

Building Better Batteries

Everyone wants to have the latest technological gadget. That is why iPods, digital cameras, smartphones, Game Boys, and the Wii have sold millions of units. These devices require lots of power and can drain batteries quickly. Battery manufacturers are constantly searching for ways to build longer-lasting batteries.

A particular manufacturer produces AA batteries that are designed to last an average of 17 hours with a standard deviation of 0.8 hours. Quality control inspectors select a random sample of 50 batteries during each hour of production, and they then drain them under conditions that mimic normal use. Here are the lifetimes (in hours) of the batteries from one such sample:

16.73 15.60 16.31 17.57 16.14 17.28 17.27 17.50 15.46 16.50 16.19 15.59 17.54
16.46 15.63 16.82 17.16 16.62 16.71 16.69 17.98 16.36 17.80 16.61 15.99 15.64
17.20 17.24 16.68 16.55 17.48 15.58 17.61 15.98 16.99 16.93 16.01 17.54 17.41
16.91 16.60 16.78 15.75 17.31 16.50 16.72 17.55 16.46 16.67 17.28

Assuming the process is working properly, the population distribution of battery lifetime has mean $\mu = 17$ hours and standard deviation $\sigma = 0.8$ hours. We do not know the shape of the population distribution.

1. Make an appropriate graph to display the sample data. Describe what you see.
2. Assume that the battery production process is working properly. Describe the shape, center, and spread of the sampling distribution of \bar{x} for random samples of 50 batteries. Justify your answers.

For the random sample of 50 batteries above, the average lifetime was $\bar{x} = 16.718$ hours.

3. Find the probability of obtaining a random sample of 50 batteries with a mean lifetime of 16.718 hours or less if the production process is working properly. Show your work. Based on your answer, do you believe that the process is working properly? Why or why not?

The plant manager also wants to know what proportion p of all batteries produced that day lasted less than 16.5 hours, which he has declared "unsuitable." From past experience, about 27% of batteries made at the plant are unsuitable. If the manager does not find convincing evidence that the proportion of unsuitable batteries p produced that day is greater than .27, the whole batch of batteries will be shipped to customers.

4. Assume that the actual proportion of unsuitable batteries produced that day is $p = 0.27$. Describe the shape, center, and spread of the sampling distribution of \hat{p} for random sample of 50 batteries. Justify your answer.

For the random sample of 50 batteries, the sample proportion with lifetimes less than 16.5 hours was $\hat{p} = 0.32$.

5. Find the probability of obtaining a random sample of 50 batteries in which 32% or more of the batteries are unsuitable if $p = 0.27$. Show your work. Based on your answer, should the entire batch of batteries be shipped to customers? Why or Why not?