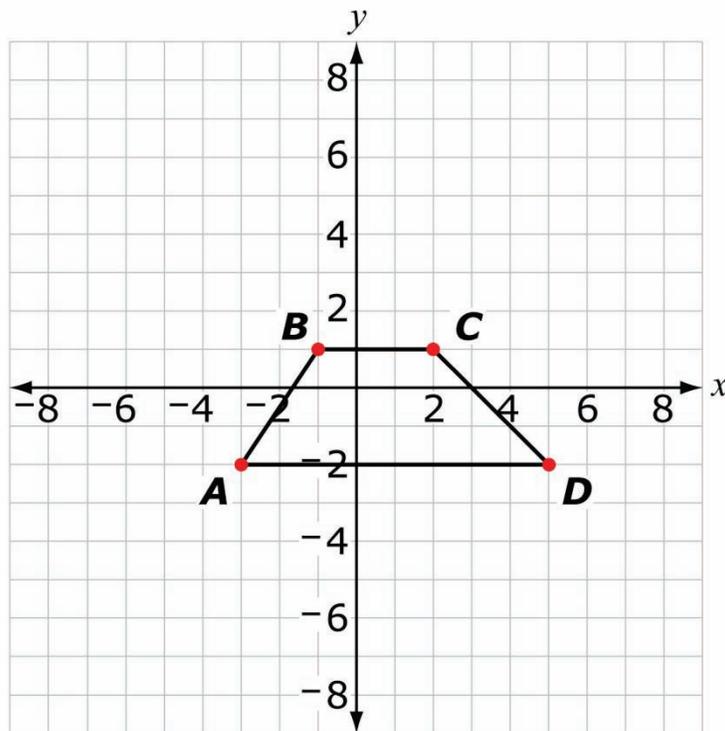
Grade 8 Mathematics Sample TE Item



MAT.08.TE.1.0000G.G.146

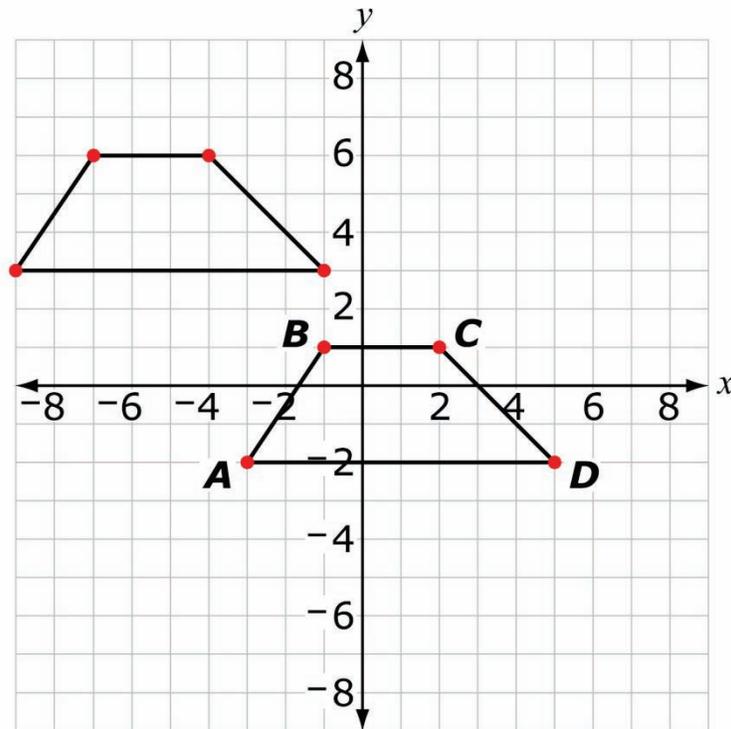
Sample Item ID:	<b>MAT.08.TE.1.0000G.G.146</b>
Grade:	08
Claim(s):	<b>Claim 1: Concepts and Procedures</b> Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Assessment Target(s):	<b>1 G:</b> Understand congruence and similarity using physical models, transparencies, or geometry software.
Content Domain:	Geometry
Standard(s):	8.G.2
Mathematical Practice(s):	6
DOK:	1
Item Type:	TE
Score Points:	1
Difficulty:	L
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Target-Specific Attributes (e.g., accessibility issues):	
Notes:	Vertex Based Quadrilaterals TE template

Trapezoid  $ABCD$  is shown on this coordinate grid. Translate trapezoid  $ABCD$  6 units to the left and 5 units up and graph the image of  $ABCD$  on the grid.



To graph the image, plot the vertices of the image on the coordinate grid and then connect the vertices with line segments.

