Chapters 5 & 6 Review

1. Steve says "I have two children, one of which is a boy". Given this information, what is the probability that Steve has two boys?

   (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{1}{4}$ (E) None of the above.

2. There are 8 different kinds of prizes given at a carnival game. If we select 4 prizes from a bin returning each one after it is selected, how many different outcomes of prizes are in the sample space?

   (A) 4^8 (B) 8^4 (C) 32 (D) 4 (E) 8

3. Which of the following statements is/are true?

   I. Independent events can be disjoint
   II. If two events are not disjoint, then they must be independent
   III. If two events are independent, then they not must be disjoint

   (A) III only. (B) II and III only. (C) I and III only. (D) I, II, and III. (E) II only

   Use the following information for the next 2 questions

A standard deck of 52 cards is acquired. For those who do not know, a deck of cards contains 4 suits (hearts, diamonds, clubs, and spades). Hearts and diamonds are red, clubs and spades are black. There are 13 denominations of cards (2,3,4,5,6,7,8,9,10, Jack, Queen, King, and Ace).

4. If you select two cards not replacing each one after it is selected, what is the probability that you select a black card and a red card?

   (A) $\left(\frac{25}{51}\right)$ (B) $\left(\frac{26}{51}\right)$ (C) $\left(\frac{13}{51}\right)$ (D) $\left(\frac{13}{102}\right)$ (E) None of the above

5. If you select two cards without replacing each one after it is selected, what is the probability that you select at least one ace?

   (A) $\frac{188}{221}$ (B) $\frac{15}{34}$ (C) $\frac{33}{221}$ (D) $1-\frac{39}{52}$ (E) None of the above

6. If events *C* and *D* are disjoint, what is $P(X|Y)$?

   (A) P(*C*) (B) P(*D*) (C) P(*C*) + P(*D*) (D) 0 (E) 1

7. Suppose *X* is a random variable with mean μ. Suppose we observe *X* many times and keep track of the average of the observed values. The law of large numbers says that

   (A) The value of μ will get larger and larger as we observe *X*.
   (B) As we observe *X* more and more, this average and the value of μ will get larger and larger.
   (C) This average will get closer and closer to μ as we observe *X* more and more often.
   (D) As we observe *X* more and more, this average will get to be a larger and larger multiple of μ.
   (E) All of the above are true

   In a population of students, the number of calculators owned is a random variable *X* with $P(X = 0) = 0.2$, $P(X = 1) = 0.6$, and $P(X = 2) = 0.2$.

8. The mean of this probability distribution is:

   (A) 0 (B) 0.5 (C) 1 (D) 1.5 (E) 2

9. The variance of this population is:

(A) 0 (B) 0.4 (C) 0.5 (D) 0.63 (E) 1

A semi-truck ships computers from Atlanta to Ft. Lauderdale. The weight of any one computer that is put on the truck follows a normal model with a mean of 56 pounds and a standard deviation of 10.4 pounds. Currently, the truck contains 500 computers. Assume the weights of the computers are independent random variables.

10. What is the expected value for the total weight of the shipment aboard the semi-truck?

(A) 56 pounds (B) 560 pounds (C) 3136 pounds (D) 28,000 pounds (E) 167.33 pounds

11. What is the standard deviation for the total weight of the shipment aboard the semi-truck?

(A) 10.4 pounds (B) 5200 pounds (C) 108.16 pounds (D) 54080 pounds (E) 232.55 pounds

12. The semi-truck approaches a weigh station. If the total weight of the shipment exceeds 28,300 pounds, the truck driver will have to pay an additional charge for the excess weight. What is the approximate probability that he will have to pay this additional charge?

(A) 0.10 (B) 0.01 (C) 0.90 (D) 0.99 (E) 0.95

13. A local street contains 7 street lights. The probability that any one street light does not work is 0.30. What is the probability that at least one does not work?

(A) 0.0002 (B) 0.9998 (C) 0.0824 (D) 0.9176 (E) 0.9563

14. In a particular game, a fair die is tossed. If the number of spots showing is either 4 or 5 you win $1, if number of spots showing is 6 you win $4, and if the number of spots showing is 1, 2, or 3 you win nothing. Let *X* be the amount that you win when playing the game once. The expected value of *X* is

(A) $0.00 (B) $1.00 (C) $2.50 (D) $4.00 (E) $6.00

15. Cans of soft drinks cost $0.30 in a certain vending machine. The characteristics of the random variable *X*, the number of cans sold per day, are given as:

$$E(X) = 125, \text{ and } Var(X) = 50$$

The expected value and variance of the daily revenue (*Y*) is:

