Probability Challenge

1. On January 28, 1986, Space Shuttle *Challenger* exploded on takeoff. All seven crew members were killed. Following the disaster, scientists and statisticians helped analyze what went wrong. They determined that the failure of O-ring joints in the shuttle's booster rockets was to blame. Under the cold conditions that day, experts estimated that the probability that an individual O-ring joint would function properly was 0.977. But there were six of these O-ring joints and all six had to function properly for the shuttle to launch safely. Assuming that O-ring joints succeed or fail independently, find the probability that the shuttle would launch safely under similar conditions.

2. Many people who come to clinics to be tested for HIV, the virus that causes AIDS, do not come back to learn the test results. Clinics now use "rapid HIV tests" that give a result while the patient waits. In a clinic in Malawi, for example, use of rapid tests increased the percent of patients who learned their test results from 69% to 99.7%.

The trade-off for fast results is that rapid tests are less accurate than slower laboratory tests. Applied to people who have no HIV antibodies, one rapid test has probability about 0.004 of producing a false positive (that is, of falsely indicating that antibodies are present).

If a clinic tests 200 people who are free of HIV antibodies, what is the chance that at least one false positive will occur?

3. In baseball, a perfect game is when a pitcher doesn't allow any hitters to reach base in all nine innings. Historically, pitchers throw a perfect inning—an inning where no hitters reach base—about 40% of the time (http://www.baseballprospectus.com/article.php?articleid=11110). So, to throw a perfect game, a pitcher needs to have nine perfect innings in a row. What is the probability that a pitcher throws nine perfect innings in a row, assuming the pitcher's performance in an inning is independent of his performance in other innings?

4. After taking college placement tests, freshmen sometimes are required to repeat high school work. Such work is called "remediation" and does not count toward a college degree. About 11% of college freshmen have to take a remedial course (in CSU system about 40% has to take remediation in Language Arts) in reading. Suppose you select two freshmen at random and check to see if they have to take remedial reading.

a) Find the probability that both freshmen have to take remedial reading.
b) What is the probability at least one of the freshmen have to take remedial reading?
c) Given that the first freshman is not in remedial reading, what is the probability that the second freshmen is in remedial reading?

5. A laboratory screening test for the detection of a certain disease gives a positive result 6% of the time for the people who do not have the disease. The test gives a negative result 0.5% of the time for people who do have the disease. Large scale studies have shown that the disease occurs in about 3% of the population.

a) What is the probability that a person selected at random tests positive for the disease?

b) What is the probability that a person selected at random who tests positive for the disease does not have the disease?

6. Lie detectors are controversial instruments, barred from use as evidence in many courts. Nonetheless, many employers use lie detector screening as part of their hiring process in the hope that they can avoid hiring people who might be dishonest. There has been some research, but no agreement, about the reliability of polygraph tests. Based on this research, suppose that a polygraph can detect 65% of lies, but incorrectly identifies 15% of true statements as lies.
A company believes that 95% of its job applicants are trustworthy. The company gives everyone a polygraph test, asking, "Have you ever stolen anything from your place of work?" Naturally, all the applicants answer "No," but the polygraph identifies some of those answers as lies, making the person ineligible for a job. What's the probability that a job applicant rejected under suspicion of dishonesty was actually trustworthy?

7. About 55% of high school students participate in a school athletic team at some level and about 5% of these athletes go on to play on a college team in the NCAA. What percent of high school students play a sport in high school *and* go on to play a sport in the NCAA?

8. Only 5% of male high school basketball, baseball, and football players go on to play at the college level. Of these, 1.7% enter major league professional sports. Also, the probability that a high school athlete reaching professional play without college is 0.01%. What is the probability that a high school athlete chosen at random, plays professional sports. Show your work.

9. In one study, doctors in Germany and the United States were asked to estimate the probability that a woman with a positive mammogram actually has breast cancer, even though she's in a low-risk group: 40 to 50 years old, with no symptoms or family history of breast cancer. To make the question specific, the doctors were told to assume the following statistics — couched in terms of percentages and probabilities — about the prevalence of breast cancer among women in this cohort, and also about the mammogram's sensitivity and rate of false positives:

The probability that one of these women has breast cancer is 0.8 percent. If a woman has breast cancer, the probability is 90 percent that she will have a positive mammogram. If a woman does not *have breast cancer, the probability is 7 percent that she will still have a positive mammogram. Imagine a woman who has a positive mammogram. What is the probability that she actually has breast cancer?*

When Gigerenzer, the researcher, asked 24 other German doctors the same question, their estimates whipsawed from 1 percent to 90 percent. Eight of them thought the chances were 10 percent or less, 8 more said 90 percent, and the remaining 8 guessed somewhere between 50 and 80 percent. Imagine how upsetting it would be as a patient to hear such divergent opinions.

As for the American doctors, 95 out of 100 estimated the woman's probability of having breast cancer to be somewhere around 75 percent.

What is the right answer?

10. Before going on vacation for a week, you ask your spacey friend to water your ailing plant. Without water, the plant has a 90 percent chance of dying. Even with proper watering, it has a 20 percent chance of dying. And the probability that your friend will forget to water it is 30 percent. (a) What's the chance that your plant will survive the week? (b) If it's dead when you return, what's the chance that your friend forgot to water it? (c) If your friend forgot to water it, what's the chance it'll be dead when you return?