Sample Top-score Response:



*TE information:*

**Item Code:** MAT.08.TE.1.0000G.G.146

**Template: Vertex Based Quadrilaterals**

**Interaction Space Parameters:**

- A. True
- B.
- C. True
- D. True
- E. A quadrilateral with the following vertices with their labels:  
A (-3, -2), B (-1, 1), C (2, 1), and D (5, -2).
- F. False

**Scoring Data (Specific to Each Item):**

- 1. False
- 2. True
  - a. (-9, 3), tolerance = 0
  - b. (-7, 6), tolerance = 0
  - c. (-4, 6), tolerance = 0
  - d. (-1, 3), tolerance = 0
- 3. False
- 4. False
- 5. False

Grade 8 Mathematics Sample TE Item

**MAT.08.TE.1.000EE.B.144**

Sample Item ID:	<b>MAT.08.TE.1.000EE.B.144</b>
Grade:	08
Claim(s):	<b>Claim 1: Concepts and Procedures</b> Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Assessment Target(s):	<b>1 B:</b> Work with radicals and integer exponents.
Content Domain:	Equations and Expressions
Standard(s):	8.EE.2
Mathematical Practice(s):	1, 2, 4, 5, 7, 8
DOK:	1
Item Type:	TE
Score Points:	1
Difficulty:	L
Key:	See TE Information.
Stimulus/Source:	
Target-Specific Attributes (e.g., accessibility issues):	Click and drag functionality will be adapted to a tab functionality for accessibility considerations.
Notes:	The item should prevent students from using numbers more than once. Calculators may not be used on this item.

Use the numbers shown to make the equations true. Each number can be used only once. To use a number, drag it to the appropriate box in an equation.

4	8	10	64	100	1,000
---	---	----	----	-----	-------

$\sqrt{\quad} \quad \square$

$\sqrt[3]{\quad} \quad \square$

*Key:*

See TE Information.

*TE Information:*

**Item Code:** MAT.08.TE.1.000EE.B.144

**Template:** Select and Order

**Interaction Space Parameters:**

- A. An image with the four rectangles. (The rectangle under the square root symbol will be considered 1, the rectangle equal to the square root of the number in rectangle 1 will be considered 2, the rectangle under the cube root symbol will be considered 3, and the rectangle equal to the cube root of the number in rectangle 3 will be considered 4.)
- B. Six images of numbers.

**Scoring Data:**

$\{64,8,1000,10\}=1$

$\{100,10,64,4\}=1$

Grade 8 Mathematics Sample TE Item

**MAT.08.TE.1.000EE.B.323**

Sample Item ID:	<b>MAT.08.TE.1.000EE.B.323</b>
Grade:	08
Claim(s):	<b>Claim 1: Concepts and Procedures</b> Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.
Assessment Target(s):	<b>1 B:</b> Work with radicals and integer exponents.
Content Domain:	Expressions and Equations
Standard(s):	8.EE.2
Mathematical Practice(s):	6
DOK:	1
Item Type:	TE
Score Points:	2
Difficulty:	M
Key:	See Sample Top-Score Response and TE Information.
Stimulus/Source:	
Target-Specific Attributes (e.g., accessibility issues):	Calculators are not available for this item. Click and drag functionality will be adapted to a tab functionality for accessibility considerations.
Notes:	TE template: Classification

Classify the numbers in the box as perfect squares and perfect cubes. To classify a number, drag it to the appropriate column in the chart. Numbers that are neither perfect squares nor perfect cubes should **not** be placed in the chart.

1    64    96    125    200    256    333    361
--

<b>Perfect Squares but Not Perfect Cubes</b>	<b>Both Perfect Squares and Perfect Cubes</b>	<b>Perfect Cubes but Not Perfect Squares</b>

*Sample Top-Score Response:*

<b>Perfect Squares but Not Perfect Cubes</b>	<b>Both Perfect Squares and Perfect Cubes</b>	<b>Perfect Cubes but Not Perfect Squares</b>
256	1	125
361	64	

*Scoring Rubric:*

See TE information.

Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student shows a thorough understanding of classifying positive integers as perfect squares, perfect cubes, or neither. The student places all the perfect squares and perfect cubes in the correct regions of the table and does not identify any of the other numbers as perfect squares or perfect cubes.

**1 point:** The student shows a partial understanding of classifying positive integers as perfect squares, perfect cubes, or neither. The student places all the perfect squares and perfect cubes in the table but makes one or two errors by not realizing that 1 and 64 are both perfect squares and perfect cubes.

**0 points:** The student shows inconsistent or no understanding of classifying positive integers as perfect squares, perfect cubes, or neither. The student misclassifies perfect squares as perfect cubes, perfect cubes as perfect squares, or identifies numbers that are neither perfect squares or perfect cubes as perfect squares or perfect cubes.

*TE Information:*

**Item Code:** MAT.08.TE.1.000EE.B.323

**Template:** Classification

**Interaction Space Parameters:**

- A. The 3 sections of the Venn diagram: perfect squares only, both perfect squares and perfect cubes, perfect cubes only
- B. The following 8 numbers: 1, 64, 96, 125, 200, 256, 333, and 361.

