Comparing two proportions

1. Many news organizations conduct polls asking adults in the United States if they approve of the job the president is doing. How did President Obama's approval rating change from August 2009 to September 2010? According to a CNN poll of 1024 randomly selected U.S. adults on September 1-2, 2010, 50% approved of Obama's job performance. A CNN poll of 1010 randomly selected U.S. adults on August 28-30, 2009 showed that 53% approved of Obama's job performance.

(a) Use the results of these polls to construct and interpret a 90% confidence interval for the change in Obama's approval rating among all US adults.

(b) Based on your interval, is there convincing evidence that Obama's job approval rating has changed?

2. Are teenagers going deaf? In a study of 3000 randomly selected teenagers in 1988-1994, 15% showed some hearing loss. In a similar study of 1800 teenagers in 2005-2006, 19.5% showed some hearing loss. (Source: *Arizona Daily Star*, 8-18-2010).

(a) Does these data give convincing evidence that the proportion of all teens with hearing loss has increased?

(b) Between the two studies, Apple introduced the iPod. If the results of the test are statistically significant, can we blame iPods for the increased hearing loss in teenagers?

3. In an effort to reduce health care costs, General Motors sponsored a study to help employees stop smoking. In the study, half of the subjects were randomly assigned to receive up to $750 for quitting smoking for a year while the other half were simply encouraged to use traditional methods to stop smoking. None of the 878 volunteers knew that there was a financial incentive when they signed up. At the end of one year, 15% of those in the financial rewards group had quit smoking while only 5% in the traditional group had quit smoking.

(a) Do the results of this study give convincing evidence that a financial incentive helps people quit smoking? Use a 1% significant level. (Source: *Arizona Daily Star*, 2-11-09).

(b) Describe a Type I and a Type II error in context. What are the consequences of each?

(c) What is the probability of a Type I error?

(d) If $\beta = 0.13$, what is the probability of the power of the test for the alternative? What does that mean in context?

1. Solution:

(a) State: We want to estimate $p_{2010} - p_{2009}$ at the 90% confidence level where p_{2010} = the true proportion of all U.S. adults who approved of President Obama's job performance in September 2010 and p_{2009} = the true proportion of all U.S. adults who approved of President Obama's job performance in August 2009.

Plan: We should use a two-sample *z* interval for $p_{2010} - p_{2009}$ if the conditions are satisfied.

- **Random: The data came from separate random samples.**
- **Normal: $n_{2010}\hat{p}_{2010}$ = 512, $n_{2010}\left(1-\hat{p}_{2010}\right)$ = 512, $n_{2009}\hat{p}_{2009}$ = 535, $n_{2009}\left(1-\hat{p}_{2009}\right)$ = 475 are all at least 10.**
- **Independent: The samples were taken independently and there are more than 10(1024) = 10,240 U.S. adults in 2010 and 10(1010) = 10,100 U.S. adults in 2009.**

Do: $(0.50-0.53) \pm 1.645\sqrt{\dfrac{0.50(1-0.50)}{1024}+\dfrac{0.53(1-0.53)}{1010}}$ **= –0.03 $\pm$ 0.036 = (–0.066, 0.006)**

Conclude: We are 95% confident that the interval from –0.066 to 0.006 captures the true change in the proportion of U.S. adults who approve of President Obama's job performance from August 2009 to September 2010. That is, it is plausible that his job approval has fallen by up to 6.6 percentage points or increased by up to 0.6 percentage points.
(b) Since 0 is included in the interval, it is plausible that there has been no change in President Obama's approval rating. Thus, we do not have convincing evidence that his approval rating has changed.

2. Solution:

(a) State: We will test $H_0: p_1 - p_2 = 0$ versus $H_a: p_1 - p_2 > 0$ at the 0.05 significance level where p_1 = the proportion of all teenagers with hearing loss in 2005-2006 and p_2 = the proportion of all teenagers with hearing loss in 1988-1994.

Plan: We should use a two-sample *z* test for $p_1 - p_2$ if the conditions are satisfied.

- **Random: The data came from separate random samples.**
- **Normal: $n_1\hat{p}_1$ = 351, $n_1\left(1-\hat{p}_1\right)$ = 1449, $n_2\hat{p}_2$ = 450, $n_2\left(1-\hat{p}_2\right)$ = 2550 are all at least 10.**
- **Independent: The samples were taken independently and there were more than 10(1800) = 18,000 teenagers in 2005-2006 and 10(3000) = 30,000 teenagers in 1988-1994.**

Do: $\hat{p}_C = \dfrac{450+351}{3000+1800}$ **= 0.167, *z* =** $\dfrac{(0.195-0.15)-0}{\sqrt{\dfrac{0.167(1-0.167)}{1800}+\dfrac{0.167(1-0.167)}{3000}}}$ **= 4.05, *P*-value $\approx$ 0**

Conclude: Since the *P*-value is less than 0.05, we reject H_0. We have convincing evidence that the proportion of all teens with hearing loss has increased from 1988-1994 to 2005-2006.
(b) No. Since we didn't do an experiment where we randomly assigned some teens to listen to iPods and other teens to avoid listening to iPods, we cannot conclude that iPods are the cause. It is possible that teens who listen to iPods also like to listen to music in their cars and perhaps the car stereos are causing the hearing loss.

3. State: We will test $H_0: p_1 - p_2 = 0$ versus $H_a: p_1 - p_2 > 0$ at the 0.05 significance level where p_1 = the true quitting rate for employees like these who get a financial incentive to quit smoking and p_2 = the true quitting rate for employees like these who don't get a financial incentive to quit smoking.

Plan: We should use a two-sample *z* test for $p_1 - p_2$ if the conditions are satisfied.

- **Random: The treatments were randomly assigned.**
- **Normal: $n_1\hat{p}_1$ = 66, $n_1\left(1-\hat{p}_1\right)$ = 373, $n_2\hat{p}_2$ = 22, $n_2\left(1-\hat{p}_2\right)$ = 417 are all at least 10.**
- **Independent: The random assignment allows us to view these two groups as independent. We must assume that each employee's decision to quit is independent of other employee's decisions.**

Do: $\hat{p}_C = \dfrac{66+22}{439+439}$ **= 0.100, *z* =** $\dfrac{(0.15-0.05)-0}{\sqrt{\dfrac{0.1(1-0.1)}{439}+\dfrac{0.1(1-0.1)}{439}}}$ **= 4.94, *P*-value $\approx$ 0**

Conclude: Since the *P*-value is less than 0.05, we reject H_0. We have convincing evidence that financial incentives help employees like these quit smoking.