AP Statistics

Five Probability Rules:

1. A probability is a number between 0 and 1: For any event A, **$0 \le P(A) \le 1$**.
Probability of 0: the event *never* occurs. Probability of 1: the event *always* occurs.

2. The Probability of the set of all possible outcomes must be 1. **P(S) = 1**
S represents the set of all possible outcomes.

3. The Probability of an event occurring is 1 minus the probability that it does not occur.
$P(A) = 1 – P(A^C)$; A^C the complement of A.

4. **Disjoint (or mutually exclusive) events have no outcomes in common**. The addition Rule states:
For two disjoint events, A and B, the probability that one or the other occurs is the sum of the probabilities of the two events. **$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$,** provided that A and B are ***disjoint***.

5. For two independent events, A and B, the probability that both A and B occur is the product of the probabilities of the two events.
$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$, provided that A and B are ***independent***.

Probability Step-By-Step using the five rules:

In 2001 Masterfoods, the manufacturer of M&M's milk chocolate candies, decided to add another color to the standard color lineup of brown, yellow, red, orange, blue, and red. To decide which color, they surveyed kids in nearly every country of the world and asked them to vote among purple, pink, and teal. The global winner was purple! In the United States, 42% of those voted said purple, 37% said teal, and only 19% said pink. But in Japan the percentages were 38% pink, 36% teal, and only 16% purple. Do these add up to 1?

1. What is the probability that a Japanese M&M's survey respondent selected at random preferred either pink or teal?
P(pink) = 0.38, P(teal) = 0.36, P(purple) = 0.16

Since event "pink" and "teal" are individual outcomes (a respondent cannot choose both colors), they are disjoint. What rule can be applied?
P(pink or teal) = P(pink) + P(teal) = 0.38 + 0.36 = 0.74

2. If we pick two respondents at random, what is the probability that they both said purple?
Check independence: it is unlikely that the choice made by one respondent affected the choice of the other, so it seems the two events are independent.
P(both purple) = P(1st respondent chose purple and 2nd respondent chose purple) = P(A) x P(B)
0.16 x 0.16 = 0.0256

3. If we pick three respondents at random, what is the probability that at least one preferred purple?
The phrase "at least…" often flags a question best answered by looking at the complement. The complement of "at least one preferred purple" is "none of them preferred purple."
P(at least one picked purple) = 1 – (none picked purple)
Independent events?
P(none picked purple) = P(1st not purple) x P(2nd picked purple) x P(3rd not purple)
= $[P(\text{not purple})]^3$.
P(not purple) = 1 – P(purple) = 1 – 0.16 = 0.84
P(none picked purple) = $(0.84)^3$ = 0.5927. So, P(at least 1 picked purple) = 1 – P(none) = 1 – 0.5927 = 0.4073
There is about a 40.7% chance that at least one of the respondents picked purple.

Example 2: Distribution Tables:

A Gallup Poll in March 2001 asked 1005 U.S. adults how the United States should deal with the current energy situation: by more production, more conservation, or both? Here are the results:

Response	Number
More Production	332
More Conservation	563
Both	80
No opinion	30
Total	**1005**

If we select a person at random from this sample of 1005 adults,

1) what is the probability that the person responded "More production?"
2) what is the probability that the person responded "Both" or had no opinion

Suppose we select three people at random from this sample:

3) What is the probability that all three responded "More conservation?"
4) What is the probability that none responded "Both?"
5) What assumption did you make in computing these probabilities?
6) Explain why you think that assumption is reasonable?

Practice:

The Masterfoods company says that before their introduction of purple, yellow candies made up 20% of their plain M&M's red another 20%, and orange, blue, and green each made up 10%. The rest were brown.

If you pick an M&M at random, what is the probability that

1) it is brown?
2) it is yellow or orange?
3) it is not green?
4) it is striped?

If you pick three M&M's in a row, what is the probability that

5) they are all brown?
6) the third one is the first one that is red?
7) none are yellow?
8) at least one is green?