**Scoring Data:**

$$\{1=1,2=AB,3=D\}=2$$

$$\{1=ABFH,3=D\}=1$$

$$\{1=1,3=ABD\}=1$$

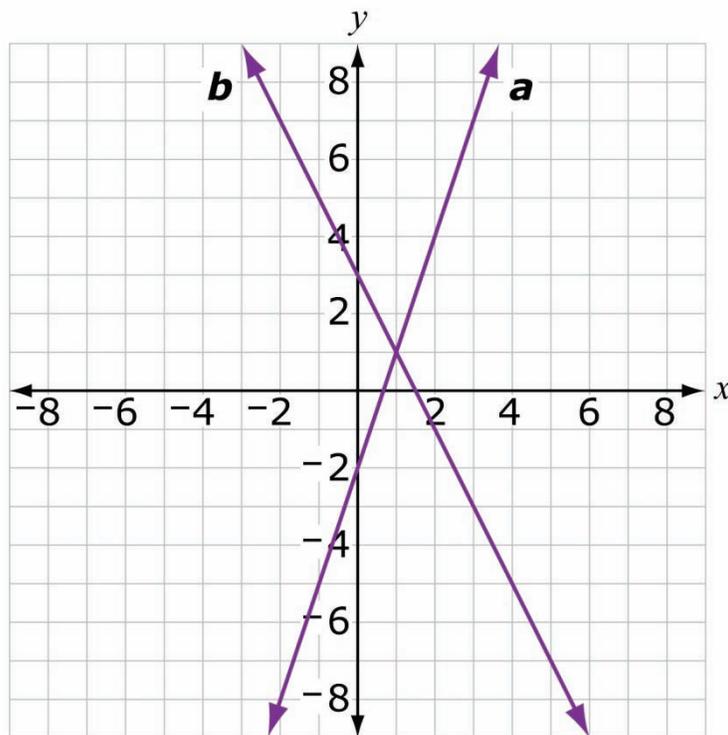
$$\{1=AFH,3=BD\}=1$$

$$\{1=BFH,3=AD\}=1$$

## MAT.08.TE.1.000EE.C.200

Sample Item ID:	<b>MAT.08.TE.1.000EE.C.200</b>
Grade:	08
Claim(s):	<b>Claim 1: Concepts and Procedures</b> Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Assessment Target(s):	<b>1 C:</b> Understand the connections between proportional relationships, lines, and linear equations.
Content Domain:	Equations and Expressions
Standard(s):	8.EE.8
Mathematical Practice(s):	1, 5, 7
DOK:	1
Item Type:	TE
Score Points:	2
Difficulty:	L
Key:	The equation of line $a$ is $y = -2x + 3$ . The equation of line $b$ is $y = 3x - 2$ .
Stimulus/Source:	
Target-Specific Attributes (e.g., accessibility issues):	Click and drag functionality will be replaced by tab-functionality as needed for accessibility.
Notes:	TE template: Select and Order

The graphs of line  $a$  and line  $b$  are shown on this coordinate grid.



Match each line with its equation. Click on an equation and then drag it to the corresponding box for each line.

The equation of line  $a$  is .

The equation of line  $b$  is .

$$y = -2x + 3$$

$$y = 2x + 3$$

$$y = 3x - 2$$

$$y = -\frac{1}{2}x + 3$$

$$y = -\frac{1}{3}x - 2$$

*TE Information:*

**Item Code:** MAT.08.TE.1.000EE.C.200

**Template: Select and Order**

**Interaction Space Parameters:**

- A. An image with the two rectangles. (The rectangle for the equation of line  $a$  will be considered 1, and the rectangle for the equation of line  $b$  will be considered 2.)
- B. Five images of equations.

