

Hypothesis Testing in a Nutshell

A hypothesis test is used to determine whether or not a claim is true. That claim could be about a single mean value, whether or not one proportion is bigger than another, whether or not the difference between two means is zero, etc. The claim is tested by comparing a sample statistic to a hypothesized value. We call the claim we are testing the "null hypothesis" and set up an "alternative" based on what we are trying to prove. Our calculations are based on figuring out the probability of observing our sample value under the assumption that the null hypothesis is correct. That is, we ask ourselves, "*Self, if I took many samples from a population where the null hypothesis was true, what distribution of sample values would I expect to see?*" The hypothesis test places the observed value on that distribution and calculates the probability of observing it assuming the null really was true...we call that probability the p-value. If the probability is small, then we are saying "*It's unlikely that I'd observe this value if the null was true...therefore, I have evidence that the null could be false.*" It is as simple as that.

A Hypothesis Test should consist of 4 main parts:

- ✓ Introduction: Parameter of Interest---Null & Alternative Hypotheses
- ✓ Conditions and Name for the Test
- ✓ Calculations—Sketch—Test Statistic—p-value
- ✓ Conclusion—Statistical and Practical

The test statistic formulas used to calculate the p-value all follow the same general format:

$$\text{test statistic} = \frac{\text{observed} - \text{null parameter}}{\text{standard deviation of sampling distribution}}$$

One Mean

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Two Means

$$t = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

One Proportion

$$z = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o(1-p_o)}{n}}}$$

Two Proportions

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_c(1-\hat{p}_c)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Note: If you have matched pairs situation,

calculate the differences, state the link, and do a one sample t-test on those differences.

Note: $\hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2}$ is the pooled proportion.

✓ **Introduction and Hypotheses:**

When you first read a problem, determine what you will be testing (means? proportions? one sample? two samples? matched pairs?). Once this is determined, state the parameter of interest, and set up a pair of hypotheses. The null is the claim in question and the alternative is what you think your sample is proving.

✓ **Conditions for the Sampling Distribution:**

What you are testing determines which test statistic you will use (z-test, t-test, etc). This test statistic will then be used to calculate the p-value...but that comes later. Choose the appropriate test based on your hypotheses and (this is important) check the conditions for that test. Remember, all of your calculations are based on a sampling distribution, so we need to make sure we satisfy the requirements for constructing that distribution!

Means

SRS or random sample

Population ten times larger than sample

$n \geq 30$ Central Limit Theorem

Note: If you have data, you must check normality by graphing data. Outliers and/or strong are bad. If using matched pairs, indicate why. If you are using a two-sample t-test, indicate the samples are independent.

Proportions

$np \geq 10, n(1-p) \geq 10$

Note: Two sample problems require you to check the assumptions for both samples (and calculate the pooled proportion)

✓ **Calculations:**

Your calculator will do most of the dirty work...your job is to interpret it. However, just because the meat-grinder does the test for you does not mean you should just copy the results from the screen to your paper. Take time to sketch the sampling distribution (draw a normal or t-distribution centered at the null value and shade in your sample observation), write the test statistic formula with the appropriate information inserted, give the z or t test value, the df (if necessary), and most importantly, the p-value.

✓ **Conclusion:**

Did your test produce a small p-value (less than .01 or .05)? If so, then you have shown that it is unlikely you would observe the sample information if the null hypothesis were true...therefore, you have evidence that the null may be false! Reject or fail-to-reject the null based on your p-value and then conclude by interpreting your test in terms of the original problem. It is as simple as that!!