(A) E(*Y*) = 37.5, Var(*Y*) = 50 (B) E(*Y*) = 37.5, Var(*Y*) = 4.5
(C) E(*Y*) = 37.5, Var(*Y*) = 15 (D) E(*Y*) = 37.5, Var(*Y*) = 10 (E) E(*Y*) = 125, Var(*Y*) = 4.5

16. Which of the following is considered a discrete random variable?

(A) The number of ounces of coke in a 12oz can
(B) The number of headaches experienced in a day at EHS
(C) The amount of time it takes for a car to start
(D) The amount of carbon monoxide in the air (in moles)
(E) All of the above are discrete

17. What is the probability that the 4^{th} student sampled is the first one who has experienced math anxiety?

(A) $(0.20)^4$ (B) $(0.20)^6$ (C) $(0.20)^3(0.80)$ (D) $(0.80)^3$ (E) $(0.80)^3(0.20)$

18. If we select 8 students, what is the probability that at least one student has experienced math anxiety?

(A) 0.80 (B) 0.20 (C) 0.8322 (D) 0.1978 (E) 0.64

19. If we select 8 students, what is the probability that exactly three have experienced math anxiety?

(A) $\binom{8}{3}(0.20)^3(0.80)^8$ (B) $\binom{8}{3}(0.20)^3(0.80)^5$ (C) $\binom{5}{3}(0.20)^3(0.80)^5$

(D) $(0.20)^3(0.80)^5$ (E) $(0.20)^5(0.80)^3$

20. If we select 8 students, what is the probability that at least two have experienced math anxiety?

(A) 0.2936 (B) 0.64 (C) 0.4967 (D) 0.5033 (E) 0.6225

21. Which of the following is not a qualification for a random variable to be classified as geometric?

(A) There are two outcomes
(B) There are a fixed number of trials
(C) The probability of success is constant
(D) The trials are independent
(E) The probability of failure is constant

22. Let's say that the probability of success for an event is 0.23 and I want to use a normal distribution to approximate the binomial distribution. What is the minimum number of trials that I need to run before this becomes appropriate?

(A) 10 (B) 35 (C) 187 (D) 44 (E) 13

23. Which of these follows a binomial model?

(A) The number of black cards drawn from a deck until an ace is found
(B) The color of the cars in a parking lot
(C) Your mom
(D) The number of people we survey until we find one who owns an iPod
(E) The number of hits a baseball player gets in 6 times at bat

24. Which of these follows a geometric model?

(A) The number of black cards present in a hand of 10 cards
(B) The color of the cars in a parking lot
(C) Seriously, your mom
(D) The number of people we survey until we find one who owns an iPod
(E) The number of hits a baseball player gets in 6 times at bat

25. Ten percent of all trucks undergoing a certain inspection will fail the inspection. Assume that trucks are independently undergoing this inspection one at a time. The expected number of trucks inspected before a truck fails inspection is

(A) 2 (B) 4 (C) 5 (D) 10 (E) 20

Solutions:

1. [new] **Answer: (B)**

If Steve says that he has two children and <u>one</u> of them is a boy, then you have to take order into account. So the sample space contains:

BG GB BB

Therefore, there is a 1/3 probability that he has two boys.

2. [new] **Answer: (B)**

Remember the rule for the number of outcomes in the sample space:

$$\text{number of possibilities}^{\text{number you select}}$$

Therefore: $8^4 = 4096$

3. [new] **Answer: (A)**

I. is FALSE $\rightarrow$ If two events are independent, they <u>cannot</u> be disjoint and vice versa
II. is FALSE $\rightarrow$ There are three types of events we cover: (1) independent, (2) disjoint, (3) neither
III. is TRUE $\rightarrow$ If two events are disjoint, they <u>cannot</u> be independent and vice versa

4. [new] **Answer: (B)**

Since I did not specify order, you have to take into account both orders that exist:

Black&Red OR Red&Black

$$\left(\frac{26}{52}\right)\left(\frac{26}{51}\right)+\left(\frac{26}{52}\right)\left(\frac{26}{51}\right)=\frac{26}{51}$$

5. [new] **Answer: (C)**

P(at least one ace) = 1 – P(no aces)

$$\text{P(at least one ace)} = 1 - \left(\frac{48}{52}\right)\left(\frac{47}{51}\right) = \frac{33}{221}$$

6. [new] **Answer: (D)**

According to your formula sheet:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

But, if two events are disjoint, then $P(A \cap B) = 0$. So: $P(A|B) = 0$.

7. [new] **Answer: (C)**

Remember that the mean (a.k.a. expected value) of any probability distribution is the long term average in all such trials. Therefore, as you observe the outcomes of any random variable more and more and more, they should steadily approach the true average value.