**Scoring Data:**

$$\{y = 3x - 2, y = -2x + 3\} = 2$$

$$\{y = 3x - 2, y = 2x + 3\} = 1$$

$$\{y = 3x - 2, y = -\frac{1}{2}x + 3\} = 1$$

$$\{y = 3x - 2, y = \frac{1}{3}x - 2\} = 1$$

$$\{y = 2x + 3, y = -2x + 3\} = 1$$

$$\{y = -\frac{1}{2}x + 3, y = -2x + 3\} = 1$$

$$\{y = \frac{1}{3}x - 2, y = -2x + 3\} = 1$$

## Grade 8 Mathematics Sample TE Item

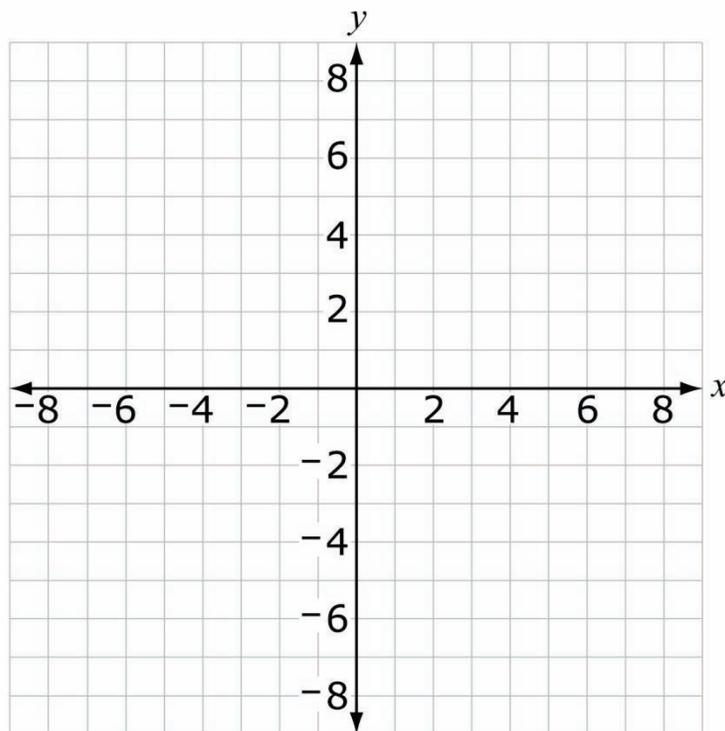


## MAT.08.TE.1.000EE.D.147

Sample Item ID:	<b>MAT.08.TE.1.000EE.D.147</b>
Grade:	08
Primary Claim:	<b>Claim 1: Concepts and Procedures</b> Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Secondary Claim(s):	
Primary Content Domain:	Expressions and Equations
Secondary Content Domain(s):	
Assessment Target(s):	1 D: Analyze and solve linear equations and pairs of simultaneous linear equations.
Standard(s):	8.EE.8
Mathematical Practice(s):	1, 2, 5
DOK:	2
Item Type:	TE
Score Points:	3
Difficulty:	M
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Target-Specific Attributes (e.g., accessibility issues):	
Notes:	TE template: Straight Lines AI will be needed to score this item.

**Part A**

The solution of a system of two linear equations is  $(-3, 1)$ . On this coordinate grid, graph two lines that could be the graphs of the two linear equations in the system.

**Part B**

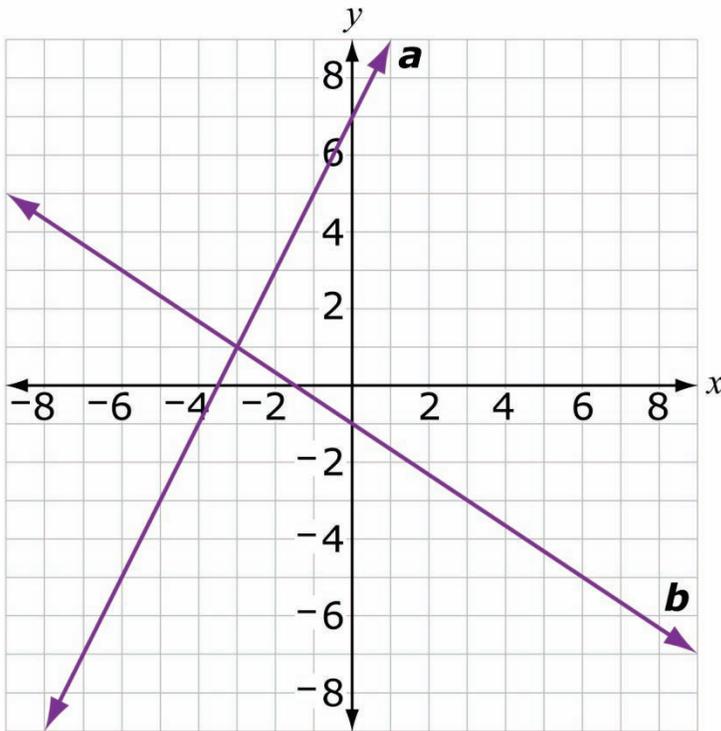
The computer has labeled the lines you graphed  $a$  and  $b$ .

What is the equation of line  $a$  ?

What is the equation of line  $b$  ?

*Sample Top-Score Response:*

**Part A**



**Part B**

Line *a*:  $y = 2x + 7$

Line *b*:  $y = -\frac{2}{3}x - 1$

*Scoring Rubric:*

Responses to this item will receive 0-3 points, based on the following:

**3 points:** The student shows a thorough understanding of the solutions of systems of equations and writing the equations of lines from a graph. The two lines intersect at the point  $(-3, 1)$  and both of the equations given are correct.

**2 points:** The student shows a partial understanding of the solutions of systems of equations and writing the equations of lines from a graph. The response shows two lines that intersect at the point  $(-3, 1)$  but one or both equations given are incorrect.

**1 point:** The student shows a limited understanding of the solutions of systems of equations and writing the equations of lines from a graph. The response shows two lines that do NOT intersect at the point  $(-3, 1)$  but both equations given are correct.

**0 points:** The student shows inconsistent or no understanding of the solutions of systems of equations and writing the equations of lines from a graph. The response shows two lines that do NOT intersect at the point  $(-3, 1)$  and there is an error in at least one of the equations of the lines.

*TE Information:*

**Item Code:** MAT.08.TE.2.000EE.B.147 Part A

**Template: Straight Lines**

**Interaction Space Parameters:**

- A. True
- B.
- C. True (visible)
- D. False
- E.
- F. True
- G. Draw lines extended across the interaction space with arrowheads at the outermost points
- H. True
- I. 2
- J. Label first and last grid increment

**Scoring Data:**

Line 1:

- a. Start point
  - i. Do not consider
- b. End point
  - i. Do not consider
- c. x-intercept
  - i. Do not consider
- d. y-intercept
  - i. Do not consider
- e. Slope
  - i. Do not consider

Line 2:

- a. Start point
    - i. Do not consider
  - b. End point
    - i. Do not consider
  - c. x-intercept
    - i. Do not consider
  - d. y-intercept
    - i. Do not consider
  - e. Slope
    - i. Do not consider
- A. Parallel=false
  - B. Perpendicular=false
  - C. Intersecting
    - i. True
    - ii. True
    - iii. (-3,1)

## MAT.08.TE.2.0000F.A.148

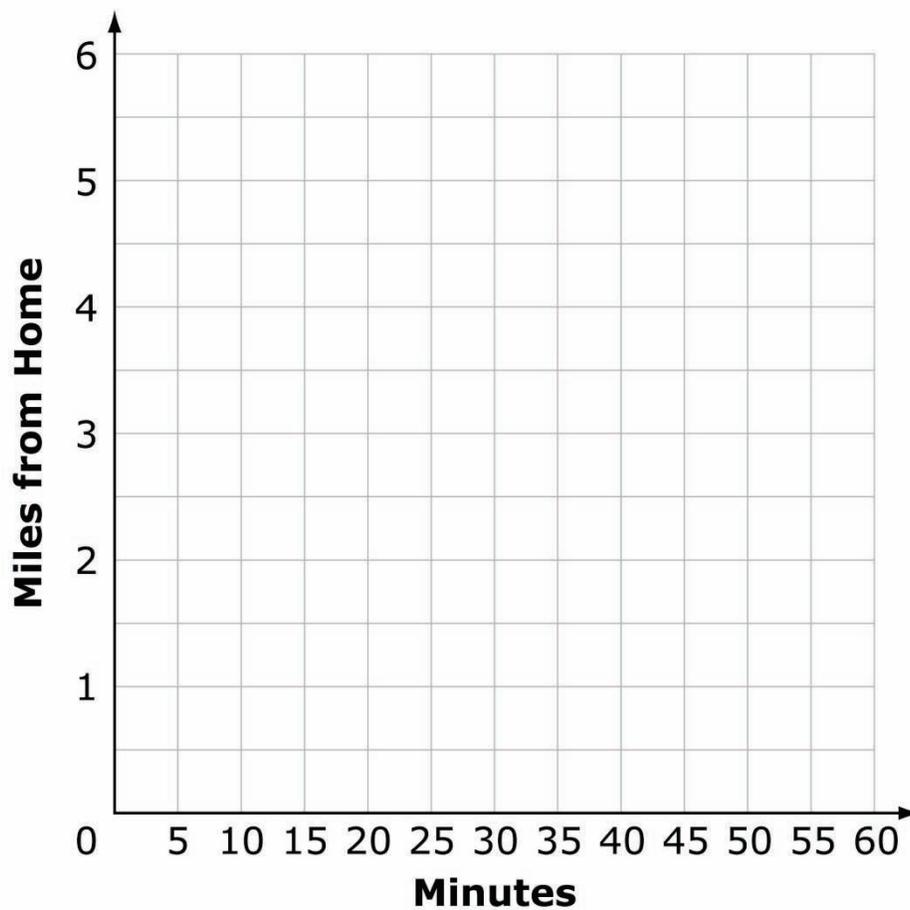
Sample Item ID:	<b>MAT.08.TE.2.0000F.A.148</b>
Grade:	08
Primary Claim:	<b>Claim 2: Problem Solving</b> Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.
Secondary Claim(s):	<b>Claim 1: Concepts and Procedures</b> Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	Functions
Secondary Content Domain(s):	
Assessment Target(s):	<b>2 A:</b> Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.  <b>2 B:</b> Select and use appropriate tools strategically.  <b>1 F:</b> Use functions to model relationships between quantities.
Standard(s):	8.F.5
Mathematical Practice(s):	1, 2, 4
DOK:	2
Item Type:	TE
Score Points:	2
Difficulty:	M
Key:	See Sample Top-Score Response.
Stimulus/Source:	
Target-Specific Attributes (e.g., accessibility issues):	
Notes:	The purpose of this item is to determine if students can sketch a graph that matches the verbal description of a function.

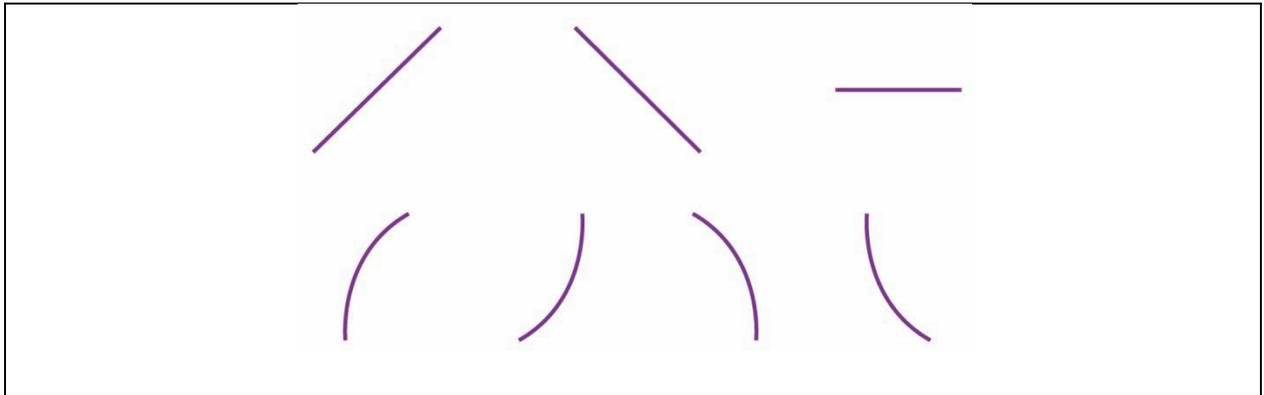
Carla rode her bike to her grandmother's house. The following information describes her trip:

- For the first 5 minutes, Carla rode fast and then slowed down. She rode 1 mile.
- For the next 15 minutes, Carla rode at a steady pace until she arrived at her grandmother's house. She rode 3 miles.
- For the next 10 minutes, Carla visited her grandmother.
- For the next 5 minutes, Carla rode slowly at first but then began to ride faster. She rode 1 mile.