8. [new] **Answer: (C)**

Do, this in your calculator or use the formula:

$$E(X) = 0(0.2) + 1(0.6) + 2(0.2) = 1$$

9. [new] **Answer: (B)**

Again, do this in your calculator or use the formula (but be careful that your calculator gives you the standard deviation… I'm asking for the variance):

$$Var(X) = (0-1)^2(0.2) + (1-1)^2(0.6) + (2-1)^2(0.2) = 0.4$$

10. [new] **Answer: (D)**

$$\mu_{\text{TOTAL}} = 500(56) = 28{,}000 \text{ pounds}$$

11. [new] **Answer: (E)**

$$\sigma_{\text{TOTAL}} = \sqrt{10.4^2(500)} = 232.55 \text{ pounds}$$

12. [new] **Answer: (A)**

Using the answers to #7 and #8, just acquire a *z*-score:

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{28300 - 28000}{232.55} = 1.29$$

[normalcdf(1.29,10000)]

P(X> 28300) = P(z > 1.29) = 0.0985 ≈ 0.10

13. [old] **Answer: (D)**

P(at least one) = 1 – P(none)

P(at least one does not work) = 1 – P(none do not work)

P(at least one does not work) = 1 – P(all work)

P(at least one does not work) = 1 – 0.70^7 = 0.9176

14. [new] **Answer: (B)**

Construct a probability distribution table to display these outcomes letting X = the amount of money you win:

X	\$1	\$4	\$0
P(X)	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{3}{6}$
	↑ 4 or 5	↑ 6	↑ 1, 2, or 3

$$E(X)=1\left(\frac{2}{6}\right)+4\left(\frac{1}{6}\right)+0\left(\frac{3}{6}\right)=\$1$$

15. [new] **Answer: (B)**

E(Y) = 0.30(125) = 37.5

SD(X) = $\sqrt{50}$ $\rightarrow$ SD(Y) = 30$\sqrt{50}$ $\rightarrow$ Var(Y) = $\left(30\sqrt{50}\right)^2$ = 4.5

16. [new] **Answer: (B)**

Remember that a discrete random variable is countable and a continuous random variable is measurable. The only choice that is countable is (B).

17. [new] **Answer: (E)**

This question is asking for the number of failures that must be endured until success. Therefore, this follows a geometric distribution. We are looking for three failures (with a probability of 0.80) and then one success (with a probability of 0.20). Therefore:

$$(0.80)(0.80)(0.80)(0.20) = (0.80)^3(0.20)$$

18. [old] **Answer: (C)**

Remember that "at least one is one minus none":

P(at least one w/ math anxiety) = 1 – P(none with math anxiety)

P(at least one w/ math anxiety) = $1 - 0.80^8 = 0.8322$

19. [new] **Answer: (B)**

Since all four qualifications are met (two outcomes, p(success) is constant, trials independent, fixed # trials), this follows a binomial distribution with parameters n=8 and p=0.20. Then, just use the formula for a binomial probability:

$$P(x = k) = \binom{n}{k} p^k (1-p)^{n-k} = \binom{8}{3} (0.20)^3 (0.80)^5$$

20. [new] **Answer: (C)**

This is one for your calculator:

P(at least two) = 1 – [P(exactly 0) + P(exactly 1)]

P(at least two) = 1 – [binompdf(8,0.20,0) + binompdf(8,0.20,1)] = 0.4967

21. [new] **Answer: (B)**

For a geometric distribution, you are trying to essentially answer the question: "How many failures do I have to endure prior to success?". Therefore, there <u>cannot</u> be a fixed number of trials because you do not know how many trials there will be… that's what you're trying to figure out.

22. [new] **Answer: (D)**

In order to use the normal approximation, you have to have at least 10 successes and at least 10 failures. Or:

$$np \geq 10 \text{ and } n(1-p) \geq 10$$

Therefore, just solve until you have accomplished that:

$np \geq 10$	$n(1-p) \geq 10$
$n(0.23) \geq 10$	$n(0.77) \geq 10$
$n \geq 43.47 \approx 44$	$n \geq 12.98 \approx 13$

However, since you need both of these to be true, you will need a sample size of at least 44.

23. [new] **Answer: (E)**

The binomial distribution discusses the probability of getting exactly a certain number of successes within a given number of trials. The only one of these choices that works appropriately is when we discuss the number of hits acquired at six times at bat.

24. [new] **Answer: (D)**

The geometric model discusses the number of failures before success, so the number of people we survey (failures) until we find someone with an iPod (success).

25. [new] **Answer: (D)**

Since this follows a geometric distribution, simply use the formula for the mean (or expected value) of a geometric distribution:

$$\mu = \frac{1}{p} = \frac{1}{0.10} = 10$$