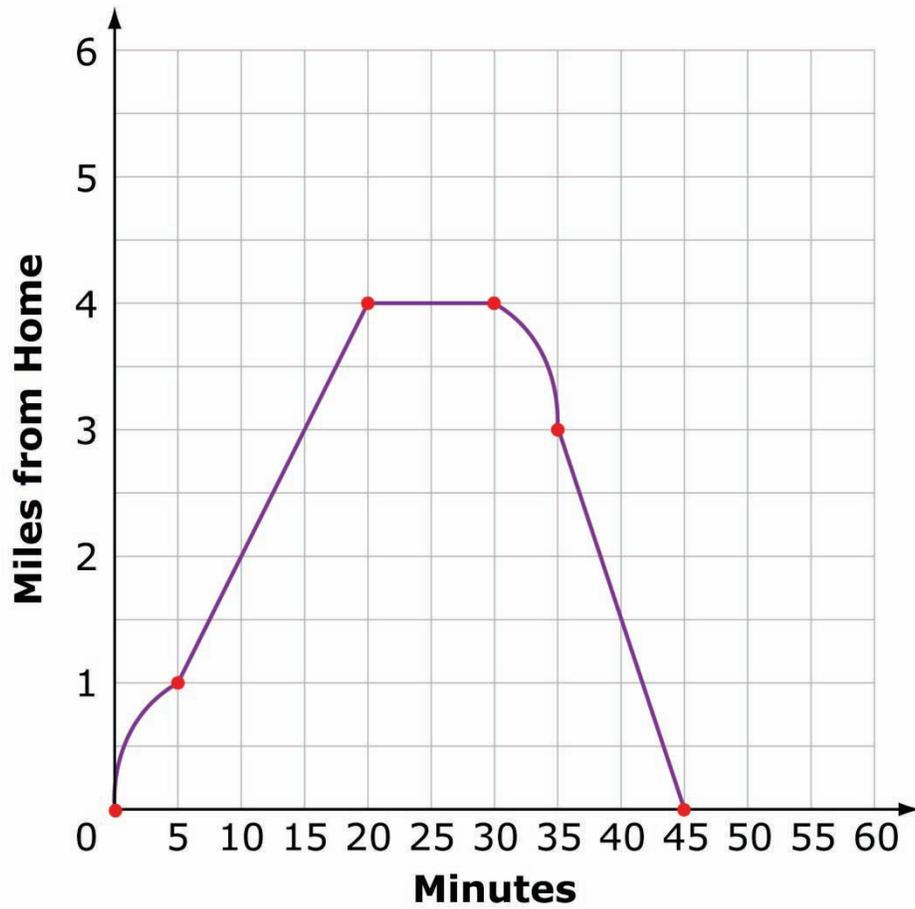
- For the last 10 minutes, Carla rode fast. She rode 3 miles at a steady pace.

Graph each part of Carla's trip. To graph part of her trip, first click the correct line type in the box. Then click in the graph to add the starting point and the ending point for that part of her trip. Repeat these steps until a graph of Carla's entire trip has been created.





*Sample Top-Score Response:*



*Scoring Rubric:*

Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student shows a thorough understanding of how to sketch the graph of a function based on a verbal description. The student's response matches the Sample Top-score Response.

**1 point:** The student shows a partial understanding of how to sketch the graph of a function based on a verbal description. The student makes 1 or 2 minor errors in the graph, such as using a straight line when a curved line should be used or using a concave-up curve when a concave-down curve should be used.

**0 points:** The student shows inconsistent or no understanding of how to sketch the graph of a function based on a verbal description.

## MAT.08.TE.2.000EE.A.145

Sample Item ID:	<b>MAT.08.TE.2.000EE.A.145</b>
Grade:	08
Primary Claim:	<b>Claim 2: Problem Solving</b> Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.
Secondary Claim(s):	<b>Claim 1: Concepts and Procedures</b> Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	Equations and Expressions
Secondary Content Domain(s):	
Assessment Target(s):	<b>2 A:</b> Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace.  <b>2 B:</b> Select and use appropriate tools strategically.  <b>1 B:</b> Work with radicals and integer exponents.
Standard(s):	8.EE.4
Mathematical Practice(s):	1, 2, 5, 6, 7, 8
DOK:	2
Item Type:	TE
Score Points:	1
Difficulty:	M
Key:	Schedar, Ruchbah
Stimulus/Source:	Ridpath, Ian. The Facts on File atlas of stars and planets: a beginner's guide to the universe. New York: Facts on File, 1993.
Target-Specific Attributes (e.g., accessibility issues):	Calculators should be available for this target.
Notes:	TE template: Select Objects

A light-year is a unit of distance. It is the distance that light travels in 1 year. For example, the distance from the North Star to Earth is about 434 light-years because it takes light about 434 years to travel from the North Star to Earth.

The table lists five stars in the constellation Cassiopeia and their approximate distances, in light-years, from Earth.

Light travels at a speed of  $3 \times 10^8$  meters per second. Highlight each star in the table that is between  $7 \times 10^{17}$  meters and  $3 \times 10^{18}$  meters from Earth. Click the name of a star to highlight it.

Star	Distance from Earth in Light Years
Schedar	228.56
Caph	54.46
Tsih	613.08
Ruchbah	99.41
Segin	441.95

*TE Information:*

**Item Code:** MAT.08.TE.2.000EE.A.145

**Template:** Select Objects

**Interaction Space Parameters:**

- A. The five rows in the table in the item
- B. False

**Scoring Data:**

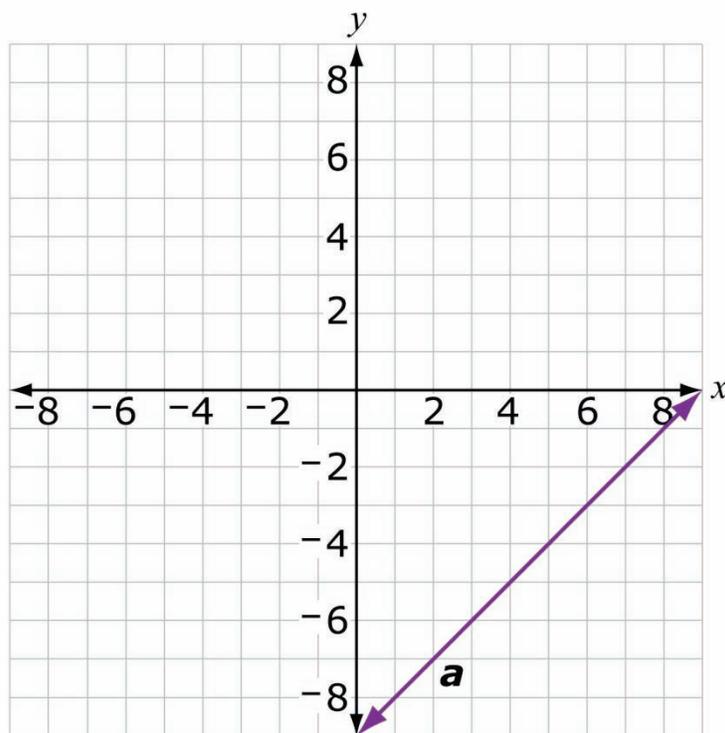
{Schedar, Ruchbah}=1

## MAT.08.TE.2.000EE.B.149

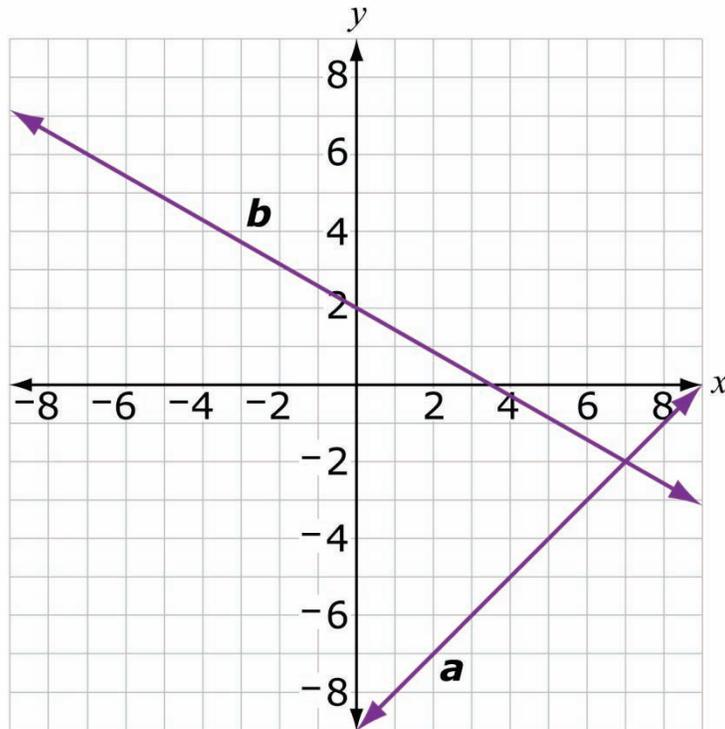
Sample Item ID:	<b>MAT.08.TE.2.000EE.B.149</b>
Grade:	08
Primary Claim:	<b>Claim 2: Problem Solving</b> Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.
Secondary Claim(s):	<b>Claim 1: Concepts and Procedures</b> Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	Equations and Expressions
Secondary Content Domain(s):	
Assessment Target(s):	<b>2 B:</b> Select and use appropriate tools strategically.  <b>1 D:</b> Analyze and solve linear equations and pairs of simultaneous linear equations.
Standard(s):	8.EE.8
Mathematical Practice(s):	1, 2, 5
DOK:	2
Item Type:	TE
Score Points:	1
Difficulty:	L
Key:	See Sample Top-Score Response and TE Information.
Stimulus/Source:	
Target-Specific Attributes (e.g., accessibility issues):	
Notes:	The purpose of this item is to determine students' understanding of the solutions of systems of linear equations.

Line  $a$  is shown on the coordinate grid. Construct line  $b$  on the coordinate grid so that

- line  $a$  and line  $b$  represent a system of linear equations with a solution of  $(7, -2)$ .
- the slope of line  $b$  is greater than  $-1$  and less than  $0$ .
- the  $y$ -intercept of line  $b$  is positive.



Sample Top-Score Response:



*TE Information:*

**Item Code:** MAT.08.TE.2.000EE.B.149

**Template:** Single Line

**Interaction Space Parameters:**

- A: Use default grid
- C: Make grid visible
- D: Use graphic overlay
- E: Graph of the equation  $y = x - 9$  labeled  $a$  with arrows on each end
- F: Support snap-to behavior
- G: Draw an extended line with arrows on each end

**Scoring Data:**

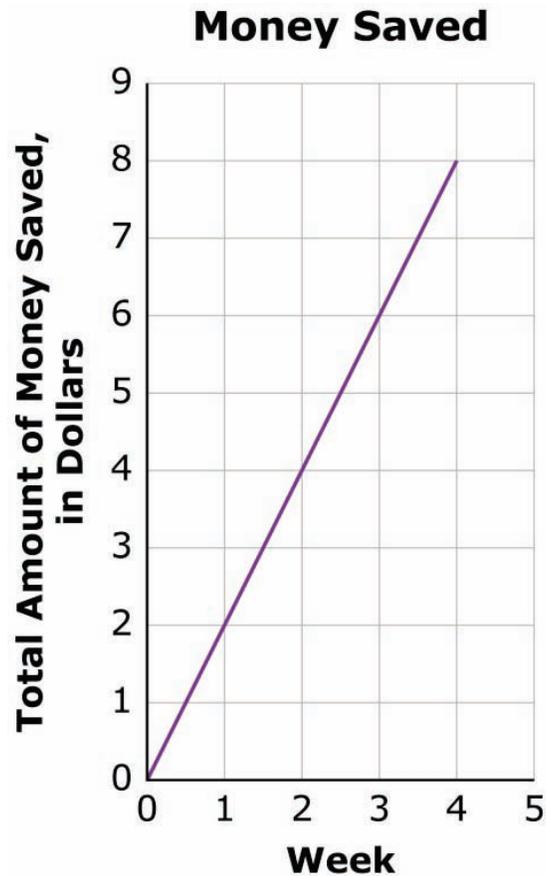
- Start Point
  - A: Do not consider
- End Point
  - A: Do not consider
- x-Intercept
  - A: Do not consider
- y-Intercept
  - A: Consider
  - B: Greater than 0
  - C: 0
- Slope
  - A: Consider
  - B: Between -1 and 0
  - C: 0

## MAT.08.TE.2.000EE.C.202 Claim 2

Sample Item ID:	MAT.08.TE.2.000EE.C.202
Grade:	08
Primary Claim:	<b>Claim 2: Problem Solving</b> Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.
Secondary Claim(s):	Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Primary Content Domain:	Equations and Expressions
Secondary Content Domain(s):	
Assessment Target(s):	2 C: Interpret results in the context of a situation.  1 C: Understand the connections between proportional relationships, lines, and linear equations.
Standard(s):	8.EE.5
Mathematical Practice(s):	1, 2, 4, 5, 7, 8
DOK:	2
Item Type:	TE
Score Points:	1
Difficulty:	L
Key:	Omar, Carla
Stimulus/Source:	
Target-specific attributes (e.g., accessibility issues):	Click and drag functionality will be adapted to a tab functionality for accessibility considerations.
Notes:	TE template: Select and Order Calculator tool should be turned on for this item.

Three students saved money for four weeks.

Antwan saved the same amount of money each week for 4 weeks. He made this graph to show how much money he saved.



Carla saved the same amount of money each week for 4 weeks. She made this table to show how much money she saved.

Week	Total Amount of Money Saved
1	\$1.75
2	\$3.50
3	\$5.25
4	\$7.00

Omar saved the same amount of money each week for 4 weeks. He wrote the equation below to show how much he saved. In the equation,  $S$  is the total amount of money saved, in dollars, and  $w$  is the number of weeks.

$$S = 2.5w$$

Identify the student who saved the greatest amount of money each week **and** the student who saved the least amount of money each week. To select a student, drag the student's name into the box next to the appropriate description.

**Antwan****Carla****Omar****Student Name****Student Description**

Student who saved the greatest amount of money each week

Student who saved the least amount of money each week

*TE Information:*

**Item Code:** MAT.08.TE.2.000EE.C.202

**Template:** Select and Order

**Interaction Space Parameters:**

- A. An image with two rectangles. (The rectangle for the student who saved the most each week will be considered 1 and the rectangle for the student who saved the least each week will be considered 2.)
- B. Three images of names.

**Scoring Data:**

{Omar,Carla}=1